

## 27. Straight Line in Space

### Exercise 27A

#### 1. Question

A line passes through the point (3, 4, 5) and is parallel to the vector  $(2\hat{i} + 2\hat{j} - 3\hat{k})$ . Find the equations of the line in the vector as well as Cartesian forms.

#### Answer

Given: line passes through point (3, 4, 5) and is parallel to  $2\hat{i} + 2\hat{j} - 3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

Explanation:

Here,  $\vec{a} = 3\hat{i} + 4\hat{j} + 5\hat{k}$  and  $\vec{b} = 2\hat{i} + 2\hat{j} - 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 3\hat{i} + 4\hat{j} + 5\hat{k} + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$$

Cartesian form:

$$\frac{x-3}{2} = \frac{y-4}{2} = \frac{z-5}{-3}$$

#### 2. Question

A line passes through the point (2, 1, -3) and is parallel to the vector  $(\hat{i} - 2\hat{j} + 3\hat{k})$ . Find the equations of the line in vector and Cartesian forms.

#### Answer

Given: line passes through (2, 1, -3) and is parallel to  $\hat{i} - 2\hat{j} + 3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda \vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

Explanation:

Here,  $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

Cartesian form:

$$\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+3}{3}$$

### 3. Question

Find the vector equation of the line passing through the point with position vector  $(2\hat{i} + \hat{j} - 5\hat{k})$  and parallel to the vector  $(\hat{i} + 3\hat{j} - \hat{k})$ . Deduce the Cartesian equations of the line.

### Answer

Given: line passes through  $2\hat{i} + \hat{j} - 5\hat{k}$  and is parallel to  $\hat{i} + 3\hat{j} - \hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda \vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

Explanation:

Here,  $\vec{a} = 2\hat{i} + \hat{j} - 5\hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} + \hat{j} - 5\hat{k} + \lambda(\hat{i} + 3\hat{j} - \hat{k})$$

Cartesian form:

$$\frac{x-2}{1} = \frac{y-1}{3} = \frac{z+5}{-1}$$

#### 4. Question

A line is drawn in the direction of  $(\hat{i} + \hat{j} - 2\hat{k})$  and it passes through a point with position vector  $(2\hat{i} - \hat{j} - 4\hat{k})$ . Find the equations of the line in the vector as well as Cartesian forms.

#### Answer

Given: line passes through  $2\hat{i} - \hat{j} - 4\hat{k}$  and is drawn in the direction of  $\hat{i} + \hat{j} - 2\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

Explanation:

Since line is drawn in the direction of  $(\hat{i} + \hat{j} - 2\hat{k})$ , it is parallel to  $(\hat{i} + \hat{j} - 2\hat{k})$

Here,  $\vec{a} = 2\hat{i} - \hat{j} - 4\hat{k}$  and  $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} - \hat{j} - 4\hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$$

Cartesian form:

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z+4}{-2}$$

#### 5. Question

The Cartesian equations of a line are  $\frac{x-3}{2} = \frac{y+2}{-5} = \frac{z-6}{4}$ . Find the vector equation of the line.

Given: Cartesian equation of line

$$\frac{x-3}{2} = \frac{y+2}{-5} = \frac{z-6}{4}$$

To find: equation of line in vector form

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda \vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

Explanation:

From the Cartesian equation of the line, we can find  $\vec{a}$  and  $\vec{b}$

Here,  $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$  and  $\vec{b} = 2\hat{i} - 5\hat{j} + 4\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} - 5\hat{j} + 4\hat{k})$$

## 6. Question

The Cartesian equations of a line are  $3x + 1 = 6y - 2 = 1 - z$ . Find the fixed point through which it passes, its direction ratios and also its vector equation.

**Answer**

Given: Cartesian equation of line are  $3x + 1 = 6y - 2 = 1 - z$

To find: fixed point through which the line passes through, its direction ratios and the vector equation.

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda \vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line and also its direction ratio.

Explanation:

The Cartesian form of the line can be rewritten as:

$$\frac{x + \frac{1}{3}}{\frac{1}{3}} = \frac{y - \frac{1}{3}}{\frac{1}{6}} = \frac{z - 1}{-1} = \lambda$$

$$\Rightarrow \frac{x + \frac{1}{3}}{2} = \frac{y - \frac{1}{3}}{1} = \frac{z - 1}{-6} = \lambda$$

Therefore,  $\vec{a} = \frac{-1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}$  and  $\vec{b} = 2\hat{i} + \hat{j} - 6\hat{k}$

So, the line passes through  $\left(\frac{-1}{3}, \frac{1}{3}, 1\right)$  and direction ratios of the line are (2, 1, -6) and vector form is:

$$\vec{r} = \frac{-1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k} + \lambda(2\hat{i} + \hat{j} - 6\hat{k})$$

## 7. Question

Find the Cartesian equations of the line which passes through the point (1, 3, -2) and is parallel to the line given by  $\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$ . Also, find the vector form of the equations so obtained.

## Answer

Given: line passes through (1, 3, -2) and is parallel to the line

$$\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

$$\text{Vector form: } \vec{r} = \vec{a} + \lambda\vec{b}$$

$$\text{Cartesian form: } \frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

Explanation:

Since the line (say  $L_1$ ) is parallel to another line (say  $L_2$ ),  $L_1$  has the same direction ratios as that of  $L_2$

$$\text{Here, } \vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$$

Since the equation of  $L_2$  is

$$\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$$

$$\vec{b} = 3\hat{i} + 5\hat{j} - 6\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 3\hat{j} - 2\hat{k} + \lambda(3\hat{i} + 5\hat{j} - 6\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{3} = \frac{y-3}{5} = \frac{z+2}{-6}$$

### 8. Question

Find the equations of the line passing through the point (1, -2, 3) and parallel to the line

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}. \text{ Also find the vector form of this equation so obtained.}$$

### Answer

Given: line passes through (1, -2, 3) and is parallel to the line

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

$$\text{Vector form: } \vec{r} = \vec{a} + \lambda\vec{b}$$

$$\text{Cartesian form: } \frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

Explanation:

Since the line (say  $L_1$ ) is parallel to another line (say  $L_2$ ),  $L_1$  has the same direction ratios as that of  $L_2$

$$\text{Here, } \vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

Since the equation of  $L_2$  is

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

$$\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} - 4\hat{j} + 5\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z-3}{5}$$

### 9. Question

Find the Cartesian and vector equations of a line which passes through the point (1, 2, 3) and is

parallel to the line  $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$ .

### Answer

Given: line passes through (1, 2, 3) and is parallel to the line

$$\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

$$\text{Vector form: } \vec{r} = \vec{a} + \lambda \vec{b}$$

$$\text{Cartesian form: } \frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

Explanation:

Since the line (say  $L_1$ ) is parallel to another line (say  $L_2$ ),  $L_1$  has the same direction ratios as that of  $L_2$

$$\text{Here, } \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Equation of  $L_2$  can be rewritten as:

$$\frac{x+2}{-1} = \frac{y+3}{7} = \frac{z-3}{\frac{3}{2}}$$

$$\Rightarrow \frac{x+2}{-2} = \frac{y+3}{14} = \frac{z-3}{3}$$

$$\vec{b} = -2\hat{i} + 14\hat{j} + 3\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-2\hat{i} + 14\hat{j} + 3\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{-2} = \frac{y-2}{14} = \frac{z-3}{3}$$

### 10. Question

Find the equations of the line passing through the point  $(-1, 3, -2)$  and perpendicular to each of the lines  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  and  $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$ .

### Answer

Given: line passes through  $(-1, 3, -2)$  and is perpendicular to each of the lines  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  and  $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda \vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

If 2 lines of direction ratios  $a_1:a_2:a_3$  and  $b_1:b_2:b_3$  are perpendicular, then  $a_1b_1 + a_2b_2 + a_3b_3 = 0$

Explanation:

Here,  $\vec{a} = -\hat{i} + 3\hat{j} - 2\hat{k}$

Let the direction ratios of the line be  $b_1:b_2:b_3$

Direction ratios of the other two lines are  $1 : 2 : 3$  and  $-3 : 2 : 5$

Since the other two line are perpendicular to the given line, we have

$$b_1 + 2b_2 + 3b_3 = 0$$

$$-3b_1 + 2b_2 + 5b_3 = 0$$

Solving,

$$\frac{b_1}{\begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix}} = \frac{-b_2}{\begin{vmatrix} 1 & 3 \\ -3 & 5 \end{vmatrix}} = \frac{b_3}{\begin{vmatrix} 1 & 2 \\ -3 & 2 \end{vmatrix}}$$

$$\Rightarrow \frac{b_1}{4} = \frac{b_2}{-14} = \frac{b_3}{8}$$

$$\Rightarrow \frac{b_1}{2} = \frac{b_2}{-7} = \frac{b_3}{4}$$

$$\vec{b} = 2\hat{i} - 7\hat{j} + 4\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$$

Cartesian form of the line is:

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$$

### 11. Question

Find the Cartesian and vector equations of the line passing through the point (1, 2, -4) and perpendicular to each of the lines  $\frac{x-8}{8} = \frac{y+19}{-16} = \frac{z-10}{7}$  and  $\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$ .

### Answer

Given: line passes through (1, 2, -4) and is perpendicular to each of the lines  $\frac{x-8}{8} = \frac{y+19}{-16} = \frac{z-10}{7}$  and  $\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  is a vector parallel to the line.

If 2 lines of direction ratios  $a_1:a_2:a_3$  and  $b_1:b_2:b_3$  are perpendicular, then  $a_1b_1 + a_2b_2 + a_3b_3 = 0$

Explanation:

Here,  $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

Let the direction ratios of the line be  $b_1:b_2:b_3$

Direction ratios of other two lines are 8 : -16 : 7 and 3 : 8 : -5

Since the other two line are perpendicular to the given line, we have

$$8b_1 - 16b_2 + 7b_3 = 0$$

$$3b_1 + 8b_2 - 5b_3 = 0$$

Solving,

$$\frac{b_1}{\begin{vmatrix} -16 & 7 \\ 8 & -5 \end{vmatrix}} = \frac{-b_2}{\begin{vmatrix} 8 & 7 \\ 3 & -5 \end{vmatrix}} = \frac{b_3}{\begin{vmatrix} 8 & -16 \\ 3 & 8 \end{vmatrix}}$$

$$\Rightarrow \frac{b_1}{24} = \frac{b_2}{61} = \frac{b_3}{112}$$

$$\vec{b} = 24\hat{i} + 61\hat{j} + 112\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(24\hat{i} + 61\hat{j} + 112\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{24} = \frac{y-2}{61} = \frac{z+4}{112}$$

## 12. Question

Prove that the lines  $\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7}$  and  $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$  intersect each other and find the point of their intersection.

## Answer

Given: The equations of the two lines are

$$\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7} \text{ and } \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

$$\text{Vector form: } \vec{r} = \vec{a} + \lambda\vec{b}$$

$$\text{Cartesian form: } \frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $b_1 : b_2 : b_3$  is the direction ratios of the line.

Proof:

Let

$$\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7} = \lambda_1$$

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda_2$$

So a point on the first line is  $(\lambda_1 + 4, 4\lambda_1 - 3, 7\lambda_1 - 1)$

A point on the second line is  $(2\lambda_2 + 1, -3\lambda_2 - 1, 8\lambda_2 - 10)$

If they intersect they should have a common point.

$$\lambda_1 + 4 = 2\lambda_2 + 1 \Rightarrow \lambda_1 - 2\lambda_2 = -3 \dots (1)$$

$$4\lambda_1 - 3 = -3\lambda_2 - 1 \Rightarrow 4\lambda_1 + 3\lambda_2 = 2 \dots (2)$$

Solving (1) and (2),

$$11\lambda_2 = 14$$

$$\lambda_2 = \frac{14}{11}$$

$$\text{Therefore, } \lambda_1 = \frac{-5}{11}$$

Substituting for the z coordinate, we get

$$7\lambda_1 - 1 = \frac{-46}{11} \text{ and } 8\lambda_2 - 10 = \frac{2}{11}$$

So, the lines do not intersect.

### 13. Question

Show that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-4}{5} = \frac{y-1}{2} = z$  intersect each other. Also, find the point of their intersection.

#### Answer

Given: The equations of the two lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-4}{5} = \frac{y-1}{2} = z$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

$$\text{Vector form: } \vec{r} = \vec{a} + \lambda \vec{b}$$

$$\text{Cartesian form: } \frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $b_1 : b_2 : b_3$  is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda_1$$

$$\frac{x-4}{5} = \frac{y-1}{2} = z = \lambda_2$$

So a point on the first line is  $(2\lambda_1 + 1, 3\lambda_1 + 2, 4\lambda_1 + 3)$

A point on the second line is  $(5\lambda_2 + 4, 2\lambda_2 + 1, \lambda_2)$

If they intersect they should have a common point.

$$2\lambda_1 + 1 = 5\lambda_2 + 4 \Rightarrow 2\lambda_1 - 5\lambda_2 = 3 \dots (1)$$

$$3\lambda_1 + 2 = 2\lambda_2 + 1 \Rightarrow 3\lambda_1 - 2\lambda_2 = -1 \dots (2)$$

Solving (1) and (2),

$$-11\lambda_2 = 11$$

$$\lambda_2 = -1$$

Therefore,  $\lambda_1 = -1$

Substituting for the z coordinate, we get

$$4\lambda_1 + 3 = -1 \text{ and } \lambda_2 = -1$$

So, the lines intersect and their point of intersection is  $(-1, -1, -1)$

#### 14. Question

Show that the lines  $\frac{x-1}{2} = \frac{y+1}{3} = z$  and  $\frac{x+1}{5} = \frac{y-2}{1}, z=2$  do not intersect each other.

#### Answer

Given: The equations of the two lines are

$$\frac{x-1}{2} = \frac{y+1}{3} = z \text{ and } \frac{x+1}{5} = \frac{y-2}{1}, z=2$$

To Prove: the lines do not intersect each other.

Formula Used: Equation of a line is

$$\text{Vector form: } \vec{r} = \vec{a} + \lambda \vec{b}$$

$$\text{Cartesian form: } \frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $b_1 : b_2 : b_3$  is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2} = \frac{y+1}{3} = z = \lambda_1$$

$$\frac{x+1}{5} = \frac{y-2}{1} = \lambda_2, z = 2$$

So a point on the first line is  $(2\lambda_1 + 1, 3\lambda_1 - 1, \lambda_1)$

A point on the second line is  $(5\lambda_2 - 1, \lambda_2 + 1, 2)$

If they intersect they should have a common point.

$$2\lambda_1 + 1 = 5\lambda_2 - 1 \Rightarrow 2\lambda_1 - 5\lambda_2 = -2 \dots (1)$$

$$3\lambda_1 - 1 = \lambda_2 + 1 \Rightarrow 3\lambda_1 - \lambda_2 = 2 \dots (2)$$

Solving (1) and (2),

$$-13\lambda_2 = -10$$

$$\lambda_2 = \frac{10}{13}$$

$$\text{Therefore, } \lambda_1 = \frac{33}{65}$$

Substituting for the z coordinate, we get

$$\lambda_1 = \frac{33}{65} \text{ and } z = 2$$

So, the lines do not intersect.

### 15. Question

Find the coordinates of the foot of the perpendicular drawn from the point  $(1, 2, 3)$  to the line

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}. \text{ Also, find the length of the perpendicular from the given point to the line.}$$

### Answer

$$\text{Given: Equation of line is } \frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}.$$

To find: coordinates of foot of the perpendicular from  $(1, 2, 3)$  to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

$$\text{Cartesian form: } \frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $b_1 : b_2 : b_3$  is the direction ratios of the line.

2. Distance between two points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Explanation:

Let

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$$

So the foot of the perpendicular is  $(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$

Direction ratio of the line is  $3 : 2 : -2$

Direction ratio of the perpendicular is

$$\Rightarrow (3\lambda + 6 - 1) : (2\lambda + 7 - 2) : (-2\lambda + 7 - 3)$$

$$\Rightarrow (3\lambda + 5) : (2\lambda + 5) : (-2\lambda + 4)$$

Since this is perpendicular to the line,

$$3(3\lambda + 5) + 2(2\lambda + 5) - 2(-2\lambda + 4) = 0$$

$$\Rightarrow 9\lambda + 15 + 4\lambda + 10 + 4\lambda - 8 = 0$$

$$\Rightarrow 17\lambda = -17$$

$$\Rightarrow \lambda = -1$$

So the foot of the perpendicular is  $(3, 5, 9)$

$$\text{Distance} = \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2}$$

$$= \sqrt{4 + 9 + 36}$$

$$= 7 \text{ units}$$

Therefore, the foot of the perpendicular is  $(3, 5, 9)$  and length of perpendicular is 7 units.

### 16. Question

Find the length and the foot of the perpendicular drawn from the point  $(2, -1, 5)$  to the line

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}.$$

**Answer**

Given: Equation of line is  $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}.$

To find: coordinates of foot of the perpendicular from  $(2, -1, 5)$  to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

$$\text{Cartesian form: } \frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $b_1 : b_2 : b_3$  is the direction ratios of the line.

2. Distance between two points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Explanation:

Let

$$\frac{x - 11}{10} = \frac{y + 2}{-4} = \frac{z + 8}{-11} = \lambda$$

So the foot of the perpendicular is  $(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$

Direction ratio of the line is  $10 : -4 : -11$

Direction ratio of the perpendicular is

$$\Rightarrow (10\lambda + 11 - 2) : (-4\lambda - 2 + 1) : (-11\lambda - 8 - 5)$$

$$\Rightarrow (10\lambda + 9) : (-4\lambda - 1) : (-11\lambda - 13)$$

Since this is perpendicular to the line,

$$10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$$

$$\Rightarrow 100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0$$

$$\Rightarrow 237\lambda = -237$$

$$\Rightarrow \lambda = -1$$

So the foot of the perpendicular is  $(1, 2, 3)$

$$\text{Distance} = \sqrt{(1 - 2)^2 + (2 + 1)^2 + (3 - 5)^2}$$

$$= \sqrt{1 + 9 + 4}$$

$$= \sqrt{14} \text{ units}$$

Therefore, the foot of the perpendicular is  $(1, 2, 3)$  and length of perpendicular is  $\sqrt{14}$  units.

### 17. Question

Find the vector and Cartesian equations of the line passing through the points  $A(3, 4, -6)$  and  $B(5, -2, 7)$ .

### Answer

Given: line passes through the points  $(3, 4, -6)$  and  $(5, -2, 7)$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

$$\text{Vector form: } \vec{r} = \vec{a} + \lambda\vec{b}$$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  with  $b_1 : b_2 : b_3$  being the direction ratios of the line.

Explanation:

Here,  $\vec{a} = 3\hat{i} + 4\hat{j} - 6\hat{k}$

The direction ratios of the line are  $(3 - 5) : (4 + 2) : (-6 - 7)$

$\Rightarrow -2 : 6 : -13$

$\Rightarrow 2 : -6 : 13$

So,  $\vec{b} = 2\hat{i} - 6\hat{j} + 13\hat{k}$

Therefore,

Vector form:

$\vec{r} = 3\hat{i} + 4\hat{j} - 6\hat{k} + \lambda(2\hat{i} - 6\hat{j} + 13\hat{k})$

Cartesian form:

$\frac{x-3}{2} = \frac{y-4}{-6} = \frac{z+6}{13}$

### 18. Question

Find the vector and Cartesian equations of the line passing through the points A(2, -3, 0) and B(-2, 4, 3).

**Answer**

Given: line passes through the points (2, -3, 0) and (-2, 4, 3)

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  with  $b_1 : b_2 : b_3$  being the direction ratios of the line.

Explanation:

Here,  $\vec{a} = 2\hat{i} - 3\hat{j}$

The direction ratios of the line are  $(2 + 2) : (-3 - 4) : (0 - 3)$

$\Rightarrow 4 : -7 : -3$

$$\Rightarrow -4 : 7 : 3$$

$$\text{So, } \vec{b} = -4\hat{i} + 7\hat{j} + 3\hat{k}$$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} - 3\hat{j} + \lambda(-4\hat{i} + 7\hat{j} + 3\hat{k})$$

Cartesian form:

$$\frac{x-2}{-4} = \frac{y+3}{7} = \frac{z}{3}$$

### 19. Question

Find the vector and Cartesian equations of the line joining the points whose position vectors are  $(\hat{i} - 2\hat{j} + \hat{k})$  and  $(\hat{i} + 3\hat{j} - 2\hat{k})$ .

### Answer

Given: line passes through the points whose position vectors are  $(\hat{i} - 2\hat{j} + \hat{k})$  and  $(\hat{i} + 3\hat{j} - 2\hat{k})$ .

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

$$\text{Vector form: } \vec{r} = \vec{a} + \lambda\vec{b}$$

$$\text{Cartesian form: } \frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  with  $b_1 : b_2 : b_3$  being the direction ratios of the line.

Explanation:

$$\text{Here, } \vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$

The direction ratios of the line are  $(1 - 1) : (-2 - 3) : (1 + 2)$

$$\Rightarrow 0 : -5 : 3$$

$$\Rightarrow 0 : 5 : -3$$

$$\text{So, } \vec{b} = -5\hat{j} + 3\hat{k}$$

Therefore,

Vector form:

$$\vec{r} = \hat{i} - 2\hat{j} + \hat{k} + \lambda(5\hat{j} - 3\hat{k})$$

Cartesian form:

$$\frac{x-1}{0} = \frac{y+2}{5} = \frac{z-1}{-3}$$

## 20. Question

Find the vector equation of a line passing through the point A(3, -2, 1) and parallel to the line joining the points B(-2, 4, 2) and C(2, 3, 3). Also, find the Cartesian equations of the line.

### Answer

Given: line passes through the point (3, -2, 1) and is parallel to the line joining points B(-2, 4, 2) and C(2, 3, 3).

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda \vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  with  $b_1 : b_2 : b_3$  being the direction ratios of the line.

Explanation:

Here,  $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$

The direction ratios of the line are  $(-2 - 2) : (4 - 3) : (2 - 3)$

$$\Rightarrow -4 : 1 : -1$$

$$\Rightarrow 4 : -1 : 1$$

So,  $\vec{b} = 4\hat{i} - \hat{j} + \hat{k}$

Therefore,

Vector form:

$$\vec{r} = 3\hat{i} - 2\hat{j} + \hat{k} + \lambda(4\hat{i} - \hat{j} + \hat{k})$$

Cartesian form:

$$\frac{x-3}{4} = \frac{y+2}{-1} = \frac{z-1}{1}$$

## 21. Question

Find the vector equation of a line passing through the point having the position vector  $(\hat{i} + 2\hat{j} - 3\hat{k})$  and parallel to the line joining the points with position vectors  $(\hat{i} - \hat{j} + 5\hat{k})$  and  $(2\hat{i} + 3\hat{j} - 4\hat{k})$ .

Also, find the Cartesian equivalents of this equation.

**Answer**

Given: line passes through the point with position vector  $\hat{i} + 2\hat{j} - 3\hat{k}$  and parallel to the line joining the points with position vectors  $\hat{i} - \hat{j} + 5\hat{k}$  and  $2\hat{i} + 3\hat{j} - 4\hat{k}$ .

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  with  $b_1 : b_2 : b_3$  being the direction ratios of the line.

Explanation:

Here,  $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

The direction ratios of the line are  $(1 - 2) : (-1 - 3) : (5 + 4)$

$\Rightarrow -1 : -4 : 9$

$\Rightarrow 1 : 4 : -9$

So,  $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$

Therefore,

Vector form:

$$\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \lambda(\hat{i} + 4\hat{j} - 9\hat{k})$$

Cartesian form:

$$\frac{x-1}{1} = \frac{y-2}{4} = \frac{z+3}{-9}$$

**22. Question**

Find the coordinates of the foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining the points B(1, 4, 6) and C(5, 4, 4).

**Answer**

Given: perpendicular drawn from point A (1, 2, 1) to line joining points B (1, 4, 6) and C (5, 4, 4)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  with  $b_1 : b_2 : b_3$  being the direction ratios of the line.

If 2 lines of direction ratios  $a_1:a_2:a_3$  and  $b_1:b_2:b_3$  are perpendicular, then  $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (1, 4, 6) is a point on the line.

Therefore,  $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

Also direction ratios of the line are (1 - 5) : (4 - 4) : (6 - 4)

$\Rightarrow -4 : 0 : 2$

$\Rightarrow -2 : 0 : 1$

So, equation of the line in Cartesian form is

$$\frac{x-1}{-2} = \frac{y-4}{0} = \frac{z-6}{1} = \lambda$$

Any point on the line will be of the form  $(-2\lambda + 1, 4, \lambda + 6)$

So the foot of the perpendicular is of the form  $(-2\lambda + 1, 4, \lambda + 6)$

The direction ratios of the perpendicular is

$(-2\lambda + 1 - 1) : (4 - 2) : (\lambda + 6 - 1)$

$\Rightarrow (-2\lambda) : 2 : (\lambda + 5)$

From the direction ratio of the line and the direction ratio of its perpendicular, we have

$$-2(-2\lambda) + 0 + \lambda + 5 = 0$$

$$\Rightarrow 4\lambda + \lambda = -5$$

$$\Rightarrow \lambda = -1$$

So, the foot of the perpendicular is (3, 4, 5)

### 23. Question

Find the coordinates of the foot of the perpendicular drawn from the point A(1, 8, 4) to the line joining the points B(0, -1, 3) and C(2, -3, -1).

**Answer**

Given: perpendicular drawn from point A (1, 8, 4) to line joining points B (0, -1, 3) and C (2, -3, -1)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  with  $b_1 : b_2 : b_3$  being the direction ratios of the line.

If 2 lines of direction ratios  $a_1:a_2:a_3$  and  $b_1:b_2:b_3$  are perpendicular, then  $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (0, -1, 3) is a point on the line.

Therefore,  $\vec{a} = -\hat{j} + 3\hat{k}$

Also direction ratios of the line are (0 - 2) : (-1 + 3) : (3 + 1)

$$\Rightarrow -2 : 2 : 4$$

$$\Rightarrow -1 : 1 : 2$$

So, equation of the line in Cartesian form is

$$\frac{x}{-1} = \frac{y+1}{1} = \frac{z-3}{2} = \lambda$$

Any point on the line will be of the form  $(-\lambda, \lambda - 1, 2\lambda + 3)$

So the foot of the perpendicular is of the form  $(-\lambda, \lambda - 1, 2\lambda + 3)$

The direction ratios of the perpendicular is

$$(-\lambda - 1) : (\lambda - 1 - 8) : (2\lambda + 3 - 4)$$

$$\Rightarrow (-\lambda - 1) : (\lambda - 9) : (2\lambda - 1)$$

From the direction ratio of the line and the direction ratio of its perpendicular, we have

$$-1(-\lambda - 1) + \lambda - 9 + 2(2\lambda - 1) = 0$$

$$\Rightarrow \lambda + 1 + \lambda - 9 + 4\lambda - 2 = 0$$

$$\Rightarrow 6\lambda = 10$$

$$\Rightarrow \lambda = \frac{5}{3}$$

So, the foot of the perpendicular is  $\left(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3}\right)$

#### 24. Question

Find the image of the point (0, 2, 3) in the line  $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ .

**Answer**

Given: Equation of line is  $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ .

To find: image of point (0, 2, 3)

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda \vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  with  $b_1 : b_2 : b_3$  being the direction ratios of the line.

If 2 lines of direction ratios  $a_1:a_2:a_3$  and  $b_1:b_2:b_3$  are perpendicular, then  $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Explanation:

Let

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$$

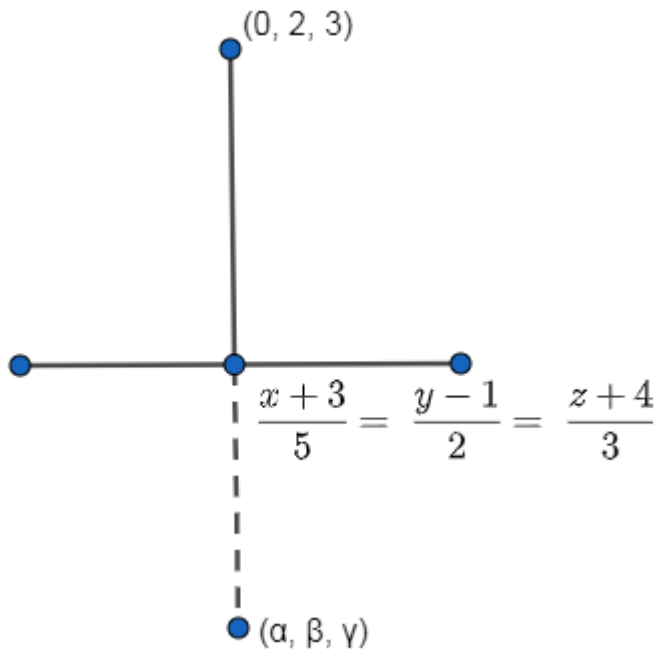
So the foot of the perpendicular is  $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

The direction ratios of the perpendicular is

$$(5\lambda - 3 - 0) : (2\lambda + 1 - 2) : (3\lambda - 4 - 3)$$

$$\Rightarrow (5\lambda - 3) : (2\lambda - 1) : (3\lambda - 7)$$

Direction ratio of the line is  $5 : 2 : 3$



From the direction ratio of the line and the direction ratio of its perpendicular, we have

$$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$$

$$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$$

$$\Rightarrow 38\lambda = 38$$

$$\Rightarrow \lambda = 1$$

So, the foot of the perpendicular is  $(2, 3, -1)$

The foot of the perpendicular is the mid-point of the line joining  $(0, 2, 3)$  and  $(\alpha, \beta, \gamma)$

So, we have

$$\frac{\alpha + 0}{2} = 2 \Rightarrow \alpha = 4$$

$$\frac{\beta + 2}{2} = 3 \Rightarrow \beta = 4$$

$$\frac{\gamma + 3}{2} = -1 \Rightarrow \gamma = -5$$

So, the image is  $(4, 4, -5)$

## 25. Question

Find the image of the point  $(5, 9, 3)$  in the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ .

**Answer**

Given: Equation of line is  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ .

To find: image of point (5, 9, 3)

Formula Used: Equation of a line is

Vector form:  $\vec{r} = \vec{a} + \lambda \vec{b}$

Cartesian form:  $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where  $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$  is a point on the line and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  with  $b_1 : b_2 : b_3$  being the direction ratios of the line.

If 2 lines of direction ratios  $a_1:a_2:a_3$  and  $b_1:b_2:b_3$  are perpendicular, then  $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Explanation:

Let

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

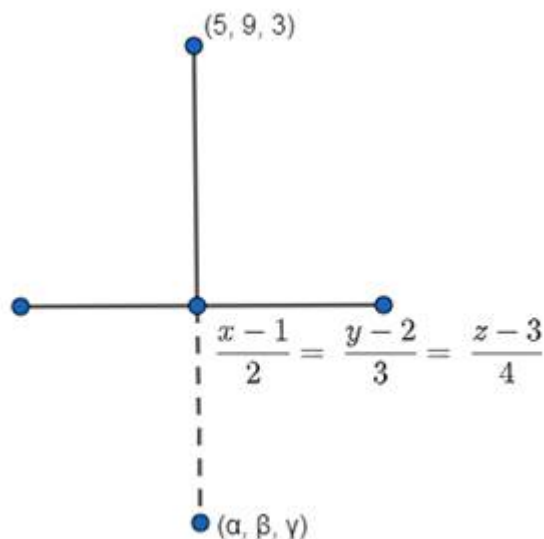
So the foot of the perpendicular is  $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

The direction ratios of the perpendicular is

$$(2\lambda + 1 - 5) : (3\lambda + 2 - 9) : (4\lambda + 3 - 3)$$

$$\Rightarrow (2\lambda - 4) : (3\lambda - 7) : (4\lambda)$$

Direction ratio of the line is 2 : 3 : 4



From the direction ratio of the line and the direction ratio of its perpendicular, we have

$$2(2\lambda - 4) + 3(3\lambda - 7) + 4(4\lambda) = 0$$

$$\Rightarrow 4\lambda - 8 + 9\lambda - 21 + 16\lambda = 0$$

$$\Rightarrow 29\lambda = 29$$

$$\Rightarrow \lambda = 1$$

So, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and ( $\alpha$ ,  $\beta$ ,  $\gamma$ )

So, we have

$$\frac{\alpha + 5}{2} = 3 \Rightarrow \alpha = 1$$

$$\frac{\beta + 9}{2} = 5 \Rightarrow \beta = 1$$

$$\frac{\gamma + 3}{2} = 7 \Rightarrow \gamma = 11$$

So, the image is (1, 1, 11)

## 26. Question

Find the image of the point (2, -1, 5) in the line

$$\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$$

## Answer

Given: Point (2, -1, 5)

$$\text{Equation of line} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$$

$$\text{The equation of line can be re-arranged as } \frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = r$$

The general point on this line is

$$(10r + 11, -4r - 2, -11r - 8)$$

Let N be the foot of the perpendicular drawn from the point P(2, 1, -5) on the given line.

Then, this point is N(10r + 11, -4r - 2, -11r - 8) for some fixed value of r.

$$\text{D.r.'s of PN are } (10r + 9, -4r - 3, -11r - 3)$$

$$\text{D.r.'s of the given line is } 10, -4, -11.$$

Since, PN is perpendicular to the given line, we have,

$$10(10r + 9) - 4(-4r - 3) - 11(-11r - 3) = 0$$

$$100r + 90 + 16r + 12 + 121r + 33 = 0$$

$$237r = 135$$

$$r = \frac{135}{237}$$

Then, the image of the point is

$$\frac{\alpha - 11}{-11} = 0, \frac{\beta + 2}{7} = 1, \frac{\gamma + 8}{9} = 1$$

Therefore, the image is (0, 5, 1).

## Exercise 27B

### 1. Question

Show that the points A(2, 1, 3), B(5, 0, 5) and C(-4, 3, -1) are collinear.

**Answer**

**Given -**

$$A = (2, 1, 3)$$

$$B = (5, 0, 5)$$

$$C = (-4, 3, -1)$$

**To prove -** A, B and C are collinear

**Formula to be used -** If P = (a, b, c) and Q = (a', b', c'), then the direction ratios of the line PQ is given by ((a'-a), (b'-b), (c'-c))

The direction ratios of the line AB can be given by

$$((5-2), (0-1), (5-3))$$

$$=(3, -1, 2)$$

Similarly, the direction ratios of the line BC can be given by

$$((-4-5), (3-0), (-1-5))$$

$$=(-9, 3, -6)$$

**Tip -** If it is shown that direction ratios of AB =  $\lambda$  times that of BC, where  $\lambda$  is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

$$=(3, -1, 2)$$

$$=(-1/3) \times (-9, 3, -6)$$

$$=(-1/3) \times \text{d.r. of BC}$$

Hence, **A, B and C are collinear**

## 2. Question

Show that the points A(2, 3, -4), B(1, -2, 3) and C(3, 8, -11) are collinear.

**Answer**

**Given -**

$$A = (2, 3, -4)$$

$$B = (1, -2, 3)$$

$$C = (3, 8, -11)$$

**To prove –** A, B and C are collinear

**Formula to be used –** If P = (a,b,c) and Q = (a',b',c'), then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

$$((1-2), (-2-3), (3+4))$$

$$=(-1, -5, 7)$$

Similarly, the direction ratios of the line BC can be given by

$$((3-1), (8+2), (-11-3))$$

$$=(2, 10, -14)$$

**Tip –** If it is shown that direction ratios of AB =  $\lambda$  times that of BC, where  $\lambda$  is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

$$=(-1, -5, 7)$$

$$=(-1/2) \times (2, 10, -14)$$

$$=(-1/2) \times \text{d.r. of BC}$$

Hence, **A, B and C are collinear**

## 3. Question

Find the value of  $\lambda$  for which the points A(2, 5, 1), B(1, 2, -1) and C(3,  $\lambda$ , 3) are collinear.

**Answer**

**Given -**

$$A = (2, 5, 1)$$

$$B = (1, 2, -1)$$

$$C = (3, \lambda, 3)$$

**To find –** The value of  $\lambda$  so that A, B and C are collinear

**Formula to be used** – If  $P = (a,b,c)$  and  $Q = (a',b',c')$ , then the direction ratios of the line PQ is given by  $((a'-a),(b'-b),(c'-c))$

The direction ratios of the line AB can be given by

$$((1-2),(2-5),(-1-1))$$

$$=(-1,-3,-2)$$

Similarly, the direction ratios of the line BC can be given by

$$((3-1),(\lambda-2),(3+1))$$

$$=(2,\lambda-2,4)$$

**Tip** – If it is shown that direction ratios of AB =  $\lambda$  times that of BC, where  $\lambda$  is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

$$=(-1,-3,-2)$$

$$=(-1/2) \times (2,\lambda-2,4)$$

$$=(-1/2) \times \text{d.r. of BC}$$

Since, A, B and C are collinear,

$$\therefore -\frac{1}{2}(\lambda-2) = -3$$

$$\Rightarrow \lambda - 2 = 6$$

$$\Rightarrow \lambda = 8$$

#### 4. Question

Find the values of  $\lambda$  and  $\mu$  so that the points  $A(3, 2, -4)$ ,  $B(9, 8, -10)$  and  $C(\lambda, \mu, -6)$  are collinear.

**Answer**

**Given -**

$$A = (3, 2, -4)$$

$$B = (9, 8, -10)$$

$$C = (\lambda, \mu, -6)$$

**To find** – The value of  $\lambda$  and  $\mu$  so that A, B and C are collinear

**Formula to be used** – If  $P = (a,b,c)$  and  $Q = (a',b',c')$ , then the direction ratios of the line PQ is given by  $((a'-a),(b'-b),(c'-c))$

The direction ratios of the line AB can be given by

$$((9-3),(8-2),(-10+4))$$

$$=(6,6,-6)$$

Similarly, the direction ratios of the line BC can be given by

$$((\lambda-9),(\mu-8),(-6+10))$$

$$=(\lambda-9,\mu-8,4)$$

**Tip** – If it is shown that direction ratios of AB =  $\lambda$  times that of BC, where  $\lambda$  is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

$$=(6,6,-6)$$

$$=(-6/4) \times (-4,-4,4)$$

$$=(-3/2) \times \text{d.r. of BC}$$

Since, A, B and C are collinear,

$$\therefore -\frac{3}{2}(\lambda-9) = 6$$

$$\Rightarrow \lambda - 9 = -4$$

$$\Rightarrow \lambda = 5$$

And,

$$\therefore -\frac{3}{2}(\mu-8) = 6$$

$$\Rightarrow \mu - 8 = -4$$

$$\Rightarrow \mu = 4$$

### 5. Question

Find the values of  $\lambda$  and  $\mu$  so that the points A(-1, 4, -2), B( $\lambda$ ,  $\mu$ , 1) and C(0, 2, -1) are collinear.

**Answer**

**Given -**

$$A = (-1, 4, -2)$$

$$B = (\lambda, \mu, 1)$$

$$C = (0, 2, -1)$$

**To find** – The value of  $\lambda$  and  $\mu$  so that A, B and C are collinear

**Formula to be used** – If P = (a, b, c) and Q = (a', b', c'), then the direction ratios of the line PQ is given by ((a'-a), (b'-b), (c'-c))

The direction ratios of the line AB can be given by

$$((\lambda+1),(\mu-4),(1+2))$$

$$=(\lambda+1,\mu-4,3)$$

Similarly, the direction ratios of the line BC can be given by

$$((0-\lambda), (2-\mu), (-1-1))$$

$$=(-\lambda, 2-\mu, -2)$$

**Tip** – If it is shown that direction ratios of AB =  $\alpha$  times that of BC, where  $\lambda$  is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

$$=(\lambda+1, \mu-4, 3)$$

Say,  $\alpha$  be an arbitrary constant such that d.r. of AB =  $\alpha$  X d.r. of BC

$$\text{So, } 3 = \alpha \times (-2)$$

$$\text{i.e. } \alpha = -3/2$$

Since, A, B and C are collinear,

$$\therefore -\frac{3}{2}(-\lambda) = \lambda + 1$$

$$\Rightarrow 3\lambda = 2\lambda + 2$$

$$\Rightarrow \lambda = 2$$

And,

$$\therefore -\frac{3}{2}(2-\mu) = \mu - 4$$

$$\Rightarrow -6 + 3\mu = 2\mu - 8$$

$$\Rightarrow \mu = -2$$

## 6. Question

The position vectors of three points A, B and C are  $\vec{a}(-4\hat{i} + 2\hat{j} - 3\hat{k})$ ,  $\vec{b}(\hat{i} + 3\hat{j} - 2\hat{k})$  and  $\vec{c}(-9\hat{i} + \hat{j} - 4\hat{k})$  respectively. show that the points A, B and C are collinear.

**Answer**

**Given -**

$$\vec{A} = -4\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$$

$$\vec{C} = -9\hat{i} + \hat{j} - 4\hat{k}$$

It can thus be written as:

$$A = (-4, 2, -3)$$

CAREER POINT

$$B = (1, 3, -2)$$

$$C = (-9, 1, -4)$$

**To prove** – A, B and C are collinear

**Formula to be used** – If  $P = (a, b, c)$  and  $Q = (a', b', c')$ , then the direction ratios of the line PQ is given by  $((a'-a), (b'-b), (c'-c))$

The direction ratios of the line AB can be given by

$$((1+4), (3-2), (-2+3))$$

$$=(5, 1, 1)$$

Similarly, the direction ratios of the line BC can be given by

$$((-9-1), (1-3), (-4+2))$$

$$=(-10, -2, -2)$$

**Tip** – If it is shown that direction ratios of  $AB = \lambda$  times that of BC, where  $\lambda$  is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

$$=(5, 1, 1)$$

$$=(-1/2) \times (-10, -2, -2)$$

$$=(-1/2) \times \text{d.r. of BC}$$

Hence, **A, B and C are collinear**

## Exercise 27C

### 1. Question

Find the angle between each of the following pairs of lines:

$$\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k}) \text{ and } \vec{r} = (2\hat{i} - \hat{j} - 5\hat{k}) + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$$

**Answer**

$$\text{Given - } \vec{L}_1 = (3\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k})$$

$$\& \vec{L}_2 = (2\hat{i} - \hat{j} - 5\hat{k}) + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$$

**To find** – Angle between the two pair of lines

$$\text{Direction ratios of } L_1 = (1, -1, -2)$$

$$\text{Direction ratios of } L_2 = (3, -5, -4)$$

**Tip** – If  $(a,b,c)$  be the direction ratios of the first line and  $(a',b',c')$  be that of the second, then the angle between these pair of lines is given by  $\cos^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1 \times 3 + (-1) \times (-5) + (-2) \times (-4)}{\sqrt{1^2 + 1^2 + 2^2} \sqrt{3^2 + 5^2 + 4^2}}\right)$$

$$= \cos^{-1}\left(\frac{3 + 5 + 8}{\sqrt{6}\sqrt{50}}\right)$$

$$= \cos^{-1}\left(\frac{16}{5\sqrt{6}\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$$

## 2. Question

Find the angle between each of the following pairs of lines:

$$\vec{r} = (3\hat{i} - 4\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 3\hat{k}) \text{ and } \vec{r} = 5\hat{i} + \mu(-\hat{i} + \hat{j} + \hat{k})$$

**Answer**

$$\text{Given - } \vec{L}_1 = (3\hat{i} - 4\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 3\hat{k})$$

$$\& \vec{L}_2 = (5\hat{i}) + \mu(-\hat{i} + \hat{j} + \hat{k})$$

**To find** – Angle between the two pair of lines

Direction ratios of  $L_1 = (1, 0, 3)$

Direction ratios of  $L_2 = (-1, 1, 1)$

**Tip** – If  $(a,b,c)$  be the direction ratios of the first line and  $(a',b',c')$  be that of the second, then the angle between these pair of lines is given by  $\cos^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1 \times (-1) + 0 \times 1 + 3 \times 1}{\sqrt{1^2 + 0^2 + 3^2} \sqrt{1^2 + 1^2 + 1^2}}\right)$$

$$= \cos^{-1}\left(\frac{-1 + 3}{\sqrt{10}\sqrt{3}}\right)$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{30}}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{30}}{15}\right)$$

### 3. Question

Find the angle between each of the following pairs of lines:

$$\vec{r} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k}) \text{ and } \vec{r} = 3\hat{k} + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$$

**Answer**

**Given** -  $\vec{L}_1 = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$

&  $\vec{L}_2 = (3\hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

**To find** - Angle between the two pair of lines

Direction ratios of  $L_1 = (2, -2, 1)$

Direction ratios of  $L_2 = (1, 2, -2)$

**Tip** - If  $(a, b, c)$  be the direction ratios of the first line and  $(a', b', c')$  be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2 \times 1 + (-2) \times 2 + 1 \times (-2)}{\sqrt{2^2 + 2^2 + 1^2} \sqrt{1^2 + 2^2 + 2^2}}\right)$$

$$= \cos^{-1}\left(\frac{2 - 4 - 2}{3 \times 3}\right)$$

$$= \cos^{-1}\left(-\frac{4}{9}\right)$$

### 4. Question

Find the angle between each of the following pairs of lines:

$$\frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2} \text{ and } \frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$$

**Answer**

**Given** -  $\vec{L}_1 = \frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

$$\vec{L_2} = \frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$$

**To find** – Angle between the two pair of lines

Direction ratios of  $L_1 = (1,1,2)$

Direction ratios of  $L_2 = (3,5,4)$

**Tip** – If  $(a,b,c)$  be the direction ratios of the first line and  $(a',b',c')$  be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$= \cos^{-1} \left( \frac{1 \times 3 + 1 \times 5 + 2 \times 4}{\sqrt{1^2 + 1^2 + 2^2} \sqrt{3^2 + 5^2 + 4^2}} \right)$$

$$= \cos^{-1} \left( \frac{3 + 5 + 8}{\sqrt{6} \times \sqrt{50}} \right)$$

$$= \cos^{-1} \left( \frac{8\sqrt{3}}{15} \right)$$

## 5. Question

Find the angle between each of the following pairs of lines:

$$\frac{x-4}{4} = \frac{y+1}{4} = \frac{z-6}{5} \text{ and } \frac{x-5}{1} = \frac{2y+5}{-2} = \frac{z-3}{1}$$

**Answer**

$$\text{Given - } \vec{L_1} = \frac{x-4}{4} = \frac{y+1}{3} = \frac{z-6}{5}$$

$$\& \vec{L_2} = \frac{x-5}{1} = \frac{y+5/2}{-1} = \frac{z-3}{1}$$

**To find** – Angle between the two pair of lines

Direction ratios of  $L_1 = (4,3,5)$

Direction ratios of  $L_2 = (1,-1,1)$

**Tip** – If  $(a,b,c)$  be the direction ratios of the first line and  $(a',b',c')$  be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$\begin{aligned}
 &= \cos^{-1} \left( \frac{4 \times 1 + 3 \times (-1) + 5 \times 1}{\sqrt{4^2 + 3^2 + 5^2} \sqrt{1^2 + 1^2 + 1^2}} \right) \\
 &= \cos^{-1} \left( \frac{4 - 3 + 5}{5\sqrt{2} \times \sqrt{3}} \right) \\
 &= \cos^{-1} \left( \frac{6}{5\sqrt{6}} \right) \\
 &= \cos^{-1} \left( \frac{2\sqrt{6}}{15} \right)
 \end{aligned}$$

## 6. Question

Find the angle between each of the following pairs of lines:

$$\frac{3-x}{-2} = \frac{y+5}{1} = \frac{1-z}{3} \text{ and } \frac{x}{3} = \frac{1-y}{-2} = \frac{z+2}{-1}$$

**Answer**

$$\text{Given - } \vec{L_1} = \frac{x-3}{2} = \frac{y+5}{1} = \frac{z-1}{-3}$$

$$\& \vec{L_2} = \frac{x}{3} = \frac{y-1}{2} = \frac{z+2}{-1}$$

**To find** – Angle between the two pair of lines

Direction ratios of  $L_1 = (2, 1, -3)$

Direction ratios of  $L_2 = (3, 2, -1)$

**Tip** – If  $(a, b, c)$  be the direction ratios of the first line and  $(a', b', c')$  be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$\begin{aligned}
 &= \cos^{-1} \left( \frac{2 \times 3 + 1 \times 2 + (-3) \times (-1)}{\sqrt{2^2 + 1^2 + 3^2} \sqrt{3^2 + 2^2 + 1^2}} \right) \\
 &= \cos^{-1} \left( \frac{6 + 2 + 3}{\sqrt{14} \times \sqrt{14}} \right) \\
 &= \cos^{-1} \left( \frac{11}{14} \right)
 \end{aligned}$$

## 7. Question

Find the angle between each of the following pairs of lines:

$$\frac{x}{1} = \frac{z}{-1}, y = 0 \text{ and } \frac{x}{3} = \frac{y}{4} = \frac{z}{5}$$

**Answer**

**Given** -  $\vec{L_1} = \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$

&  $\vec{L_2} = \frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

**To find** - Angle between the two pair of lines

Direction ratios of  $L_1 = (1, 0, -1)$

Direction ratios of  $L_2 = (3, 4, 5)$

**Tip** - If  $(a, b, c)$  be the direction ratios of the first line and  $(a', b', c')$  be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$= \cos^{-1} \left( \frac{1 \times 3 + 0 \times 4 + (-1) \times 5}{\sqrt{1^2 + 0^2 + 1^2} \sqrt{3^2 + 4^2 + 5^2}} \right)$$

$$= \cos^{-1} \left( \frac{3 - 5}{5\sqrt{2} \times \sqrt{2}} \right)$$

$$= \cos^{-1} \left( \frac{1}{5} \right)$$

## 8. Question

Find the angle between each of the following pairs of lines:

$$\frac{5-x}{3} = \frac{y+3}{-2}, z=5 \text{ and } \frac{x-1}{1} = \frac{1-y}{3} = \frac{z-5}{2}$$

**Answer**

**Given** -  $\vec{L_1} = \frac{x-5}{-3} = \frac{y+3}{-2} = \frac{z-5}{0}$

&  $\vec{L_2} = \frac{x-1}{1} = \frac{y-1}{-3} = \frac{z-5}{2}$

**To find** - Angle between the two pair of lines

Direction ratios of  $L_1 = (-3, -2, 0)$

Direction ratios of  $L_2 = (1, -3, 2)$

**Tip** – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$= \cos^{-1} \left( \frac{(-3) \times 1 + (-2) \times (-3) + 0 \times 2}{\sqrt{3^2 + 2^2 + 0^2} \sqrt{1^2 + 3^2 + 2^2}} \right)$$

$$= \cos^{-1} \left( \frac{-3 + 6}{\sqrt{13} \times \sqrt{14}} \right)$$

$$= \cos^{-1} \left( \frac{3}{\sqrt{182}} \right)$$

### 9. Question

Show that the lines  $\frac{x-3}{2} = \frac{y+1}{-3} = \frac{z-2}{4}$  and  $\frac{x+2}{2} = \frac{y-4}{4} = \frac{z+5}{2}$  are perpendicular to each other.

**Answer**

**Given** –  $\vec{L}_1 = \frac{x-3}{2} = \frac{y+1}{-3} = \frac{z-2}{4}$

&  $\vec{L}_2 = \frac{x+2}{2} = \frac{y-4}{4} = \frac{z+5}{2}$

**To prove** – The lines are perpendicular to each other

Direction ratios of  $L_1 = (2, -3, 4)$

Direction ratios of  $L_2 = (2, 4, 2)$

**Tip** – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$= \cos^{-1} \left( \frac{2 \times 2 + (-3) \times 4 + 4 \times 2}{\sqrt{2^2 + 3^2 + 4^2} \sqrt{2^2 + 4^2 + 2^2}} \right)$$

$$= \cos^{-1} \left( \frac{4 - 12 + 8}{\sqrt{29} \times \sqrt{24}} \right)$$

$$= \cos^{-1}(0)$$

$$= \frac{\pi}{2}$$

Hence, **the lines are perpendicular to each other.**

### 10. Question

If the lines  $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$  and  $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{5}$  are perpendicular to each other then find the value of  $\lambda$ .

**Answer**

**Given** -  $\vec{L}_1 = \frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$

&  $\vec{L}_2 = \frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-5}$

**To find** - The value of  $\lambda$

Direction ratios of  $L_1 = (-3, 2\lambda, 2)$

Direction ratios of  $L_2 = (3\lambda, 1, -5)$

**Tip** - If  $(a, b, c)$  be the direction ratios of the first line and  $(a', b', c')$  be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

Since the lines are perpendicular to each other,

The angle between the lines

$$\Rightarrow \cos^{-1} \left( \frac{(-3) \times 3\lambda + 2\lambda \times 1 + 2 \times (-5)}{\sqrt{3^2 + (2\lambda)^2 + 2^2} \sqrt{(3\lambda)^2 + 1^2 + 5^2}} \right) = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} \left( \frac{-9\lambda + 2\lambda - 10}{\sqrt{13 + 4\lambda^2} \sqrt{9\lambda^2 + 26}} \right) = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} \left( \frac{-7\lambda - 10}{\sqrt{13 + 4\lambda^2} \sqrt{9\lambda^2 + 26}} \right) = \frac{\pi}{2}$$

$$\Rightarrow \left( \frac{-7\lambda - 10}{\sqrt{13 + 4\lambda^2} \sqrt{9\lambda^2 + 26}} \right) = \cos \frac{\pi}{2} = 0$$

$$\Rightarrow -7\lambda - 10 = 0$$

$$\Rightarrow \lambda = -\frac{10}{7}$$

### 11. Question

Show that the lines  $x = -y = 2z$  and  $x + 2 = 2y - 1 = -z + 1$  are perpendicular to each other.

**HINT:** The given lines are  $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$  and  $\frac{x+2}{1} = \frac{y-\frac{1}{2}}{1} = \frac{z-1}{-2}$ .

**Answer**

**Given** –  $\vec{L_1} = \frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$

&  $\vec{L_2} = \frac{x+2}{2} = \frac{y-\frac{1}{2}}{1} = \frac{z-1}{-2}$

**To prove** – The lines are perpendicular to each other

Direction ratios of  $L_1 = (2, -2, 1)$

Direction ratios of  $L_2 = (2, 1, -2)$

**Tip** – If  $(a, b, c)$  be the direction ratios of the first line and  $(a', b', c')$  be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$= \cos^{-1} \left( \frac{2 \times 2 + (-2) \times 1 + 1 \times (-2)}{\sqrt{2^2 + 2^2 + 1^2} \sqrt{1^2 + 1^2 + 2^2}} \right)$$

$$= \cos^{-1} \left( \frac{4 - 2 - 2}{\sqrt{29} \times \sqrt{24}} \right)$$

$$= \cos^{-1}(0)$$

$$= \frac{\pi}{2}$$

Hence, **the lines are perpendicular to each other.**

## 12. Question

Find the angle between two lines whose direction ratios are

i. 2, 1, 2 and 4, 8, 1

ii. 5, -12, 13 and -3, 4, 5

iii. 1, 1, 2 and  $(\sqrt{3}-1), (-\sqrt{3}-1), 4$

iv. a, b, c and  $(b-c), (c-a), (a-b)$

**Answer**

**(i): Given** – Direction ratios of  $L_1 = (2, 1, 2)$  & Direction ratios of  $L_2 = (4, 8, 1)$

**To find** – Angle between the two pair of lines  
CAREER POINT

**Tip** – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$= \cos^{-1} \left( \frac{2 \times 4 + 1 \times 8 + 2 \times 1}{\sqrt{2^2 + 1^2 + 2^2} \sqrt{4^2 + 8^2 + 1^2}} \right)$$

$$= \cos^{-1} \left( \frac{8 + 8 + 2}{3 \times 9} \right)$$

$$= \cos^{-1} \left( \frac{18}{27} \right)$$

$$= \cos^{-1} \left( \frac{2}{3} \right)$$

**(ii): Given** – Direction ratios of  $L_1 = (5, -12, 13)$  & Direction ratios of  $L_2 = (-3, 4, 5)$

**To find** – Angle between the two pair of lines

**Tip** – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$= \cos^{-1} \left( \frac{5 \times (-3) + (-12) \times 4 + 13 \times 5}{\sqrt{5^2 + 12^2 + 13^2} \sqrt{3^2 + 4^2 + 5^2}} \right)$$

$$= \cos^{-1} \left( \frac{-15 - 48 + 65}{13\sqrt{2} \times 5\sqrt{2}} \right)$$

$$= \cos^{-1} \left( \frac{2}{130} \right)$$

$$= \cos^{-1} \left( \frac{1}{65} \right)$$

**(iii) Given** – Direction ratios of  $L_1 = (1, 1, 2)$  & Direction ratios of  $L_2 = (\sqrt{3}-1, -\sqrt{3}-1, 4)$

**To find** – Angle between the two pair of lines

**Tip** – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$\begin{aligned}
 &= \cos^{-1} \left( \frac{1 \times (\sqrt{3} - 1) + 1 \times (-\sqrt{3} - 1) + 2 \times 4}{\sqrt{1^2 + 1^2 + 2^2} \sqrt{(\sqrt{3} - 1)^2 + (-\sqrt{3} - 1)^2 + 4^2}} \right) \\
 &= \cos^{-1} \left( \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{\sqrt{6} \sqrt{24}} \right) \\
 &= \cos^{-1} \left( \frac{1}{2} \right) \\
 &= \frac{\pi}{3}
 \end{aligned}$$

**(iv) Given** – Direction ratios of  $L_1 = (a, b, c)$  & Direction ratios of  $L_2 = ((b-c), (c-a), (a-b))$

**To find** – Angle between the two pair of lines

**Tip** – If  $(a, b, c)$  be the direction ratios of the first line and  $(a', b', c')$  be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$\begin{aligned}
 &= \cos^{-1} \left( \frac{a \times (b - c) + b \times (c - a) + c \times (a - b)}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b - c)^2 + (c - a)^2 + (a - b)^2}} \right) \\
 &= \cos^{-1} \left( \frac{0}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b - c)^2 + (c - a)^2 + (a - b)^2}} \right) \\
 &= \cos^{-1}(0) \\
 &= \frac{\pi}{2}
 \end{aligned}$$

### 13. Question

If  $A(1, 2, 3)$ ,  $B(4, 5, 7)$ ,  $C(-4, 3, -6)$  and  $D(2, 9, 2)$  are four given points then find the angle between the lines AB and CD.

**Answer**

**Given -**

$$A = (1, 2, 3)$$

$$B = (4, 5, 7)$$

$$C = (-4, 3, -6)$$

$$D = (2, 9, 2)$$

**Formula to be used** – If  $P = (a,b,c)$  and  $Q = (a',b',c')$ , then the direction ratios of the line PQ is given by  $((a'-a),(b'-b),(c'-c))$

The direction ratios of the line AB can be given by

$$((4-1),(5-2),(7-3))$$

$$=(3,3,4)$$

Similarly, the direction ratios of the line CD can be given by

$$((2+4),(9-3),(2+6))$$

$$=(6,6,8)$$

**To find** – Angle between the two pair of lines AB and CD

**Tip** – If  $(a,b,c)$  be the direction ratios of the first line and  $(a',b',c')$  be that of the second, then the

angle between these pair of lines is given by  $\cos^{-1} \left( \frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}} \right)$

The angle between the lines

$$= \cos^{-1} \left( \frac{3 \times 6 + 3 \times 6 + 4 \times 8}{\sqrt{3^2 + 3^2 + 4^2} \sqrt{6^2 + 6^2 + 8^2}} \right)$$

$$= \cos^{-1} \left( \frac{18 + 18 + 32}{\sqrt{34} \times 2\sqrt{34}} \right)$$

$$= \cos^{-1} \left( \frac{68}{2 \times 34} \right)$$

$$= \cos^{-1} 1$$

$$= 0$$

## Exercise 27D

### 1. Question

Find the shortest distance between the given lines.

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}),$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$

**Answer**

**Given equations :**

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

**To Find :** d

**Formula :**

### 1. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

### 2. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 3. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$  and

$\vec{r} = \vec{a_2} + \lambda\vec{b_2}$  is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

**Answer :**

For given lines,

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

Here,

$$\vec{a_1} = \hat{i} + \hat{j}$$

$$\vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

Therefore,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$= \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3)$$

$$\therefore \vec{b_1} \times \vec{b_2} = 3\hat{i} - \hat{j} - 7\hat{k}$$

$$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{3^2 + (-1)^2 + (-7)^2}$$

$$= \sqrt{9 + 1 + 49}$$

$$= \sqrt{59}$$

$$\vec{a_2} - \vec{a_1} = (2 - 1)\hat{i} + (1 - 1)\hat{j} + (-1 - 0)\hat{k}$$

$$\therefore \vec{a_2} - \vec{a_1} = \hat{i} + 0\hat{j} - \hat{k}$$

Now,

$$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (3\hat{i} - \hat{j} - 7\hat{k}) \cdot (\hat{i} + 0\hat{j} - \hat{k})$$

$$= (3 \times 1) + ((-1) \times 0) + ((-7) \times (-1))$$

$$= 3 + 0 + 7$$

$$= 10$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$\therefore d = \left| \frac{10}{\sqrt{59}} \right|$$

## 2. Question

Find the shortest distance between the given lines.

$$\vec{r} = (-4\hat{i} + 4\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k}),$$

$$\vec{r} = (-3\hat{i} - 8\hat{j} - 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 3\hat{k})$$

**Answer**

**Given equations :**

$$\vec{r} = (-4\hat{i} + 4\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r} = (-3\hat{i} - 8\hat{j} - 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 3\hat{k})$$

**To Find : d**

**Formula :**

**1. Cross Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines  $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$  and

$\vec{r} = \vec{a_2} + \lambda\vec{b_2}$  is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

**Answer :**

For given lines,

$$\vec{r} = (-4\hat{i} + 4\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r} = (-3\hat{i} - 8\hat{j} - 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 3\hat{k})$$

Here,

$$\vec{a}_1 = -4\hat{i} + 4\hat{j} + \hat{k}$$

$$\vec{b}_1 = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{a}_2 = -3\hat{i} - 8\hat{j} - 3\hat{k}$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{j} + 3\hat{k}$$

Therefore,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 3 & 3 \end{vmatrix}$$

$$= \hat{i}(3 + 3) - \hat{j}(3 + 2) + \hat{k}(3 - 2)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = 6\hat{i} - 5\hat{j} + \hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{6^2 + (-5)^2 + 1^2}$$

$$= \sqrt{36 + 25 + 1}$$

$$= \sqrt{62}$$

$$\vec{a}_2 - \vec{a}_1 = (-3 + 4)\hat{i} + (-8 - 4)\hat{j} + (-3 - 1)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = \hat{i} - 12\hat{j} - 4\hat{k}$$

Now,

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (6\hat{i} - 5\hat{j} + \hat{k}) \cdot (\hat{i} - 12\hat{j} - 4\hat{k})$$

$$= (6 \times 1) + ((-5) \times (-12)) + (1 \times (-4))$$

$$= 6 + 60 - 4$$

$$= 62$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{62}{\sqrt{62}} \right|$$

$$d = \sqrt{62} \text{ units}$$

### 3. Question

Find the shortest distance between the given lines.

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}),$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k}).$$

**Answer**

**Given equations :**

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

**To Find : d**

**Formula :**

**1. Cross Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and

$\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

**Answer :**

CAREER POINT

For given lines,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

Here,

$$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$$

Therefore,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(-3 - 6) - \hat{j}(1 - 4) + \hat{k}(3 + 6)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-9)^2 + 3^2 + 9^2}$$

$$= \sqrt{81 + 9 + 81}$$

$$= \sqrt{171}$$

$$\vec{a}_2 - \vec{a}_1 = (4 - 1)\hat{i} + (5 - 2)\hat{j} + (6 - 3)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Now,

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (-9\hat{i} + 3\hat{j} + 9\hat{k}) \cdot (3\hat{i} + 3\hat{j} + 3\hat{k})$$

$$= ((-9) \times 3) + (3 \times 3) + (9 \times 3)$$

$$= -27 + 9 + 27$$

$$= 9$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{9}{\sqrt{171}} \right|$$

$$\therefore d = \frac{9}{\sqrt{19} \cdot \sqrt{9}}$$

$$\therefore d = \frac{3}{\sqrt{19}}$$

$$\therefore d = \frac{3\sqrt{19}}{19}$$

#### 4. Question

Find the shortest distance between the given lines.

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}),$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}).$$

**Answer**

**Given equations :**

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

**To Find : d**

**Formula :**

##### 1. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

##### 2. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

CAREER POINT

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 3. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and

$\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$  is given by,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

**Answer :**

For given lines,

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Here,

$$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$$

Therefore,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(-2 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 2)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 0\hat{j} + 3\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 0^2 + 3^2}$$

$$= \sqrt{9 + 0 + 9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$$\vec{a}_2 - \vec{a}_1 = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 1)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$$

Now,

$$\begin{aligned}
 (\overline{b_1} \times \overline{b_2}) \cdot (\overline{a_2} - \overline{a_1}) &= (-3\hat{i} + 0\hat{j} + 3\hat{k}) \cdot (\hat{i} - 3\hat{j} - 2\hat{k}) \\
 &= ((-3) \times 1) + (0 \times (-3)) + (3 \times (-2)) \\
 &= -3 + 0 - 6 \\
 &= -9
 \end{aligned}$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) \cdot (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{-9}{3\sqrt{2}} \right|$$

$$\therefore d = \frac{3}{\sqrt{2}}$$

$$\therefore d = \frac{3\sqrt{2}}{2}$$

## 5. Question

Find the shortest distance between the given lines.

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}),$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k}).$$

**Answer**

**Given equations :**

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

**To Find :** d

**Formula :**

### 1. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

## 2. Dot Product :

If  $\bar{a}$  &  $\bar{b}$  are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \cdot \bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

## 3. Shortest distance between two lines :

The shortest distance between the skew lines  $\bar{r} = \bar{a}_1 + \lambda\bar{b}_1$  and

$\bar{r} = \bar{a}_2 + \lambda\bar{b}_2$  is given by,

$$d = \left| \frac{(\bar{b}_1 \times \bar{b}_2) \cdot (\bar{a}_2 - \bar{a}_1)}{|\bar{b}_1 \times \bar{b}_2|} \right|$$

## Answer :

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

Here,

$$\bar{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\bar{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\bar{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\bar{b}_2 = -2\hat{i} + 3\hat{j} + 8\hat{k}$$

Therefore,

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$$

$$= \hat{i}(24 - 18) - \hat{j}(16 + 12) + \hat{k}(6 - 6)$$

$$\therefore \bar{b}_1 \times \bar{b}_2 = 6\hat{i} - 28\hat{j} + 0\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{6^2 + (-28)^2 + 0^2}$$

$$= \sqrt{36 + 784 + 0}$$

$$= \sqrt{820}$$

$$\vec{a}_2 - \vec{a}_1 = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$$

Now,

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (6\hat{i} - 28\hat{j} + 0\hat{k}) \cdot (2\hat{i} + \hat{j} - \hat{k})$$

$$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$$

$$= 12 - 28 + 0$$

$$= -16$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{-16}{\sqrt{820}} \right|$$

$$d = \frac{16}{\sqrt{820}} \text{ units}$$

## 6. Question

Find the shortest distance between the given lines.

$$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k}),$$

$$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k}).$$

**Answer**

**Given equations :**

$$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$$

$$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$$

**To Find : d**

**Formula :**

**1. Cross Product :**

CAREER POINT

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

## 2. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

## 3. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and

$\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

## Answer :

For given lines,

$$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$$

$$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$$

Here,

$$\vec{a}_1 = 6\hat{i} + 3\hat{k}$$

$$\vec{b}_1 = 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\vec{a}_2 = -9\hat{i} + \hat{j} - 10\hat{k}$$

$$\vec{b}_2 = 4\hat{i} + \hat{j} + 6\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$$

$$= \hat{i}(-6 - 4) - \hat{j}(12 - 16) + \hat{k}(2 + 4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -10\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-10)^2 + 4^2 + 6^2}$$

$$= \sqrt{100 + 16 + 36}$$

$$= \sqrt{152}$$

$$\overline{a_2} - \overline{a_1} = (-9 - 6)\hat{i} + (1 - 0)\hat{j} + (6 - 3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = -15\hat{i} + \hat{j} + 3\hat{k}$$

Now,

$$(\overline{b_1} \times \overline{b_2}) \cdot (\overline{a_2} - \overline{a_1}) = (-10\hat{i} + 4\hat{j} + 6\hat{k}) \cdot (-15\hat{i} + \hat{j} + 3\hat{k})$$

$$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$$

$$= 150 + 4 + 18$$

$$= 172$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) \cdot (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{172}{\sqrt{152}} \right|$$

$$\therefore d = \frac{172}{2\sqrt{38}}$$

$$\therefore d = \frac{86}{\sqrt{38}}$$

$$d = \frac{86}{\sqrt{38}} \text{ units}$$

## 7. Question

Find the shortest distance between the given lines.

$$\vec{r} = (3 - t)\hat{i} + (4 + 2t)\hat{j} + (t - 2)\hat{k},$$

$$\vec{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}.$$

**Answer**

**Given equations :**

$$\vec{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$$

$$\vec{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}$$

**To Find : d**

**Formula :**

**1. Cross Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and

$\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

**Answer :**

Given lines,

$$\vec{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$$

CAREER POINT

$$\bar{r} = (1 + s)\hat{i} + (3s - 7)\hat{j} + (2s - 2)\hat{k}$$

Above equations can be written as

$$\bar{r} = (3\hat{i} + 4\hat{j} - 2\hat{k}) + t(-\hat{i} + 2\hat{j} + \hat{k})$$

$$\bar{r} = (\hat{i} - 7\hat{j} - 2\hat{k}) + s(\hat{i} + 3\hat{j} + 2\hat{k})$$

Here,

$$\bar{a}_1 = 3\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\bar{b}_1 = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\bar{a}_2 = \hat{i} - 7\hat{j} - 2\hat{k}$$

$$\bar{b}_2 = \hat{i} + 3\hat{j} + 2\hat{k}$$

Therefore,

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$= \hat{i}(4 - 3) - \hat{j}(-2 - 1) + \hat{k}(-3 - 2)$$

$$\therefore \bar{b}_1 \times \bar{b}_2 = \hat{i} + 3\hat{j} - 5\hat{k}$$

$$\therefore |\bar{b}_1 \times \bar{b}_2| = \sqrt{1^2 + 3^2 + (-5)^2}$$

$$= \sqrt{1 + 9 + 25}$$

$$= \sqrt{35}$$

$$\bar{a}_2 - \bar{a}_1 = (1 - 3)\hat{i} + (-7 - 4)\hat{j} + (-2 + 2)\hat{k}$$

$$\therefore \bar{a}_2 - \bar{a}_1 = -2\hat{i} - 11\hat{j} + 0\hat{k}$$

Now,

$$(\bar{b}_1 \times \bar{b}_2) \cdot (\bar{a}_2 - \bar{a}_1) = (\hat{i} + 3\hat{j} - 5\hat{k}) \cdot (-2\hat{i} - 11\hat{j} + 0\hat{k})$$

$$= (1 \times (-2)) + (3 \times (-11)) + ((-5) \times 0)$$

$$= -2 - 33 + 0$$

$$= -35$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\bar{b}_1 \times \bar{b}_2) \cdot (\bar{a}_2 - \bar{a}_1)}{|\bar{b}_1 \times \bar{b}_2|} \right|$$

$$\therefore d = \left| \frac{-35}{\sqrt{35}} \right|$$

$$\therefore d = \sqrt{35}$$

$$d = \sqrt{35} \text{ units}$$

### 8. Question

Find the shortest distance between the given lines.

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k},$$

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}.$$

**Answer**

**Given equations :**

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k}$$

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

**To Find : d**

**Formula :**

#### 1. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

#### 2. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 3. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$  and

$\vec{r} = \vec{a_2} + \lambda \vec{b_2}$  is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

**Answer :**

Given lines,

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k}$$

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

Above equations can be written as

$$\vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k})$$

Here,

$$\vec{a_1} = -\hat{i} + \hat{j} - \hat{k}$$

$$\vec{b_1} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{a_2} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b_2} = -\hat{i} + 2\hat{j} + \hat{k}$$

Therefore,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1 + 2) - \hat{j}(1 - 1) + \hat{k}(2 + 1)$$

$$\therefore \vec{b_1} \times \vec{b_2} = 3\hat{i} - 0\hat{j} + 3\hat{k}$$

$$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{3^2 + 0^2 + 3^2}$$

$$= \sqrt{9 + 0 + 9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$$\vec{a_2} - \vec{a_1} = (1 + 1)\hat{i} + (-1 - 1)\hat{j} + (2 + 1)\hat{k}$$

$$\therefore \vec{a_2} - \vec{a_1} = 2\hat{i} - 2\hat{j} + 3\hat{k}$$

Now,

$$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (3\hat{i} - 0\hat{j} + 3\hat{k}) \cdot (2\hat{i} - 2\hat{j} + 3\hat{k})$$

$$= (3 \times 2) + (0 \times (-2)) + (3 \times 3)$$

$$= 6 + 0 + 9$$

$$= 15$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$\therefore d = \left| \frac{15}{3\sqrt{2}} \right|$$

$$\therefore d = \frac{5}{\sqrt{2}}$$

$$\therefore d = \frac{5\sqrt{2}}{2}$$

$$d = \frac{5\sqrt{2}}{2} \text{ units}$$

### 9. Question

Compute the shortest distance between the lines  $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{k})$  and

$\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$ . Determine whether these lines intersect or not.

**Answer**

**Given equations :**

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$$

**To Find : d**

**Formula :**

#### 1. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

## 2. Dot Product :

If  $\bar{a}$  &  $\bar{b}$  are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \cdot \bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

## 3. Shortest distance between two lines :

The shortest distance between the skew lines  $\bar{r} = \bar{a}_1 + \lambda\bar{b}_1$  and

$\bar{r} = \bar{a}_2 + \lambda\bar{b}_2$  is given by,

$$d = \left| \frac{(\bar{b}_1 \times \bar{b}_2) \cdot (\bar{a}_2 - \bar{a}_1)}{|\bar{b}_1 \times \bar{b}_2|} \right|$$

## Answer :

For given lines,

$$\bar{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{k})$$

$$\bar{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$$

Here,

$$\bar{a}_1 = \hat{i} - \hat{j}$$

$$\bar{b}_1 = 2\hat{i} - \hat{k}$$

$$\bar{a}_2 = 2\hat{i} - \hat{j}$$

$$\bar{b}_2 = \hat{i} - \hat{j} - \hat{k}$$

Therefore,

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= \hat{i}(0 - 1) - \hat{j}(-2 + 1) + \hat{k}(-2 - 0)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + \hat{j} - 2\hat{k}$$

$$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-1)^2 + 1^2 + (-2)^2}$$

$$= \sqrt{1 + 1 + 4}$$

$$= \sqrt{6}$$

$$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (-1 + 1)\hat{j} + (0 - 0)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 0\hat{j} + 0\hat{k}$$

Now,

$$(\overline{b_1} \times \overline{b_2}) \cdot (\overline{a_2} - \overline{a_1}) = (-\hat{i} + \hat{j} - 2\hat{k}) \cdot (\hat{i} + 0\hat{j} + 0\hat{k})$$

$$= ((-1) \times 1) + (1 \times 0) + ((-2) \times 0)$$

$$= -1 + 0 + 0$$

$$= -1$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) \cdot (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{-1}{\sqrt{6}} \right|$$

$$\therefore d = \frac{1}{\sqrt{6}}$$

$$\therefore d = \frac{\sqrt{6}}{6}$$

$$d = \frac{\sqrt{6}}{6} \text{ units}$$

As  $d \neq 0$

Hence, the given lines do not intersect.

### 10. Question

Show that the lines  $\vec{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda (2\hat{i} - 7\hat{j} + 5\hat{k})$ , and  $\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu (2\hat{i} + \hat{j} - 3\hat{k})$  do not intersect.

**Answer**

**Given equations :**  
CAREER POINT

$$\vec{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$$

$$\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$$

**To Find : d**

**Formula :**

### 1. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

### 2. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 3. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$  and

$\vec{r} = \vec{a_2} + \lambda\vec{b_2}$  is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

**Answer :**

For given lines,

$$\vec{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$$

$$\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$$

Here,

$$\vec{a_1} = 3\hat{i} - 15\hat{j} + 9\hat{k}$$

$$\vec{b_1} = 2\hat{i} - 7\hat{j} + 5\hat{k}$$

$$\vec{a_2} = -\hat{i} + \hat{j} + 9\hat{k}$$

$$\vec{b_2} = 2\hat{i} + \hat{j} - 3\hat{k}$$

Therefore,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= \hat{i}(21 - 5) - \hat{j}(-6 - 10) + \hat{k}(2 + 14)$$

$$\therefore \vec{b_1} \times \vec{b_2} = 17\hat{i} + 16\hat{j} + 16\hat{k}$$

$$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{17^2 + 16^2 + 16^2}$$

$$= \sqrt{289 + 256 + 289}$$

$$= \sqrt{834}$$

$$\vec{a_2} - \vec{a_1} = (-1 - 3)\hat{i} + (1 + 15)\hat{j} + (9 - 9)\hat{k}$$

$$\therefore \vec{a_2} - \vec{a_1} = -4\hat{i} + 16\hat{j} + 0\hat{k}$$

Now,

$$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (17\hat{i} + 16\hat{j} + 16\hat{k}) \cdot (-4\hat{i} + 16\hat{j} + 0\hat{k})$$

$$= (17 \times (-4)) + (16 \times 16) + (16 \times 0)$$

$$= -68 + 256 + 0$$

$$= 188$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$\therefore d = \left| \frac{188}{\sqrt{834}} \right|$$

$$\therefore d = \frac{188}{\sqrt{834}} \text{ units}$$

As  $d \neq 0$

Hence, the given lines do not intersect.

## 11. Question

Show that the lines  $\vec{r} = (2\hat{i} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$  and  $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$  intersect.

Also, find their point of intersection.

**Answer**

**Given equations :**

$$\vec{r} = (2\hat{i} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines  $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$  and

$\vec{r} = \vec{a_2} + \lambda\vec{b_2}$  is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

**Answer :**

For given lines,

$$\vec{r} = (2\hat{i} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$$

Here,

$$\vec{a}_1 = 2\hat{i} - 3\hat{k}$$

$$\vec{b}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a}_2 = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Therefore,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(12 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = 3\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{3^2 + 2^2 + (-1)^2}$$

$$= \sqrt{9 + 4 + 1}$$

$$= \sqrt{14}$$

$$\vec{a}_2 - \vec{a}_1 = (2 - 2)\hat{i} + (6 - 0)\hat{j} + (3 + 3)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 0\hat{i} + 6\hat{j} + 6\hat{k}$$

Now,

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (3\hat{i} + 2\hat{j} - \hat{k}) \cdot (0\hat{i} + 6\hat{j} + 6\hat{k})$$

$$= (3 \times 0) + (2 \times 6) + ((-1) \times 6)$$

$$= 0 + 12 - 6$$

$$= 6$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{6}{\sqrt{14}} \right|$$

$$\therefore d = \frac{6}{\sqrt{14}} \text{ units}$$

As  $d \neq 0$

Hence, the given lines do not intersect.

## 12. Question

Show that the lines  $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (2\hat{i} + 3\hat{j} + 4\hat{k})$  and  $\vec{r} = (4\hat{i} + \hat{j}) + \mu (5\hat{i} + 2\hat{j} + \hat{k})$  intersect.

Also, find their point of intersection.

## Answer

**Given equations :**

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

**To Find : d**

**Formula :**

### 1. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

### 2. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 3. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$  and

$\vec{r} = \vec{a_2} + \lambda \vec{b_2}$  is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

**Answer :**

For given lines,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

Here,

$$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{a_2} = 4\hat{i} + \hat{j}$$

$$\vec{b_2} = 5\hat{i} + 2\hat{j} + \hat{k}$$

Therefore,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(3 - 8) - \hat{j}(2 - 20) + \hat{k}(4 - 15)$$

$$\therefore \vec{b_1} \times \vec{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$$

$$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$$

$$= \sqrt{25 + 324 + 121}$$

$$= \sqrt{470}$$

$$\vec{a_2} - \vec{a_1} = (4 - 1)\hat{i} + (1 - 2)\hat{j} + (0 - 3)\hat{k}$$

$$\therefore \vec{a_2} - \vec{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$$

Now,

$$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (-5\hat{i} + 18\hat{j} - 11\hat{k}) \cdot (3\hat{i} - \hat{j} - 3\hat{k})$$

$$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$$

$$= -15 - 18 + 33$$

$$= 0$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{0}{\sqrt{470}} \right|$$

$$\therefore d = 0 \text{ units}$$

$$\text{As } d = 0$$

Hence, the given lines not intersect each other.

Now, to find point of intersection, let us convert given vector equations into Cartesian equations.

For that substituting  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  in given equations,

$$\therefore L1 : x\hat{i} + y\hat{j} + z\hat{k} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\therefore L2 : x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

$$\therefore L1 : (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$$

$$\therefore L2 : (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$$

$$\therefore L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\therefore L2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$$

General point on L1 is

$$x_1 = 2\lambda + 1, y_1 = 3\lambda + 2, z_1 = 4\lambda + 3$$

let,  $P(x_1, y_1, z_1)$  be point of intersection of two given lines.

Therefore, point P satisfies equation of line L2.

$$\therefore \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = \frac{4\lambda + 3 - 0}{1}$$

$$\therefore \frac{2\lambda - 3}{5} = \frac{3\lambda + 1}{2}$$

$$\Rightarrow 4\lambda - 6 = 15\lambda + 5$$

$$\Rightarrow 11\lambda = -11$$

$$\Rightarrow \lambda = -1$$

Therefore,  $x_1 = 2(-1)+1$  ,  $y_1 = 3(-1)+2$  ,  $z_1 = 4(-1)+3$

$$\Rightarrow x_1 = -1 , y_1 = -1 , z_1 = -1$$

Hence point of intersection of given lines is  $(-1, -1, -1)$ .

### 13. Question

Find the shortest distance between the lines  $L_1$  and  $L_2$  whose vector equations are

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ and } \vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k}).$$

**HINT:** The given lines are parallel.

**Answer**

**Given equations :**

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

**To Find :** d

**Formula :**

#### 1. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

#### 2. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

#### 3. Shortest distance between two parallel lines :

The shortest distance between the parallel lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}$  and

$\bar{r} = \bar{a}_2 + \lambda \bar{b}$  is given by,

$$d = \left| \frac{|(\bar{a}_2 - \bar{a}_1) \times \bar{b}|}{|\bar{b}|} \right|$$

**Answer :**

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Here,

$$\bar{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\bar{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\bar{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\bar{b}_2 = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

As  $\bar{b}_1 = \bar{b}_2 = \bar{b}$  (say), given lines are parallel to each other.

Therefore,

$$\bar{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\therefore |\bar{b}| = \sqrt{2^2 + 3^2 + 6^2}$$

$$= \sqrt{4 + 9 + 36}$$

$$= \sqrt{49}$$

$$= 7$$

$$\bar{a}_2 - \bar{a}_1 = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$$

$$\therefore \bar{a}_2 - \bar{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$$

$$(\bar{a}_2 - \bar{a}_1) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(6 + 3) - \hat{j}(12 + 2) + \hat{k}(6 - 2)$$

$$\therefore (\bar{a}_2 - \bar{a}_1) \times \bar{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$\therefore |(\bar{a}_2 - \bar{a}_1) \times \bar{b}| = \sqrt{9^2 + (-14)^2 + 4^2}$$

$$= \sqrt{81 + 196 + 16}$$

$$= \sqrt{293}$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|} \right|$$

$$\therefore d = \left| \frac{\sqrt{293}}{7} \right|$$

$$d = \frac{\sqrt{293}}{7} \text{ units}$$

#### 14. Question

Find the distance between the parallel lines  $L_1$  and  $L_2$  whose vector equations are

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}), \text{ and } \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k}).$$

**Answer**

**Given equations :**

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

**To Find : d**

**Formula :**

#### 1. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

#### 2. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 3. Shortest distance between two parallel lines :

The shortest distance between the parallel lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}$  and

$\vec{r} = \vec{a}_2 + \lambda \vec{b}$  is given by,

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$$

**Answer :**

For given lines,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

Here,

$$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\therefore |\vec{b}| = \sqrt{1^2 + (-1)^2 + 1^2}$$

$$= \sqrt{1 + 1 + 1}$$

$$= \sqrt{3}$$

$$\vec{a}_2 - \vec{a}_1 = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 3)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 4\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(-3 - 4) - \hat{j}(1 + 4) + \hat{k}(-1 + 3)$$

$$\therefore (\vec{a}_2 - \vec{a}_1) \times \vec{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\therefore |(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = \sqrt{(-7)^2 + (-5)^2 + 2^2}$$

$$= \sqrt{49 + 25 + 4}$$

$$= \sqrt{78}$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|} \right|$$

$$\therefore d = \left| \frac{\sqrt{78}}{\sqrt{3}} \right|$$

$$\therefore d = \sqrt{26}$$

$$d = \sqrt{26} \text{ units}$$

### 15. Question

Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line

$\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$ . Also, find the distance between these lines.

HINT: The given line is

$$L_1 : \vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k}).$$

The required line is

$$L_2 : \vec{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k}).$$

Now, find the distance between the parallel lines  $L_1$  and  $L_2$ .

### Answer

**Given :** point A  $\equiv (2, 3, 2)$

Equation of line :  $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$

**To Find :** i) equation of line

ii) distance d

### Formulae :

#### 1. Equation of line :

Equation of line passing through point A ( $a_1, a_2, a_3$ ) and parallel to vector  $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Where,  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

#### 2. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

### 3. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 4. Shortest distance between two parallel lines :

The shortest distance between the parallel lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}$  and

$\vec{r} = \vec{a}_2 + \lambda\vec{b}$  is given by,

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

### Answer :

As the required line is parallel to the line

$$\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$$

Therefore, the vector parallel to the required line is

$$\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

Given point A  $\equiv$  (2, 3, 2)

$$\therefore \vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

Therefore, equation of line passing through A and parallel to  $\vec{b}$  is

$$\vec{r} = \vec{a} + \mu\vec{b}$$

$$\therefore \vec{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$$

Now, to calculate distance between above line and given line,

$$\vec{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$$

$$\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$$

Here,

$$\vec{a}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = -2\hat{i} + 3\hat{j}$$

$$\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\therefore |\vec{b}| = \sqrt{2^2 + (-3)^2 + 6^2}$$

$$= \sqrt{4 + 9 + 36}$$

$$= \sqrt{49}$$

$$= 7$$

$$\vec{a}_2 - \vec{a}_1 = (-2 - 2)\hat{i} + (3 - 3)\hat{j} + (0 - 2)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = -4\hat{i} + 0\hat{j} - 2\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 0 & -2 \\ 2 & -3 & 6 \end{vmatrix}$$

$$= \hat{i}(0 - 6) - \hat{j}(-24 + 4) + \hat{k}(12 - 0)$$

$$\therefore (\vec{a}_2 - \vec{a}_1) \times \vec{b} = -6\hat{i} + 20\hat{j} + 12\hat{k}$$

$$\therefore |(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = \sqrt{(-6)^2 + 20^2 + 12^2}$$

$$= \sqrt{36 + 400 + 144}$$

$$= \sqrt{580}$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|} \right|$$

$$\therefore d = \left| \frac{\sqrt{580}}{7} \right|$$

$$\therefore d = \frac{\sqrt{580}}{7}$$

$$d = \frac{\sqrt{580}}{7} \text{ units}$$

## 16. Question

Write the vector equation of each of the following lines and hence determine the distance between them :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} \text{ and } \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}.$$

HINT: The given lines are

$$L_1 : \vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$$

$$L_2 : \vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + 2\mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Now, find the distance between the parallel lines  $L_1$  and  $L_2$ .

**Answer**

**Given :** Cartesian equations of lines

$$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

$$L_2 : \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

**To Find :** i) vector equations of given lines

ii) distance  $d$

**Formulae :**

**1. Equation of line :**

Equation of line passing through point A ( $a_1, a_2, a_3$ ) and having direction ratios ( $b_1, b_2, b_3$ ) is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\text{Where, } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\text{And } \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

**2. Cross Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

### 3. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 4. Shortest distance between two parallel lines :

The shortest distance between the parallel lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}$  and

$\vec{r} = \vec{a}_2 + \lambda\vec{b}$  is given by,

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

### Answer :

Given Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

Line L1 is passing through point (1, 2, -4) and has direction ratios (2, 3, 6)

Therefore, vector equation of line L1 is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

And

$$L2 : \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

Line L2 is passing through point (3, 3, -5) and has direction ratios (4, 6, 12)

Therefore, vector equation of line L2 is

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

$$\therefore \vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + 2\mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Now, to calculate distance between the lines,

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + 2\mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Here,

$$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

As  $\vec{b}_1 = \vec{b}_2 = \vec{b}$  (say), given lines are parallel to each other.

Therefore,

$$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\therefore |\vec{b}| = \sqrt{2^2 + 3^2 + 6^2}$$

$$= \sqrt{4 + 9 + 36}$$

$$= \sqrt{49}$$

$$= 7$$

$$\vec{a}_2 - \vec{a}_1 = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(6 + 3) - \hat{j}(12 + 2) + \hat{k}(6 - 2)$$

$$\therefore (\vec{a}_2 - \vec{a}_1) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$\therefore |(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = \sqrt{9^2 + (-14)^2 + 4^2}$$

$$= \sqrt{81 + 196 + 16}$$

$$= \sqrt{293}$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|} \right|$$

$$\therefore d = \left| \frac{\sqrt{293}}{7} \right|$$

$$d = \frac{\sqrt{293}}{7} \text{ units}$$

### 17. Question

Write the vector equation of the following lines and hence find the shortest distance between them :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}.$$

### Answer

**Given :** Cartesian equations of lines

$$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$L_2 : \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$$

**To Find :** i) vector equations of given lines

ii) distance d

### Formulae :

#### 1. Equation of line :

Equation of line passing through point A ( $a_1, a_2, a_3$ ) and having direction ratios ( $b_1, b_2, b_3$ ) is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Where,  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

#### 2. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

#### 3. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

#### 4. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$  and

$\vec{r} = \vec{a_2} + \lambda \vec{b_2}$  is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

#### Answer :

Given Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

Line L1 is passing through point (1, 2, 3) and has direction ratios (2, 3, 4)

Therefore, vector equation of line L1 is

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

And

$$L2 : \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$$

Line L2 is passing through point (2, 3, 5) and has direction ratios (3, 4, 5)

Therefore, vector equation of line L2 is

$$\vec{r} = (3\hat{i} + 3\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$$

Now, to calculate distance between the lines,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r} = (3\hat{i} + 3\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$$

Here,

$$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{a_2} = 3\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\vec{b_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$$

$$= \sqrt{1 + 4 + 1}$$

$$= \sqrt{6}$$

$$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (5 - 3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} + 2\hat{k}$$

Now,

$$(\overline{b_1} \times \overline{b_2}) \cdot (\overline{a_2} - \overline{a_1}) = (-\hat{i} + 2\hat{j} - \hat{k}) \cdot (2\hat{i} + \hat{j} + 2\hat{k})$$

$$= ((-1) \times 2) + (2 \times 1) + ((-1) \times 2)$$

$$= -2 + 2 - 2$$

$$= -2$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) \cdot (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{-2}{\sqrt{6}} \right|$$

$$\therefore d = \frac{2}{\sqrt{3} \cdot \sqrt{2}}$$

$$\therefore d = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\therefore d = \sqrt{\frac{2}{3}}$$

$$d = \sqrt{\frac{2}{3}} \text{ units}$$

## 18. Question

Find the shortest distance between the lines given below:

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} \text{ and } \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}.$$

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$$

$$L2 : \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$$

**To Find :** distance d

**Formulae :**

**1. Equation of line :**

Equation of line passing through point A ( $a_1, a_2, a_3$ ) and having direction ratios ( $b_1, b_2, b_3$ ) is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Where,  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

**2. Cross Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**3. Dot Product :**

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

#### 4. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$  and

$\vec{r} = \vec{a_2} + \lambda \vec{b_2}$  is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

**Answer :**

Given Cartesian equations of lines

$$L1 : \frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$$

Line L1 is passing through point (1, -2, 3) and has direction ratios (-1, 1, -2)

Therefore, vector equation of line L1 is

$$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

And

$$L2 : \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$$

Line L2 is passing through point (1, -1, -1) and has direction ratios (2, 2, -2)

Therefore, vector equation of line L2 is

$$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$$

Now, to calculate distance between the lines,

$$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

$$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$$

Here,

$$\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{a_2} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{b_2} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

Therefore,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 2 & 2 & -2 \end{vmatrix}$$

$$= \hat{i}(-2 + 4) - \hat{j}(2 + 4) + \hat{k}(-2 - 2)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = 2\hat{i} - 6\hat{j} - 4\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{2^2 + (-6)^2 + (-4)^2}$$

$$= \sqrt{4 + 36 + 16}$$

$$= \sqrt{56}$$

$$\vec{a}_2 - \vec{a}_1 = (1 - 1)\hat{i} + (-1 + 2)\hat{j} + (-1 - 3)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 0\hat{i} + \hat{j} - 4\hat{k}$$

Now,

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (2\hat{i} - 6\hat{j} - 4\hat{k}) \cdot (0\hat{i} + \hat{j} - 4\hat{k})$$

$$= (2 \times 0) + ((-6) \times 1) + ((-4) \times (-4))$$

$$= 0 - 6 + 16$$

$$= 10$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{10}{\sqrt{56}} \right|$$

$$\therefore d = \frac{10}{\sqrt{56}}$$

$$d = \frac{10}{\sqrt{56}} \text{ units}$$

### 19. Question

Find the shortest distance between the lines given below:

$$\frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2} \text{ and } \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-25}{3}.$$

**HINT:** Change the given equations in vector form.

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2}$$

$$L2 : \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-23}{3}$$

**To Find :** distance d

**Formulae :**

### 1. Equation of line :

Equation of line passing through point A ( $a_1, a_2, a_3$ ) and having direction ratios ( $b_1, b_2, b_3$ ) is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Where,  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

### 2. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

### 3. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 4. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and

$\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$  is given by,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

**Answer :**

Given Cartesian equations of lines

$$L_1 : \frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2}$$

Line L1 is passing through point (12, 1, 5) and has direction ratios (-9, 4, 2)

Therefore, vector equation of line L1 is

$$\vec{r} = (12\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-9\hat{i} + 4\hat{j} + 2\hat{k})$$

And

$$L_2 : \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-23}{3}$$

Line L2 is passing through point (23, 10, 23) and has direction ratios (-6, -4, 3)

Therefore, vector equation of line L2 is

$$\vec{r} = (23\hat{i} + 10\hat{j} + 23\hat{k}) + \mu(-6\hat{i} - 4\hat{j} + 3\hat{k})$$

Now, to calculate distance between the lines,

$$\vec{r} = (12\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-9\hat{i} + 4\hat{j} + 2\hat{k})$$

$$\vec{r} = (23\hat{i} + 10\hat{j} + 23\hat{k}) + \mu(-6\hat{i} - 4\hat{j} + 3\hat{k})$$

Here,

$$\vec{a}_1 = 12\hat{i} + \hat{j} + 5\hat{k}$$

$$\vec{b}_1 = -9\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = 23\hat{i} + 10\hat{j} + 23\hat{k}$$

$$\vec{b}_2 = -6\hat{i} - 4\hat{j} + 3\hat{k}$$

Therefore,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -9 & 4 & 2 \\ -6 & -4 & 3 \end{vmatrix}$$

$$= \hat{i}(12 + 8) - \hat{j}(-27 + 12) + \hat{k}(36 + 24)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = 20\hat{i} + 15\hat{j} + 60\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{20^2 + 15^2 + 60^2}$$

$$= \sqrt{400 + 225 + 3600}$$

$$= \sqrt{4225}$$

$$= 65$$

$$\vec{a_2} - \vec{a_1} = (23 - 12)\hat{i} + (10 - 1)\hat{j} + (23 - 5)\hat{k}$$

$$\therefore \vec{a_2} - \vec{a_1} = 11\hat{i} + 9\hat{j} + 18\hat{k}$$

Now,

$$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (20\hat{i} + 15\hat{j} + 60\hat{k}) \cdot (11\hat{i} + 9\hat{j} + 18\hat{k})$$

$$= (20 \times 11) + (15 \times 9) + (60 \times 18)$$

$$= 220 + 135 + 1080$$

$$= 1435$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$\therefore d = \left| \frac{1435}{65} \right|$$

$$\therefore d = \frac{287}{13}$$

$$d = \frac{287}{13} \text{ units}$$

## Exercise 27E

### 1. Question

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x-3}{3} = \frac{y-8}{-1} = z-3 \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}.$$

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

$$L2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

**Formulae :**

### 1. Condition for perpendicularity :

If line L1 has direction ratios  $(a_1, a_2, a_3)$  and that of line L2 are  $(b_1, b_2, b_3)$  then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

## 2. Distance formula :

Distance between two points  $A \equiv (a_1, a_2, a_3)$  and  $B \equiv (b_1, b_2, b_3)$  is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

## 3. Equation of line :

Equation of line passing through points  $A \equiv (x_1, y_1, z_1)$  and  $B \equiv (x_2, y_2, z_2)$  is given by,

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

## Answer :

Given equations of lines

$$L1 : \frac{x - 3}{3} = \frac{y - 8}{-1} = \frac{z - 3}{1}$$

$$L2 : \frac{x + 3}{-3} = \frac{y + 7}{2} = \frac{z - 6}{4}$$

Direction ratios of L1 and L2 are  $(3, -1, 1)$  and  $(-3, 2, 4)$  respectively.

Let, general point on line L1 is  $P \equiv (x_1, y_1, z_1)$

$$x_1 = 3s + 3, y_1 = -s + 8, z_1 = s + 3$$

and let, general point on line L2 is  $Q \equiv (x_2, y_2, z_2)$

$$x_2 = -3t - 3, y_2 = 2t - 7, z_2 = 4t + 6$$

$$\begin{aligned} \therefore \overline{PQ} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ &= (-3t - 3 - 3s - 3)\hat{i} + (2t - 7 + s - 8)\hat{j} + (4t + 6 - s - 3)\hat{k} \\ \therefore \overline{PQ} &= (-3t - 3s - 6)\hat{i} + (2t + s - 15)\hat{j} + (4t - s + 3)\hat{k} \end{aligned}$$

Direction ratios of  $\overline{PQ}$  are  $((-3t - 3s - 6), (2t + s - 15), (4t - s + 3))$

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

$$3(-3t - 3s - 6) - 1(2t + s - 15) + 1(4t - s + 3) = 0 \text{ and}$$

$$-3(-3t - 3s - 6) + 2(2t + s - 15) + 4(4t - s + 3) = 0$$

$$\Rightarrow -9t - 9s - 18 - 2t - s + 15 + 4t - s + 3 = 0 \text{ and}$$

$$9t + 9s + 18 + 4t + 2s - 30 + 16t - 4s + 12 = 0$$

$$\Rightarrow -7t - 11s = 0 \text{ and}$$

$$29t + 7s = 0$$

Solving above two equations, we get,

$$t = 0 \text{ and } s = 0$$

therefore,

$$P \equiv (3, 8, 3) \text{ and } Q \equiv (-3, -7, 6)$$

Now, distance between points P and Q is

$$\begin{aligned} d &= \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2} \\ &= \sqrt{(6)^2 + (15)^2 + (-3)^2} \\ &= \sqrt{36 + 225 + 9} \\ &= \sqrt{270} \\ &= 3\sqrt{30} \end{aligned}$$

Therefore, the shortest distance between two given lines is

$$d = 3\sqrt{30} \text{ units}$$

Now, equation of line passing through points P and Q is,

$$\begin{aligned} \frac{x-x_1}{x_1-x_2} &= \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2} \\ \therefore \frac{x-3}{3+3} &= \frac{y-8}{8+7} = \frac{z-3}{3-6} \\ \therefore \frac{x-3}{6} &= \frac{y-8}{15} = \frac{z-3}{-3} \\ \therefore \frac{x-3}{2} &= \frac{y-8}{5} = \frac{z-3}{-1} \end{aligned}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

## 2. Question

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1} \text{ and } \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}.$$

## Answer

**Given :** Cartesian equations of lines

$$L1 : \frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$$

$$L2 : \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$$

**Formulae :**

### 1. Condition for perpendicularity :

If line L1 has direction ratios  $(a_1, a_2, a_3)$  and that of line L2 are  $(b_1, b_2, b_3)$  then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

### 2. Distance formula :

Distance between two points  $A \equiv (a_1, a_2, a_3)$  and  $B \equiv (b_1, b_2, b_3)$  is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

### 3. Equation of line :

Equation of line passing through points  $A \equiv (x_1, y_1, z_1)$  and  $B \equiv (x_2, y_2, z_2)$  is given by,

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

**Answer :**

Given equations of lines

$$L1 : \frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$$

$$L2 : \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$$

Direction ratios of L1 and L2 are  $(-1, 2, 1)$  and  $(1, 3, 2)$  respectively.

Let, general point on line L1 is  $P \equiv (x_1, y_1, z_1)$

$$x_1 = -s+3, y_1 = 2s+4, z_1 = s-2$$

and let, general point on line L2 is  $Q \equiv (x_2, y_2, z_2)$

$$x_2 = t+1, y_2 = 3t-7, z_2 = 2t-2$$

$$\therefore \overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= (t+1+s-3)\hat{i} + (3t-7-2s-4)\hat{j} + (2t-2-s+2)\hat{k}$$

$$\therefore \overrightarrow{PQ} = (t + s - 2)\hat{i} + (3t - 2s - 11)\hat{j} + (2t - s)\hat{k}$$

Direction ratios of  $\overrightarrow{PQ}$  are  $((t + s - 2), (3t - 2s - 11), (2t - s))$

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

$$-1(t + s - 2) + 2(3t - 2s - 11) + 1(2t - s) = 0 \text{ and}$$

$$1(t + s - 2) + 3(3t - 2s - 11) + 2(2t - s) = 0$$

$$\Rightarrow -t - s + 2 + 6t - 4s - 22 + 2t - s = 0 \text{ and}$$

$$t + s - 2 + 9t - 6s - 33 + 4t - 2s = 0$$

$$\Rightarrow 7t - 6s = 20 \text{ and}$$

$$14t - 7s = 35$$

Solving above two equations, we get,

$$t = 2 \text{ and } s = -1$$

therefore,

$$P \equiv (4, 2, -3) \text{ and } Q \equiv (3, -1, 2)$$

Now, distance between points P and Q is

$$\begin{aligned} d &= \sqrt{(4-3)^2 + (2+1)^2 + (-3-2)^2} \\ &= \sqrt{(1)^2 + (3)^2 + (-5)^2} \\ &= \sqrt{1+9+25} \\ &= \sqrt{35} \end{aligned}$$

Therefore, the shortest distance between two given lines is

$$d = \sqrt{35} \text{ units}$$

Now, equation of line passing through points P and Q is,

$$\begin{aligned} \frac{x-x_1}{x_1-x_2} &= \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2} \\ \therefore \frac{x-4}{4-3} &= \frac{y-2}{2+1} = \frac{z+3}{-3-2} \\ \therefore \frac{x-4}{1} &= \frac{y-2}{3} = \frac{z+3}{-5} \\ \therefore \frac{x-4}{-1} &= \frac{y-2}{-3} = \frac{z+3}{5} \end{aligned}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-4}{-1} = \frac{y-2}{-3} = \frac{z+3}{5}$$

### 3. Question

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and } \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

### Answer

**Given :** Cartesian equations of lines

$$L1 : \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$

$$L2 : \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$$

### Formulae :

#### 1. Condition for perpendicularity :

If line L1 has direction ratios  $(a_1, a_2, a_3)$  and that of line L2 are  $(b_1, b_2, b_3)$  then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

#### 2. Distance formula :

Distance between two points  $A \equiv (a_1, a_2, a_3)$  and  $B \equiv (b_1, b_2, b_3)$  is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

#### 3. Equation of line :

Equation of line passing through points  $A \equiv (x_1, y_1, z_1)$  and  $B \equiv (x_2, y_2, z_2)$  is given by,

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

### Answer :

Given equations of lines

$$L1 : \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$

$$L2 : \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$$

Direction ratios of L1 and L2 are  $(2, 1, -3)$  and  $(2, -7, 5)$  respectively.

Let, general point on line L1 is  $P \equiv (x_1, y_1, z_1)$

$$x_1 = 2s-1, y_1 = s+1, z_1 = -3s+9$$

and let, general point on line L2 is  $Q \equiv (x_2, y_2, z_2)$

$$x_2 = 2t+3, y_2 = -7t-15, z_2 = 5t+9$$

$$\begin{aligned}\therefore \overline{PQ} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ &= (5t+9-2s+1)\hat{i} + (-7t-15-s-1)\hat{j} + (5t+9+3s-9)\hat{k} \\ \therefore \overline{PQ} &= (5t-2s+10)\hat{i} + (-7t-s-16)\hat{j} + (5t+3s)\hat{k}\end{aligned}$$

Direction ratios of  $\overline{PQ}$  are  $((5t-2s+10), (-7t-s-16), (5t+3s))$

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

$$2(5t-2s+10) + 1(-7t-s-16) - 3(5t+3s) = 0 \text{ and}$$

$$2(5t-2s+10) - 7(-7t-s-16) + 5(5t+3s) = 0$$

$$\Rightarrow 10t - 4s + 20 - 7t - s - 16 - 15t - 9s = 0 \text{ and}$$

$$10t - 4s + 20 + 49t + 7s + 112 + 25t + 15s = 0$$

$$\Rightarrow -12t - 14s = -4 \text{ and}$$

$$84t + 18s = -132$$

Solving above two equations, we get,

$$t = -2 \text{ and } s = 2$$

therefore,

$$P \equiv (3, 3, 3) \text{ and } Q \equiv (-1, -1, -1)$$

Now, distance between points P and Q is

$$\begin{aligned}d &= \sqrt{(3+1)^2 + (3+1)^2 + (3+1)^2} \\ &= \sqrt{(4)^2 + (4)^2 + (4)^2} \\ &= \sqrt{16 + 16 + 16} \\ &= \sqrt{48} \\ &= 4\sqrt{3}\end{aligned}$$

Therefore, the shortest distance between two given lines is

$$d = 4\sqrt{3} \text{ units}$$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x - 3}{3 + 1} = \frac{y - 3}{3 + 1} = \frac{z - 3}{3 + 1}$$

$$\therefore \frac{x - 3}{4} = \frac{y - 3}{4} = \frac{z - 3}{4}$$

$$\therefore x - 3 = y - 3 = z - 3$$

$$\Rightarrow x = y = z$$

Therefore, equation of line of shortest distance between two given lines is

$$x = y = z$$

#### 4. Question

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x - 6}{3} = \frac{y - 7}{-1} = \frac{z - 4}{1} \text{ and } \frac{x}{-3} = \frac{y + 9}{2} = \frac{z - 2}{4}.$$

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x - 6}{3} = \frac{y - 7}{-1} = \frac{z - 4}{1}$$

$$L2 : \frac{x}{-3} = \frac{y + 9}{2} = \frac{z - 2}{4}$$

**Formulae :**

##### 1. Condition for perpendicularity :

If line L1 has direction ratios  $(a_1, a_2, a_3)$  and that of line L2 are  $(b_1, b_2, b_3)$  then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

##### 2. Distance formula :

Distance between two points  $A \equiv (a_1, a_2, a_3)$  and  $B \equiv (b_1, b_2, b_3)$  is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

##### 3. Equation of line :

Equation of line passing through points  $A \equiv (x_1, y_1, z_1)$  and  $B \equiv (x_2, y_2, z_2)$  is given by,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

**Answer :**

Given equations of lines

$$L1 : \frac{x - 6}{3} = \frac{y - 7}{-1} = \frac{z - 4}{1}$$

$$L2 : \frac{x}{-3} = \frac{y + 9}{2} = \frac{z - 2}{4}$$

Direction ratios of L1 and L2 are (3, -1, 1) and (-3, 2, 4) respectively.

Let, general point on line L1 is  $P \equiv (x_1, y_1, z_1)$

$$x_1 = 3s + 6, y_1 = -s + 7, z_1 = s + 4$$

and let, general point on line L2 is  $Q \equiv (x_2, y_2, z_2)$

$$x_2 = -3t, y_2 = 2t - 9, z_2 = 4t + 2$$

$$\begin{aligned} \therefore \overline{PQ} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ &= (-3t - 3s - 6)\hat{i} + (2t - 9 + s - 7)\hat{j} + (4t + 2 - s - 4)\hat{k} \\ \therefore \overline{PQ} &= (-3t - 3s - 6)\hat{i} + (2t + s - 16)\hat{j} + (4t - s - 2)\hat{k} \end{aligned}$$

Direction ratios of  $\overline{PQ}$  are  $((-3t - 3s - 6), (2t + s - 16), (4t - s - 2))$

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

$$3(-3t - 3s - 6) - 1(2t + s - 16) + 1(4t - s - 2) = 0 \text{ and}$$

$$-3(-3t - 3s - 6) + 2(2t + s - 16) + 4(4t - s - 2) = 0$$

$$\Rightarrow -9t - 9s - 18 - 2t - s + 16 + 4t - s - 2 = 0 \text{ and}$$

$$9t + 9s + 18 + 4t + 2s - 32 + 16t - 4s - 8 = 0$$

$$\Rightarrow -7t - 11s = 4 \text{ and}$$

$$29t + 7s = -22$$

Solving above two equations, we get,

$$t = 1 \text{ and } s = -1$$

therefore,

$$P \equiv (3, 8, 3) \text{ and } Q \equiv (-3, -7, 6)$$

Now, distance between points P and Q is

$$\begin{aligned}
 d &= \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2} \\
 &= \sqrt{(6)^2 + (15)^2 + (-3)^2} \\
 &= \sqrt{36 + 225 + 9} \\
 &= \sqrt{270} \\
 &= 3\sqrt{30}
 \end{aligned}$$

Therefore, the shortest distance between two given lines is

$$d = 3\sqrt{30} \text{ units}$$

Now, equation of line passing through points P and Q is,

$$\begin{aligned}
 \frac{x-x_1}{x_1-x_2} &= \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2} \\
 \therefore \frac{x-3}{3+3} &= \frac{y-8}{8+7} = \frac{z-3}{3-6} \\
 \therefore \frac{x-3}{6} &= \frac{y-8}{15} = \frac{z-3}{-3} \\
 \therefore \frac{x-3}{2} &= \frac{y-8}{5} = \frac{z-3}{-1}
 \end{aligned}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

## 5. Question

Show that the lines  $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$  and  $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$  intersect and find their point of intersection.

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$$

$$L2 : \frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$$

**To Find :** distance d

**Formulae :**

**1. Equation of line :**

Equation of line passing through point A ( $a_1, a_2, a_3$ ) and having direction ratios ( $b_1, b_2, b_3$ ) is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Where,  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

## 2. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

## 3. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

## 4. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$  and

$\vec{r} = \vec{a_2} + \lambda \vec{b_2}$  is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

## Answer :

Given Cartesian equations of lines

$$L1 : \frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$$

Line L1 is passing through point (0, 2, -3) and has direction ratios (1, 2, 3)

Therefore, vector equation of line L1 is

$$\vec{r} = (0\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

And

$$L2 : \frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$$

Line L2 is passing through point (2, 6, 3) and has direction ratios (2, 3, 4)

Therefore, vector equation of line L2 is

$$\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$$

Now, to calculate distance between the lines,

$$\vec{r} = (0\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$$

Here,

$$\vec{a}_1 = 0\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{b}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a}_2 = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Therefore,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(8 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$$

$$= \sqrt{1 + 4 + 1}$$

$$= \sqrt{6}$$

$$\vec{a}_2 - \vec{a}_1 = (2 - 0)\hat{i} + (6 - 2)\hat{j} + (3 - (-3))\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 2\hat{i} + 4\hat{j} + 6\hat{k}$$

Now,

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (-\hat{i} + 2\hat{j} - \hat{k}) \cdot (2\hat{i} + 4\hat{j} + 6\hat{k})$$

$$= ((-1) \times 2) + (2 \times 4) + ((-1) \times 6)$$

$$= -2 + 8 - 6$$

$$= 0$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{0}{\sqrt{14}} \right|$$

$$\therefore d = 0 \text{ units}$$

As  $d = 0$

Hence, given lines intersect each other.

Now, general point on  $L_1$  is

$$x_1 = \lambda, y_1 = 2\lambda + 2, z_1 = 3\lambda - 3$$

let,  $P(x_1, y_1, z_1)$  be point of intersection of two given lines.

Therefore, point  $P$  satisfies equation of line  $L_2$ .

$$\therefore \frac{\lambda - 2}{2} = \frac{2\lambda + 2 - 6}{3} = \frac{3\lambda - 3 - 3}{4}$$

$$\therefore \frac{\lambda - 2}{2} = \frac{2\lambda - 4}{3}$$

$$\Rightarrow 3\lambda - 6 = 4\lambda - 8$$

$$\Rightarrow \lambda = 2$$

Therefore,  $x_1 = 2, y_1 = 2(2) + 2, z_1 = 3(2) - 3$

$$\Rightarrow x_1 = 2, y_1 = 6, z_1 = 3$$

Hence point of intersection of given lines is  $(2, 6, 3)$ .

## 6. Question

Show that the lines  $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$  and  $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$  do not intersect each other.

## Answer

**Given :** Cartesian equations of lines

$$L_1 : \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$$

$$L2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$$

**To Find :** distance d

**Formulae :**

### 1. Equation of line :

Equation of line passing through point A ( $a_1, a_2, a_3$ ) and having direction ratios ( $b_1, b_2, b_3$ ) is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Where,  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

### 2. Cross Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

### 3. Dot Product :

If  $\vec{a}$  &  $\vec{b}$  are two vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\vec{a} \cdot \vec{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

### 4. Shortest distance between two lines :

The shortest distance between the skew lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and

$\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$  is given by,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

**Answer :**

Given Cartesian equations of lines

$$L1 : \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$$

Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Therefore, vector equation of line L1 is

$$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$$

And

$$L2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$$

Line L2 is passing through point (2, 1, -1) and has direction ratios (2, 3, -2)

Therefore, vector equation of line L2 is

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$$

Now, to calculate distance between the lines,

$$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$$

Here,

$$\vec{a}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{b}_1 = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{j} - 2\hat{k}$$

Therefore,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$$

$$= \hat{i}(-4 - 15) - \hat{j}(-6 - 10) + \hat{k}(9 - 4)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = -19\hat{i} + 16\hat{j} + 5\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-19)^2 + 16^2 + 5^2}$$

$$= \sqrt{361 + 256 + 25}$$

$$= \sqrt{642}$$

$$\vec{a_2} - \vec{a_1} = (2 - 1)\hat{i} + (1 + 1)\hat{j} + (-1 - 1)\hat{k}$$

$$\therefore \vec{a_2} - \vec{a_1} = \hat{i} + 2\hat{j} - 2\hat{k}$$

Now,

$$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (-19\hat{i} + 16\hat{j} + 5\hat{k}) \cdot (\hat{i} + 2\hat{j} - 2\hat{k})$$

$$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$$

$$= -19 + 32 - 10$$

$$= 3$$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$\therefore d = \left| \frac{3}{\sqrt{642}} \right|$$

$$\therefore d = \frac{3}{\sqrt{642}} \text{ units}$$

As  $d \neq 0$

Hence, given lines do not intersect each other.

## Exercise 27F

### 1. Question

If a line has direction ratios 2, -1, -2 then what are its direction cosines?

**Answer**

Given : A line has direction ratios 2, -1, -2

To find : Direction cosines of the line

Formula used : If  $(l, m, n)$  are the direction ratios of a given line then direction cosines are given by

$$\frac{l}{\sqrt{l^2 + m^2 + n^2}}, \frac{m}{\sqrt{l^2 + m^2 + n^2}}, \frac{n}{\sqrt{l^2 + m^2 + n^2}}$$

Here  $l = 2$ ,  $m = -1$ ,  $n = -2$

Direction cosines of the line with direction ratios 2, -1, -2 is

$$\frac{2}{\sqrt{2^2 + (-1)^2 + (-2)^2}}, \frac{-1}{\sqrt{2^2 + (-1)^2 + (-2)^2}}, \frac{-2}{\sqrt{2^2 + (-1)^2 + (-2)^2}}$$

$$= \frac{2}{\sqrt{4 + 1 + 4}}, \frac{-1}{\sqrt{4 + 1 + 4}}, \frac{-2}{\sqrt{4 + 1 + 4}} = \frac{2}{\sqrt{9}}, \frac{-1}{\sqrt{9}}, \frac{-2}{\sqrt{9}}$$

$$= \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

Direction cosines of the line with direction ratios 2, -1, -2 is  $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

## 2. Question

Find the direction cosines of the line  $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ .

### Answer

Given : A line  $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ .

To find : Direction cosines of the line

Formula used : If a line is given by  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$  then direction cosines are given by  $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is  $\frac{x-4}{-2} = \frac{y-0}{6} = \frac{z-1}{-3}$

Here  $l = -2, m = 6, n = -3$

Direction cosines of the line  $\frac{x-4}{-2} = \frac{y-0}{6} = \frac{z-1}{-3}$  is

$$\begin{aligned} & \frac{-2}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}}, \frac{6}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}}, \frac{-3}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}} \\ &= \frac{-2}{\sqrt{4+36+9}}, \frac{6}{\sqrt{4+36+9}}, \frac{-3}{\sqrt{4+36+9}} = \frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}}, \frac{-3}{\sqrt{49}} \\ &= \frac{-2}{7}, \frac{6}{7}, \frac{-3}{7} \end{aligned}$$

Direction cosines of the line  $\frac{x-4}{-2} = \frac{y-0}{6} = \frac{z-1}{-3}$  is  $\frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$

## 3. Question

If the equations of a line are  $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$ , find the direction cosines of a line parallel to the given line.

### Answer

Given : A line  $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$ ,

To find : Direction cosines of the line parallel to  $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$ ,

Formula used : If a line is given by  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$  then direction cosines are given by  $\frac{1}{\sqrt{l^2+m^2+n^2}}$ ,  $\frac{l}{\sqrt{l^2+m^2+n^2}}$ ,  $\frac{m}{\sqrt{l^2+m^2+n^2}}$ ,  $\frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is  $\frac{x-3}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$

Parallel lines have same direction ratios and direction cosines

Here  $l = 3$ ,  $m = -2$ ,  $n = 6$

Direction cosines of the line  $\frac{x-3}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$  is

$$\begin{aligned} & \frac{3}{\sqrt{(3)^2 + (-2)^2 + (6)^2}}, \frac{-2}{\sqrt{(3)^2 + (-2)^2 + (6)^2}}, \frac{6}{\sqrt{(3)^2 + (-2)^2 + (6)^2}} \\ &= \frac{3}{\sqrt{9+4+36}}, \frac{-2}{\sqrt{9+4+36}}, \frac{6}{\sqrt{9+4+36}} = \frac{3}{\sqrt{49}}, \frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}} \\ &= \frac{3}{7}, \frac{-2}{7}, \frac{6}{7} \end{aligned}$$

Direction cosines of the line parallel to the line  $\frac{x-3}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$  is

$$\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$$

#### 4. Question

Write the equations of a line parallel to the line  $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$  and passing through the point (1, -2, 3).

**Answer**

Given : A line  $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$

To find : equations of a line parallel to the line  $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$  and passing through the point (1, -2, 3).

Formula used : If a line is given by  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$  then equation of parallel

line passing through the point (p,q,r) is given by  $\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

Here  $l = -3$ ,  $m = 2$ ,  $n = 6$  and  $p = 1$ ,  $q = -2$ ,  $r = 3$

The line parallel to the line  $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$  and passing through the point (1,-2,3)

is given by

$$\frac{x-1}{-3} = \frac{y-(-2)}{2} = \frac{z-3}{6}$$

$$\frac{x-1}{-3} = \frac{y+2}{2} = \frac{z-3}{6}$$

The line parallel to the line  $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$  and passing through the point

$$(1, -2, 3) \text{ is given by } \frac{x-1}{-3} = \frac{y+2}{2} = \frac{z-3}{6}$$

### 5. Question

Find the Cartesian equations of the line which passes through the point  $(-2, 4, -5)$  and which is parallel to the line  $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ .

### Answer

Given : A line  $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ .

To find : equations of a line parallel to the line  $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ .

and passing through the point  $(-2, 4, -5)$ .

Formula used : If a line is given by  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$  then equation of parallel

line passing through the point  $(p, q, r)$  is given by  $\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

The given line is  $\frac{x+3}{3} = \frac{y-4}{-5} = \frac{z+8}{6}$

Here  $l = 3$ ,  $m = -5$ ,  $n = 6$  and  $p = -2$ ,  $q = 4$ ,  $r = -5$

The line parallel to the line  $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$  and passing through the point

$(-2, 4, -5)$  is given by

$$\frac{x-(-2)}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$$

$$\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$$

The line parallel to the line  $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$  and passing through the point

$$(-2, 4, -5) \text{ is given by } \frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$$

## 6. Question

Write the vector equation of a line whose Cartesian equations are  $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$ .

### Answer

Given : A line  $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$ .

To find : vector equation of a line  $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$ .

Formula used : If a line is given by  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n} = \lambda$  then vector equation of the line is given by  $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda (l\vec{i} + m\vec{j} + n\vec{k})$

Here  $a = 5$  ,  $b = -4$  ,  $c = 6$  and  $l = 3$  ,  $m = 7$  ,  $n = -2$

Substituting the above values, we get

$$\vec{r} = 5\vec{i} - 4\vec{j} + 6\vec{k} + \lambda (3\vec{i} + 7\vec{j} - 2\vec{k})$$

The vector equation of a line  $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$  is given by

$$\vec{r} = 5\vec{i} - 4\vec{j} + 6\vec{k} + \lambda (3\vec{i} + 7\vec{j} - 2\vec{k})$$

## 7. Question

The Cartesian equations of a line are  $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$ . Write the vector equation of the line.

### Answer

Given : A line  $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$ .

To find : vector equation of a line  $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$ .

Formula used : If a line is given by  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n} = \lambda$  then vector equation of the line is given by  $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda (l\vec{i} + m\vec{j} + n\vec{k})$

The given line is  $\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$

Here  $a = 3$  ,  $b = -4$  ,  $c = 3$  and  $l = -5$  ,  $m = 7$  ,  $n = 2$

Substituting the above values, we get

$$\vec{r} = 3\vec{i} - 4\vec{j} + 3\vec{k} + \lambda (-5\vec{i} + 7\vec{j} + 2\vec{k})$$

The vector equation of a line is given by  $\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$

$$\vec{r} = 3\vec{i} - 4\vec{j} + 3\vec{k} + \lambda (-5\vec{i} + 7\vec{j} + 2\vec{k})$$

### 8. Question

Write the vector equation of a line passing through the point (1, -1, 2) and parallel to the line whose

equations are  $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$ .

### Answer

Given : A line  $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$ .

To find : vector equation of a line passing through the point (1, -1, 2) and parallel

to the line whose equations are  $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$ .

Formula used : If a line is parallel to  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$  and passing through the point (p,q,r) then vector equation of the line is given by  $\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda (l\vec{i} + m\vec{j} + n\vec{k})$

The given line is  $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$

Here p = 1 , q = -1 , c = 2 and l = 1 , m = 2 , n = 2

Substituting the above values, we get

$$\vec{r} = 1\vec{i} - 1\vec{j} + 2\vec{k} + \lambda (1\vec{i} + 2\vec{j} + 2\vec{k})$$

The vector equation of a line passing through the point (1, -1, 2) and

parallel to the line whose equations are  $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$ , is given by

$$\vec{r} = \vec{i} - \vec{j} + 2\vec{k} + \lambda (\vec{i} + 2\vec{j} + 2\vec{k})$$

### 9. Question

If P(1, 5, 4) and Q(4, 1, -2) be two given points, find the direction ratios of PQ.

### Answer

Given : P(1, 5, 4) and Q(4, 1, -2) be two given points

To find : direction ratios of PQ

Formula used : if  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  be two given points then direction

ratios of PQ is given by  $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$$x_1 = 1, y_1 = 5, z_1 = 4 \text{ and } x_2 = 4, y_2 = 1, z_2 = -2$$

Direction ratios of PQ is given by  $x_2 - x_1, y_2 - y_1, z_2 - z_1$

Direction ratios of PQ is given by  $4 - 1, 1 - 5, -2 - 4$

Direction ratios of PQ is given by  $3, -4, -6$

Direction ratios of PQ is given by  $3, -4, -6$

## 10. Question

The equations of a line are  $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$ . Find the direction cosines of a line parallel to this line.

## Answer

Given : A line  $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$ .

To find : Direction cosines of the line parallel to  $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$ .

Formula used : If a line is given by  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$  then direction cosines are given by  $\frac{1}{\sqrt{l^2 + m^2 + n^2}}, \frac{l}{\sqrt{l^2 + m^2 + n^2}}, \frac{m}{\sqrt{l^2 + m^2 + n^2}}, \frac{n}{\sqrt{l^2 + m^2 + n^2}}$

The line is  $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$

Parallel lines have same direction ratios and direction cosines

Here  $l = -2, m = 2, n = 1$

Direction cosines of the line  $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$  is

$$\begin{aligned} & \frac{-2}{\sqrt{(-2)^2 + (2)^2 + (1)^2}}, \frac{2}{\sqrt{(-2)^2 + (2)^2 + (1)^2}}, \frac{1}{\sqrt{(-2)^2 + (2)^2 + (1)^2}} \\ &= \frac{-2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{1}{\sqrt{4+4+1}} = \frac{-2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}} \\ &= \frac{-2}{3}, \frac{2}{3}, \frac{1}{3} \end{aligned}$$

Direction cosines of the line parallel to the line  $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$  is

$$\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$$

### 11. Question

The Cartesian equations of a line are  $\frac{x-1}{2} = \frac{y+2}{3} = \frac{5-z}{1}$ . Find its vector equation.

#### Answer

Given : A line  $\frac{x-1}{2} = \frac{y+2}{3} = \frac{5-z}{1}$ .

To find : vector equation of a line  $\frac{x-1}{2} = \frac{y+2}{3} = \frac{5-z}{1}$ .

Formula used : If a line is given by  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n} = \lambda$  then vector equation of the line is given by  $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda (l\vec{i} + m\vec{j} + n\vec{k})$

The given line is  $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1}$

Here  $a = 1$ ,  $b = -2$ ,  $c = 5$  and  $l = 2$ ,  $m = 3$ ,  $n = -1$

Substituting the above values, we get

$$\vec{r} = 1\vec{i} - 2\vec{j} + 5\vec{k} + \lambda (2\vec{i} + 3\vec{j} - 1\vec{k})$$

The vector equation of a line  $\frac{x-1}{2} = \frac{y+2}{3} = \frac{5-z}{1}$  is given by

$$\vec{r} = 1\vec{i} - 2\vec{j} + 5\vec{k} + \lambda (2\vec{i} + 3\vec{j} - 1\vec{k})$$

### 12. Question

Find the vector equation of a line passing through the point  $(1, 2, 3)$  and parallel to the vector  $(3\hat{i} + 2\hat{j} - 2\hat{k})$ .

#### Answer

Given : A vector  $(3\hat{i} + 2\hat{j} - 2\hat{k})$ .

To find : vector equation of a line passing through the point  $(1, 2, 3)$  and parallel to the vector  $(3\hat{i} + 2\hat{j} - 2\hat{k})$ .

Formula used : If a line is parallel to the vector  $(l\vec{i} + m\vec{j} + n\vec{k})$

and passing through the point  $(p, q, r)$  then vector equation of the line is given by

$$\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda (l\vec{i} + m\vec{j} + n\vec{k})$$

Here  $p = 1$ ,  $q = 2$ ,  $c = 3$  and  $l = 3$ ,  $m = 2$ ,  $n = -2$

Substituting the above values, we get

$$\vec{r} = 1\vec{i} + 2\vec{j} + 3\vec{k} + \lambda (3\vec{i} + 2\vec{j} - 2\vec{k})$$

The vector equation of a line passing through the point  $(1, 2, 3)$  and

parallel to the vector  $(3\hat{i} + 2\hat{j} - 2\hat{k})$  is  $\vec{r} = \vec{i} + 2\vec{j} + 3\vec{k} + \lambda (3\vec{i} + 2\vec{j} - 2\vec{k})$

### 13. Question

The vector equation of a line is  $\vec{r} = (2\hat{i} + \hat{j} - 4\hat{k}) + \lambda(\hat{i} - \hat{j} - \hat{k})$ . Find its Cartesian equation.

### Answer

Given : The vector equation of a line is  $\vec{r} = (2\hat{i} + \hat{j} - 4\hat{k}) + \lambda(\hat{i} - \hat{j} - \hat{k})$ .

To find : Cartesian equation of the line

Formula used : If the vector equation of the line is given by

$\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda (l\vec{i} + m\vec{j} + n\vec{k})$  then its Cartesian equation is given by

$$\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$$

The vector equation of a line is  $\vec{r} = (2\hat{i} + \hat{j} - 4\hat{k}) + \lambda(\hat{i} - \hat{j} - \hat{k})$ .

Here  $p = 2$ ,  $q = 1$ ,  $r = -4$  and  $l = 1$ ,  $m = -1$ ,  $n = -1$

Cartesian equation is given by

$$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z-(-4)}{-1}$$

$$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z+4}{-1}$$

Cartesian equation of the line is given by  $\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z+4}{-1}$

### 14. Question

Find the Cartesian equation of a line which passes through the point  $(-2, 4, -5)$  and which is parallel

to the line  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ .

### Answer

Given : A line  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ .

To find : cartesian equations of a line parallel to the line  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ .

and passing through the point  $(-2, 4, -5)$ .

Formula used : If a line is given by  $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$  then equation of parallel

line passing through the point  $(p,q,r)$  is given by  $\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

The given line is  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

Here  $l = 3$  ,  $m = 5$  ,  $n = 6$  and  $p = -2$  ,  $q = 4$  ,  $r = -5$

The line parallel to the line  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$  and passing through the point

$(-2,4,-5)$  is given by

$$\frac{x-(-2)}{3} = \frac{y-4}{5} = \frac{z-(-5)}{6}$$

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

The line parallel to the line  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$  and passing through the point

$(-2,4,-5)$  is given by  $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

### 15. Question

Find the Cartesian equation of a line which passes through the point having position vector  $(2\hat{i} - \hat{j} + 4\hat{k})$  and is in the direction of the vector  $(\hat{i} + 2\hat{j} - \hat{k})$ .

### Answer

Given : A line which passes through the point having position vector  $(2\hat{i} - \hat{j} + 4\hat{k})$

and is in the direction of the vector  $(\hat{i} + 2\hat{j} - \hat{k})$ .

To find : cartesian equations of a line

Formula used : If a line which passes through the point having position vector

$p\vec{i} + q\vec{j} + r\vec{k}$  and is in the direction of the vector  $l\vec{i} + m\vec{j} + n\vec{k}$  then its Cartesian

equation is given by

$$\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$$

A line which passes through the point having position vector  $(2\hat{i} - \hat{j} + 4\hat{k})$

and is in the direction of the vector  $(\hat{i} + 2\hat{j} - \hat{k})$ .

Here  $l = 1$ ,  $m = 2$ ,  $n = -1$  and  $p = 2$ ,  $q = -1$ ,  $r = 4$

$$\frac{x-2}{1} = \frac{y-(-1)}{2} = \frac{z-4}{-1}$$

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

The Cartesian equation of a line which passes through the point having

position vector  $(2\hat{i} - \hat{j} + 4\hat{k})$  and is in the direction of the vector  $(\hat{i} + 2\hat{j} - \hat{k})$ . is

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

## 16. Question

Find the angle between the lines  $\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$  and  $\vec{r} = (7\hat{i} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ .

## Answer

Given : the lines  $\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$  and  $\vec{r} = (7\hat{i} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ .

To find : angle between the lines

Formula used : If the lines are  $a\vec{i} + b\vec{j} + c\vec{k} + \lambda(p\vec{i} + q\vec{j} + r\vec{k})$  and  $d\vec{i} + e\vec{j} + f\vec{k} +$

$\lambda(l\vec{i} + m\vec{j} + n\vec{k})$  then the angle between the lines 'θ' is given by

$$\theta = \cos^{-1} \frac{pl + qm + rn}{\sqrt{p^2 + q^2 + r^2} \sqrt{l^2 + m^2 + n^2}}$$

the lines  $\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$  and  $\vec{r} = (7\hat{i} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ .

Here  $p = 3$ ,  $q = 2$ ,  $r = 6$  and  $l = 1$ ,  $m = 2$ ,  $n = 2$

$$\theta = \cos^{-1} \frac{3(1) + 2(2) + 6(2)}{\sqrt{3^2 + 2^2 + 6^2} \sqrt{1^2 + 2^2 + 2^2}} = \cos^{-1} \frac{3 + 4 + 12}{\sqrt{9 + 4 + 36} \sqrt{1 + 4 + 4}}$$

$$\theta = \cos^{-1} \frac{3 + 4 + 12}{\sqrt{49} \sqrt{9}} = \cos^{-1} \frac{19}{7 \times 3} = \cos^{-1} \frac{19}{21}$$

$$\theta = \cos^{-1} \frac{19}{21}$$

The angle between the lines  $\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$  and  $\vec{r} = (7\hat{i} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$  is  $\cos^{-1} \frac{19}{21}$

### 17. Question

Find the angle between the lines  $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$  and  $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$ .

### Answer

Given : the lines  $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$  and  $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$ .

To find : angle between the lines

Formula used : If the lines are  $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$  and  $\frac{x-d}{l} = \frac{y-e}{m} = \frac{z-f}{n}$

then the angle between the lines ' $\theta$ ' is given by

$$\theta = \cos^{-1} \frac{pl + qm + rn}{\sqrt{p^2 + q^2 + r^2} \sqrt{l^2 + m^2 + n^2}}$$

The lines are  $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$  and  $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$ .

Here  $p = 3$ ,  $q = 5$ ,  $r = 4$  and  $l = 1$ ,  $m = 1$ ,  $n = 2$

$$\theta = \cos^{-1} \frac{3(1) + 5(1) + 4(2)}{\sqrt{3^2 + 5^2 + 4^2} \sqrt{1^2 + 1^2 + 2^2}} = \cos^{-1} \frac{3 + 5 + 8}{\sqrt{9 + 25 + 16} \sqrt{1 + 1 + 4}}$$

$$\theta = \cos^{-1} \frac{3 + 5 + 8}{\sqrt{50} \sqrt{6}} = \cos^{-1} \frac{16}{10\sqrt{3}} = \cos^{-1} \frac{8}{5\sqrt{3}}$$

$$\theta = \cos^{-1} \frac{8\sqrt{3}}{15}$$

The angle between the lines  $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$  and  $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$ .

$$\text{is } \cos^{-1} \frac{8\sqrt{3}}{15}$$

### 18. Question

Show that the lines  $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$  and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  are at right angles.

Given : the lines  $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$  and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ .

To prove : the lines are at right angles.

Formula used : If the lines are  $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$  and  $\frac{x-c}{l} = \frac{y-d}{m} = \frac{z-e}{n}$

then the angle between the lines 'θ' is given by

$$\theta = \cos^{-1} \frac{pl + qm + rn}{\sqrt{p^2 + q^2 + r^2} \sqrt{l^2 + m^2 + n^2}}$$

The lines  $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$  and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ .

Here  $p = 7$ ,  $q = -5$ ,  $r = 1$  and  $l = 1$ ,  $m = 2$ ,  $n = 3$

$$\theta = \cos^{-1} \frac{7(1) + (-5)(2) + 1(3)}{\sqrt{7^2 + (-5)^2 + 1^2} \sqrt{1^2 + 2^2 + 3^2}} = \cos^{-1} \frac{7 - 10 + 3}{\sqrt{49 + 25 + 1} \sqrt{1 + 4 + 9}}$$

$$\theta = \cos^{-1} \frac{0}{\sqrt{75} \sqrt{14}} = \cos^{-1} 0 = 90^\circ$$

$$\theta = 90^\circ$$

The Lines  $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$  and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  are at right angles.

### 19. Question

The direction ratios of a line are 2, 6, -9. What are its direction cosines?

### Answer

Given : A line has direction ratios 2, 6, -9

To find : Direction cosines of the line

Formula used : If (l,m,n) are the direction ratios of a given line then direction cosines are given by

$$\frac{l}{\sqrt{l^2 + m^2 + n^2}}, \frac{m}{\sqrt{l^2 + m^2 + n^2}}, \frac{n}{\sqrt{l^2 + m^2 + n^2}}$$

Here  $l = 2$ ,  $m = 6$ ,  $n = -9$

Direction cosines of the line with direction ratios 2, 6, -9 is

$$\begin{aligned} & \frac{2}{\sqrt{2^2 + 6^2 + (-9)^2}}, \frac{6}{\sqrt{2^2 + 6^2 + (-9)^2}}, \frac{-9}{\sqrt{2^2 + 6^2 + (-9)^2}} \\ &= \frac{2}{\sqrt{4 + 36 + 81}}, \frac{6}{\sqrt{4 + 36 + 81}}, \frac{-9}{\sqrt{4 + 36 + 81}} = \frac{2}{\sqrt{121}}, \frac{6}{\sqrt{121}}, \frac{-9}{\sqrt{121}} \\ &= \frac{2}{11}, \frac{6}{11}, \frac{-9}{11} \end{aligned}$$

Direction cosines of the line with direction ratios 2, 6, -9 is  $\frac{2}{11}, \frac{6}{11}, \frac{-9}{11}$

## 20. Question

A line makes angles  $90^\circ$ ,  $135^\circ$  and  $45^\circ$  with the positive directions of x-axis, y-axis and z-axis respectively. what are the direction cosines of the line?

### Answer

Given : A line makes angles  $90^\circ$ ,  $135^\circ$  and  $45^\circ$  with the positive directions of x-axis, y-axis and z-axis respectively.

To find : Direction cosines of the line

Formula used : If a line makes angles  $\alpha^\circ$ ,  $\beta^\circ$  and  $\gamma^\circ$  with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by  $\cos \alpha, \cos(180^\circ - \beta), \cos(180^\circ - \gamma)$

$$\alpha = 90^\circ, \beta = 135^\circ \text{ and } \gamma = 45^\circ$$

Direction cosines of the line is

$$\cos 90^\circ, \cos(180^\circ - 135^\circ), \cos(180^\circ - 45^\circ)$$

$$\cos 90^\circ, \cos 45^\circ, \cos(135^\circ)$$

$$0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$$

Direction cosines of the line is  $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

## 21. Question

What are the direction cosines of the y-axis?

### Answer

To find : Direction cosines of the y- axis

Formula used : If a line makes angles  $\alpha^\circ$ ,  $\beta^\circ$  and  $\gamma^\circ$  with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by  $\cos \alpha, \cos \beta, \cos \gamma$

y-axis makes  $90^\circ$  with the x and z axes

$$\alpha = 90^\circ, \beta = 0^\circ \text{ and } \gamma = 90^\circ$$

Direction cosines of the line is

$$\cos 90^\circ, \cos 0^\circ, \cos 90^\circ$$

$$0, 1, 0$$

Direction cosines of the line is  $0, 1, 0$

## 22. Question

What are the direction cosines of the vector  $(2\hat{i} + \hat{j} - 2\hat{k})$ ?

**Answer**

Given : A vector  $(2\hat{i} + \hat{j} - 2\hat{k})$

To find : Direction cosines of the vector

Formula used : If a vector is  $l\vec{i} + m\vec{j} + n\vec{k}$  then direction cosines are given by  $\frac{l}{\sqrt{l^2 + m^2 + n^2}}$ ,  $\frac{m}{\sqrt{l^2 + m^2 + n^2}}$ ,  $\frac{n}{\sqrt{l^2 + m^2 + n^2}}$

Here  $l = 2$ ,  $m = 1$ ,  $n = -2$

Direction cosines of the line with direction ratios 2, 1, -2 is

$$\begin{aligned} & \frac{2}{\sqrt{2^2 + (1)^2 + (-2)^2}}, \frac{1}{\sqrt{2^2 + (1)^2 + (-2)^2}}, \frac{-2}{\sqrt{2^2 + (1)^2 + (-2)^2}} \\ &= \frac{2}{\sqrt{4 + 1 + 4}}, \frac{1}{\sqrt{4 + 1 + 4}}, \frac{-2}{\sqrt{4 + 1 + 4}} = \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}, \frac{-2}{\sqrt{9}} \\ &= \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \end{aligned}$$

Direction cosines of the vector is  $\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$

### 23. Question

What is the angle between the vector  $\vec{r} = (4\hat{i} + 8\hat{j} + \hat{k})$  and the x-axis?

**Answer**

Given : the vector  $\vec{r} = (4\hat{i} + 8\hat{j} + \hat{k})$

To find : angle between the vector and the x-axis

Formula used : If the vector  $l\vec{i} + m\vec{j} + n\vec{k}$  and x-axis then the angle between the lines ' $\theta$ ' is given by

$$\theta = \cos^{-1} \frac{l}{\sqrt{l^2 + m^2 + n^2}}$$

Here  $l = 4$ ,  $m = 8$ ,  $n = 1$

$$\theta = \cos^{-1} \frac{4}{\sqrt{4^2 + 8^2 + 1^2}} = \cos^{-1} \frac{4}{\sqrt{16 + 64 + 1}}$$

$$\theta = \cos^{-1} \frac{4}{\sqrt{81}} = \cos^{-1} \frac{4}{9}$$

$$\theta = \cos^{-1} \frac{4}{9}$$

The angle between the vector and the x-axis is  $\cos^{-1} \frac{4}{9}$

## Objective Questions

### 1. Question

The direction ratios of two lines are 3, 2, -6 and 1, 2, 2, respectively. The acute angle between these lines is

A.  $\cos^{-1} \left( \frac{5}{18} \right)$

B.  $\cos^{-1} \left( \frac{3}{20} \right)$

C.  $\cos^{-1} \left( \frac{5}{21} \right)$

D.  $\cos^{-1} \left( \frac{8}{21} \right)$

### Answer

Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be  $|\vec{a}| = 3\hat{i} + 2\hat{j} - 6\hat{k}$  and second parallel vector be  $|\vec{b}| = \hat{i} + 2\hat{j} + 2\hat{k}$ .

For the angle, we can use the formula  $\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$

For that, we need to find the magnitude of these vectors

$$|\vec{a}| = \sqrt{3^2 + 2^2 + (-6)^2}$$

$$= 7$$

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 2^2}$$

$$= 3$$

$$\cos \alpha = \frac{(3\hat{i} + 2\hat{j} - 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{7 \times 3}$$

$$\cos \alpha = \frac{3 + 4 - 12}{21}$$

$$\cos \alpha = \frac{-5}{21}$$

$$\alpha = \cos^{-1}\left(-\frac{5}{21}\right)$$

The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant.

$$\alpha = \cos^{-1}\left(\frac{5}{21}\right)$$

## 2. Question

The direction ratios of two lines are  $a, b, c$  and  $(b - c), (c - a), (a - b)$  respectively. The angle between these lines is

- A.  $\frac{\pi}{3}$
- B.  $\frac{\pi}{2}$
- C.  $\frac{\pi}{4}$
- D.  $\frac{3\pi}{4}$

## Answer

Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be  $|\vec{a}| = a\hat{i} + b\hat{j} + c\hat{k}$  and second parallel vector be

$$|\vec{b}| = (b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k}$$

For the angle, we can use the formula  $\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$

For that, we need to find the magnitude of these vectors

$$|\vec{a}| = \sqrt{a^2 + b^2 + c^2}$$

$$= \sqrt{a^2 + b^2 + c^2}$$

$$|\vec{b}| = \sqrt{(b - c)^2 + (c - a)^2 + (a - b)^2}$$

$$= \sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)}$$

$$\cos \alpha = \frac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot ((b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k})}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\cos \alpha = \frac{ab - ac + bc - ba + ca - cb}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\cos \alpha = \frac{0}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\alpha = \cos^{-1}(0)$$

$$\alpha = \frac{\pi}{2}$$

### 3. Question

The angle between the lines  $\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3}$  and  $\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4}$  is

A.  $\frac{\pi}{6}$

B.  $\frac{\pi}{3}$

C.  $\frac{\pi}{2}$

D.  $\cos^{-1}\left(\frac{3}{8}\right)$

### Answer

Direction ratio are given implies that we can write the parallel vector towards those line, lets consider first parallel vector to be  $|\vec{a}| = 2\hat{i} + 7\hat{j} - 3\hat{k}$  and second parallel vector be  $|\vec{b}| = -\hat{i} + 2\hat{j} + 4\hat{k}$ .

For the angle we can use the formula  $\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$

For that we need to find magnitude of these vectors

$$|\vec{a}| = \sqrt{3^2 + 2^2 + (7)^2}$$

$$= \sqrt{62}$$

$$|\vec{b}| = \sqrt{1 + 2^2 + 4^2}$$

$$= \sqrt{21}$$

$$\cos \alpha = \frac{(2\hat{i} + 7\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} + 4\hat{k})}{\sqrt{21} \times \sqrt{62}}$$

$$\cos \alpha = \frac{-2 + 14 - 12}{\sqrt{21} \times \sqrt{62}}$$

$$\cos \alpha = \frac{0}{\sqrt{21} \times \sqrt{62}}$$

$$\alpha = \cos^{-1} 0$$

Negative sign does not affect anything in cosine as cosine is positive in fourth quadrant

$$\alpha = \frac{\pi}{2}$$

#### 4. Question

If the lines  $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$  and  $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$  are perpendicular to each other then  $k = ?$

A.  $\frac{-5}{7}$

B.  $\frac{5}{7}$

C.  $\frac{10}{7}$

D.  $\frac{-10}{7}$

#### Answer

If the lines are perpendicular to each other then the angle between these lines will be

$\frac{\pi}{2}$ , then the cosine will be 0

$$\vec{a} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$$

$$|\vec{a}| = \sqrt{3^2 + (2k)^2 + 2^2}$$

$$= \sqrt{13 + 4k^2}$$

$$\vec{b} = 3k\hat{i} + \hat{j} - 5\hat{k}$$

$$|\vec{b}| = \sqrt{(3k)^2 + 1 + 5^2}$$

$$= \sqrt{9k^2 + 26}$$

$$\cos\left(\frac{\pi}{2}\right) = \frac{(3k\hat{i} + \hat{j} - 5\hat{k}) \cdot (-3\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$$

$$0 = \frac{-9k + 2k - 10}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$$

$$k = -\frac{10}{7}$$

### 5. Question

A line passes through the points A(2, -1, 4) and B(1, 2, -2). The equations of the line AB are

A.  $\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6}$

B.  $\frac{x+2}{-1} = \frac{y+1}{2} = \frac{z-4}{6}$

C.  $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{6}$

D. none of these

### Answer

To write the equation of a line we need a parallel vector and a fixed point through which the line is passing

$$\text{Parallel vector} = ((2-1)\hat{i} + (-1-2)\hat{j} + (4+2)\hat{k})$$

$$= \hat{i} - 3\hat{j} + 6\hat{k}$$

$$\text{Or} = -(\hat{i} - 3\hat{j} + 6\hat{k})$$

$$\text{Fixed point is } 2\hat{i} - \hat{j} + 4\hat{k}$$

Equation

$$\frac{x-2}{1} = \frac{y-(-1)}{-3} = \frac{z-4}{6}$$

$$\frac{x-2}{1} = \frac{y+1}{-3} = \frac{z-4}{6}$$

Or

$$\frac{x-2}{-1} = \frac{y+1}{3} = \frac{z-4}{-6}$$

### 6. Question

The angle between the lines  $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$  and  $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$  is

A.  $\cos^{-1}\left(\frac{3}{4}\right)$

B.  $\cos^{-1}\left(\frac{5}{6}\right)$

C.  $\cos^{-1}\left(\frac{2}{3}\right)$

D.  $\frac{\pi}{3}$

### Answer

Direction cosine of the lines are given  $2\hat{i} + 2\hat{j} + \hat{k}$  and  $4\hat{i} + \hat{j} + 8\hat{k}$

$$\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$|\vec{a}| = \sqrt{2^2 + 2^2 + 1}$$

$$|\vec{a}| = 3$$

$$\vec{b} = 4\hat{i} + \hat{j} + 8\hat{k}$$

$$|\vec{b}| = \sqrt{4^2 + 1 + 8^2}$$

$$= 9$$

$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$\cos \alpha = \frac{(2\hat{i} + 2\hat{j} + \hat{k}) \cdot (4\hat{i} + \hat{j} + 8\hat{k})}{3 \times 9}$$

$$\cos \alpha = \frac{8 + 8 + 2}{27}$$

$$\cos \alpha = \frac{2}{3}$$

### 7. Question

The angle between the lines  $\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k})$  and  $\vec{r} = (2\hat{i} - \hat{j} - 5\hat{k}) + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$  is

A.  $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

B.  $\cos^{-1}\left(\frac{6\sqrt{2}}{5}\right)$

C.  $\cos^{-1}\left(\frac{5\sqrt{3}}{8}\right)$

D.  $\cos^{-1}\left(\frac{5\sqrt{2}}{6}\right)$

### Answer

Let  $\vec{a} = \hat{i} - \hat{j} - 2\hat{k}$  and  $\vec{b} = 3\hat{i} - 5\hat{j} - 4\hat{k}$  and  $|\vec{a}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$

$$|\vec{b}| = \sqrt{3^2 + 5^2 + 4^2} = 5\sqrt{2}$$

$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$\cos \alpha = \frac{(3\hat{i} - 5\hat{j} - 4\hat{k}) \cdot (\hat{i} - \hat{j} - 2\hat{k})}{5\sqrt{2} \times \sqrt{6}}$$

$$\cos \alpha = \frac{3 + 5 + 8}{5\sqrt{12}}$$

$$\cos \alpha = \frac{8\sqrt{3}}{15}$$

### 8. Question

A line is perpendicular to two lines having direction ratios 1, -2, -2 and 0, 2, 1. The direction cosines of the line are

A.  $\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}$

B.  $\frac{2}{3}, \frac{1}{3}, \frac{-1}{3}$

C.  $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$

D. none of these

### Answer

If a line is perpendicular to two given lines we can find out the parallel vector by cross product of the given two vectors.

$$\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$$

$$\vec{b} = 2\hat{j} + \hat{k}$$

$$\vec{a} \times \vec{b} = (\hat{i} - 2\hat{j} - 2\hat{k}) \times (2\hat{j} + \hat{k})$$

$$= 2\hat{i} - \hat{j} + 2\hat{k}$$

So the direction cosines are

$$\hat{n} = \frac{1}{\sqrt{2^2 + 1 + 2^2}}$$

$$\hat{n} = \frac{1}{3}$$

Direction cosine

$$\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$$

### 9. Question

A line passes through the point A(5, -2, 4) and it is parallel to the vector  $(2\hat{i} - \hat{j} + 3\hat{k})$ . The vector equation of the line is

A.  $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(5\hat{i} - 2\hat{j} + 4\hat{k})$

B.  $\vec{r} = (5\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

C.  $\vec{r} \cdot (5\hat{i} - 2\hat{j} + 4\hat{k}) = \sqrt{14}$

D. none of these

### Answer

CAREER POINT

Fixed point is  $5\hat{i} - 2\hat{j} + 4\hat{k}$  and parallel vector is  $2\hat{i} - \hat{j} + 3\hat{k}$

Equation  $5\hat{i} - 2\hat{j} + 4\hat{k} + \alpha(2\hat{i} - \hat{j} + 3\hat{k})$

### 10. Question

The Cartesian equations of a line are  $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1}$ . Its vector equation is

A.  $\vec{r} = (-\hat{i} + 2\hat{j} - 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$

B.  $\vec{r} = (2\hat{i} + 3\hat{j} - \hat{k}) + \lambda(\hat{i} - 2\hat{j} + 5\hat{k})$

C.  $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 4\hat{k})$

D. none of these

### Answer

Fixed point  $(1, -2, 5)$  and the parallel vector is  $2\hat{i} + 3\hat{j} - \hat{k}$

Equation  $(\hat{i} - 2\hat{j} + 5\hat{k}) + \alpha(2\hat{i} + 3\hat{j} - \hat{k})$

### 11. Question

A line passes through the point A(-2, 4, -5) and is parallel to the line  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ . The vector equation of the line is

A.  $\vec{r} = (-3\hat{i} + 4\hat{j} - 8\hat{k}) + \lambda(-2\hat{i} + 4\hat{j} - 5\hat{k})$

B.  $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$

C.  $\vec{r} = (3\hat{i} + 5\hat{j} + 6\hat{k}) + \lambda(-2\hat{i} + 4\hat{j} - 5\hat{k})$

D. none of these

### Answer

Fixed point is  $-2\hat{i} + 4\hat{j} - 5\hat{k}$  and the parallel vector is  $3\hat{i} + 5\hat{j} + 6\hat{k}$

Equation is  $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$

### 12. Question

The coordinates of the point where the line through the points A(5, 1, 6) and B(3, 4, 1) crosses the yz-plane is

A.  $(0, 17, -13)$

B.  $\left(0, \frac{-17}{2}, \frac{13}{2}\right)$

C.  $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

D. none of these

### Answer

We first need to find the equation of a line passing through the two given points

taking fixed point as  $5\hat{i} + \hat{j} + 6\hat{k}$

and the parallel vector will be  $(5 - 3)\hat{i} + (1 - 4)\hat{j} + (6 - 1)\hat{k} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

equation of the line in cartesian form

$$\frac{x - 5}{2} = \frac{y - 1}{-3} = \frac{z - 6}{5}$$

Assume above equation to be equal to  $k$ , a constant

$$\frac{x - 5}{2} = \frac{y - 1}{-3} = \frac{z - 6}{5} = k$$

And  $y$ - $z$  plane have  $x$ -coordinate as zero we may get

$$\frac{0 - 5}{2} = \frac{y - 1}{-3} = \frac{z - 6}{5} = k$$

$$k = -\frac{5}{2}$$

Now we can find  $y$  and  $z$

$$\frac{y - 1}{-3} = -\frac{5}{2}$$

$$y - 1 = \frac{15}{2}$$

$$y = \frac{17}{2}$$

$$\frac{z - 6}{5} = -\frac{5}{2}$$

$$z - 6 = -\frac{25}{2}$$

$$z = -\frac{13}{2}$$

The coordinate where the line meets y-z plane is  $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$

### 13. Question

The vector equation of the x-axis is given by

A.  $\vec{r} = \hat{i}$

B.  $\vec{r} = \hat{j} + \hat{k}$

C.  $\vec{r} = \lambda \hat{i}$

D. none of these

### Answer

Vector equation need a fixed point and a parallel vector

For x-axis fixed point can be anything ranging from negative to positive including origin

And parallel vector is  $\hat{i}$

Equation would be  $\lambda \hat{i}$

### 14. Question

The Cartesian equations of a lines are  $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}$ . What is its vector equation?

A.  $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

B.  $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 2\hat{k})$

C.  $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k})$

D. none of these

### Answer

Fixed point is  $2\hat{i} - \hat{j} + 3\hat{k}$  and the vector is  $2\hat{i} + 3\hat{j} - 2\hat{k}$

Equation  $(2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 2\hat{k})$

### 15. Question

The angle between two lines having direction ratios 1, 1, 2 and  $(\sqrt{3}-1), (-\sqrt{3}-1), 4$  is

A.  $\frac{\pi}{6}$

B.  $\frac{\pi}{2}$

C.  $\frac{\pi}{3}$

D.  $\frac{\pi}{4}$

**Answer**

Let  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$  and  $\vec{b} = (\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k}$

$$|\vec{a}| = \sqrt{6} \quad |\vec{b}| = \sqrt{(4 - 2\sqrt{3}) + (4 + 2\sqrt{3}) + 16} = 2\sqrt{6}$$

$$\cos \alpha = \frac{(\hat{i} + \hat{j} + 2\hat{k}) \cdot ((\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k})}{\sqrt{6} \times 2\sqrt{6}}$$

$$\cos \alpha = \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{12}$$

$$\cos \alpha = \frac{1}{2}$$

$$\alpha = 60^\circ$$

**16. Question**

The straight line  $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{0}$  is

A. parallel to the x-axis

B. parallel to the y-axis

C. parallel to the z-axis

D. perpendicular to the z-axis

**Answer**

It is perpendicular to z-axis because  $\cos 90^\circ$  is 0 which implies that it makes  $90^\circ$  with z-axis

**17. Question**

If a line makes angles  $\alpha$ ,  $\beta$  and  $\gamma$  with the x-axis, y-axis and z-axis respectively then  $(\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma) = ?$

- A. 1
- B. 3
- C. 2
- D.  $\frac{3}{2}$

**Answer**

$$\begin{aligned}\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma &= 1 - \cos^2 \alpha + 1 - \cos^2 \beta + 1 - \cos^2 \gamma \\ &= 3 - (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma)\end{aligned}$$

$(\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma)$  is the square of the direction ratios of all three axes which is always equal to 1

$$= 3 - 1$$

$$= 2$$

### 18. Question

If  $(a_1, b_1, c_1)$  and  $(a_2, b_2, c_2)$  be the direction ratios of two parallel lines then

A.  $a_1 = a_2, b_1 = b_2, c_1 = c_2$

B.  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

C.  $a_1^2 + b_1^2 + c_1^2 = a_2^2 + b_2^2 + c_2^2$

D.  $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

**Answer**

We know that if there is two parallel lines then their direction ratios must have a relation

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

### 19. Question

If the points  $A(-1, 3, 2)$ ,  $B(-4, 2, -2)$  and  $C(5, 5, \lambda)$  are collinear then the value of  $\lambda$  is

- A. 5
- B. 7

C. 8

D. 10

**Answer**

Determinant of these point should be zero

$$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$$

$$-1(2\lambda + 10) - 3(-4\lambda + 10) + 2(-20 - 10) = 0$$

$$10\lambda - 10 - 30 - 60 = 0$$

$$\lambda = 10$$