EXERCISE 28B

1. Find the vector equation of a plane which is at a distance of 5 units from the origin and which has $\widehat{k}$ as the unit vector normal to it.
Solution:

It is given that
d = 5
$\widehat{n} = \widehat{k}$
Here the equation of plane at 5 units distance from the origin and $\widehat{n}$as a unit vector normal to it

$\overline{r}.\widehat{n} = d\, where\, \overline{r} = x\widehat{i} + y\widehat{j} + z\widehat{k}$

$We\, know\, that\, d = 5\, and\, \widehat{n} = \widehat{k}$

$So\, the\, equation\, of\, plane\, is$

$\overline{r}.\widehat{n} = d$

$\overline{r}.\widehat{k} = 5\, which\, is\, the\, vector\, equation\, of\, the\, plane$

$Here$

$\overline{r}.\widehat{k} = (x\widehat{i} + y\widehat{j} + z\widehat{k}).\widehat{k}$

$So\, we\, get$

$= (x \times 0) + (y \times 0) + (z \times 1)$

$\overline{r}.\widehat{k} = z$

Hence, the equation of plane is $\overline{r}.\widehat{k} = 5$
Cartesian equation of the plane is z = 5.

2. Find the vector and Cartesian equations of a plane which is at a distance of 7 units from the origin and whose normal vector from the origin is $(3\widehat{i} + 5\widehat{j} - 6\widehat{k})$.
Solution:

It is given that

d = 7

$\overline{n} = 3\widehat{i} + 5\widehat{j} - 6\widehat{k}$

For the normal vector the unit vector normal to the plane is

$$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$$

By substituting the values

$$\hat{n} = \frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{3^2 + 5^2 + (-6)^2}}$$

On further calculation

$$\hat{n} = \frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{9 + 25 + 36}}$$

So we get

$$\hat{n} = \frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{70}}$$

Here the equation of the plane is

$$\bar{r}.\hat{n} = d$$

By substituting the values

$$\bar{r}.\left(\frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{70}}\right) = 7$$

So we get

$\bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}) = 7\sqrt{70}$ which is the vector equation of the plane

Here

$$\bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k})$$

We can write it as

= (x × 3) + (y × 5) + (z × (-6))

= 3x + 5y – 6z

So the Cartesian equation of the plane is

$$3x + 5y - 6z = 7\sqrt{70}$$

3. Find the vector and Cartesian equations of a plane which is at a distance of 6/√29 from the origin and whose normal vector from the origin is $(2\hat{i} - 3\hat{j} + 4\hat{k})$

Solution:

It is given that

$$d = \frac{6}{\sqrt{29}}$$

$$\bar{n} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$$

For the normal vector the unit vector normal to the plane is

$$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$$

By substituting the values

$$\hat{n} = \frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{2^2 + (-3)^2 + 4^2}}$$

On further calculation

$$\hat{n} = \frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{4 + 9 + 16}}$$

So we get

$$\hat{n} = \frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{29}}$$

Here the equation of the plane is

$$\bar{r}.\hat{n} = d$$

By substituting the values

$$\bar{r}.\left(\frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$$

So we get

$\bar{r}.\left(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right) = 6$ which is the vector equation of the plane

Here

$$\bar{r}.\left(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right)$$

We can write it as

= (x × 2) + (y × (-3)) + (z × 4)
= 2x - 3y + 4z
So the Cartesian equation of the plane is
2x – 3y + 4z = 6

4. Find the vector and Cartesian equations of a plane which is at a distance of 6 units from the origin and which has a normal with direction ratios 2, -1, -2.
Solution:

It is given that

d = 6

Direction ratios of $\overline{n}$ are (2, -1, -2) where

$$\overline{n} = 2\hat{i} - \hat{j} - 2\hat{k}$$

For the normal vector the unit vector normal to the plane is

$$\hat{n} = \frac{\overline{n}}{|\overline{n}|}$$

By substituting the values

$$\hat{n} = \frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{\sqrt{2^2 + (-1)^2 + (-2)^2}}$$

On further calculation

$$\hat{n} = \frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{\sqrt{4 + 1 + 4}}$$

So we get

$$\hat{n} = \frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{\sqrt{9}} = \frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{3}$$

Here the equation of the plane is

$$\bar{r}.\hat{n} = d$$

By substituting the values

$$\bar{r}.\left(\frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{3}\right) = 6$$

So we get

$\bar{r}.\left(2\hat{\imath}-\hat{\jmath}-2\hat{k}\right)=18$ which is the vector equation of the plane

Here

$\bar{r}.\left(2\hat{\imath}-\hat{\jmath}-2\hat{k}\right)=\left(x\hat{\imath}+y\hat{\jmath}+z\hat{k}\right).\left(2\hat{\imath}-\hat{\jmath}-2\hat{k}\right)$

We can write it as

= (x × 2) + (y × (-1)) + (z × (-2))

= 2x - y - 2z

So the Cartesian equation of the plane is

2x – y - 2z = 18

5. Find the vector and Cartesian equations of a plane which passes through the point (1, 4, 6) and normal vector to the plane is $(\hat{i}-2\hat{j}+\hat{k})$.

Solution:

It is given that

A = (1, 4, 6)

$\bar{n}=\hat{\imath}-2\hat{\jmath}+\hat{k}$

For the position vector of A = (1, 4, 6)

$\bar{a}=\hat{\imath}+4\hat{\jmath}+6\hat{k}$

Here

$\bar{a}.\bar{n}=\left(\hat{\imath}+4\hat{\jmath}+6\hat{k}\right).\left(\hat{\imath}-2\hat{\jmath}+\hat{k}\right)$

It can be written as

= (1 × 1) + (4 × (-2)) + (6 × 1)

= 1 – 8 + 6

= - 1

Here the equation of plane is

$\bar{r}.\bar{n}=\bar{a}.\bar{n}$

Where $\bar{r}.\left(\hat{\imath}-2\hat{\jmath}+\hat{k}\right)=-1$

Which is the vector equation of the plane

We know that

$$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

It can be written as

$$\bar{r}.\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$$

On further calculation

= (x × 1) + (y × (-2)) + (z × 1)\

= x – 2y + z

So the Cartesian equation of the plane is

x – 2y + z = - 1