

## 25. Product of Three Vectors

### Exercise 25A

#### Q. 1

Prove that

$$\text{i. } [\hat{i} \ \hat{j} \ \hat{k}] = [\hat{j} \ \hat{k} \ \hat{i}] = [\hat{k} \ \hat{i} \ \hat{j}] = 1$$

$$\text{ii. } [\hat{i} \ \hat{k} \ \hat{j}] = [\hat{k} \ \hat{j} \ \hat{i}] = [\hat{j} \ \hat{i} \ \hat{k}] = -1$$

Answer :

$$\text{i. } [\hat{i} \ \hat{j} \ \hat{k}] = [\hat{j} \ \hat{k} \ \hat{i}] = [\hat{k} \ \hat{i} \ \hat{j}] = 1$$

Let,  $\hat{i}, \hat{j}, \hat{k}$  be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$$\text{Magnitude of } \hat{i} \text{ is } 1 \Rightarrow |\hat{i}| = 1$$

$$\text{Magnitude of } \hat{j} \text{ is } 1 \Rightarrow |\hat{j}| = 1$$

$$\text{Magnitude of } \hat{k} \text{ is } 1 \Rightarrow |\hat{k}| = 1$$

To Prove :

$$[\hat{i} \ \hat{j} \ \hat{k}] = [\hat{j} \ \hat{k} \ \hat{i}] = [\hat{k} \ \hat{i} \ \hat{j}] = 1$$

Formulae :

a) Dot Products :

$$\text{i) } \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\text{ii) } \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

b) Cross Products :

$$\text{i) } \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$\text{ii) } \hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$$

$$\text{iii) } \hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$$

c) Scalar Triple Product :

$$[\vec{a} \ \vec{b} \ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$$

Now,

$$\begin{aligned}
 \text{(i)} \quad [\hat{i} \quad \hat{j} \quad \hat{k}] &= \hat{i} \cdot (\hat{j} \times \hat{k}) \\
 &= \hat{i} \cdot \hat{i} \quad \dots\dots\dots (\because \hat{j} \times \hat{k} = \hat{i}) \\
 &= 1 \quad \dots\dots\dots (\because \hat{i} \cdot \hat{i} = 1) \\
 \therefore [\hat{i} \quad \hat{j} \quad \hat{k}] &= 1 \quad \dots\dots\dots \text{eq(1)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad [\hat{j} \quad \hat{k} \quad \hat{i}] &= \hat{j} \cdot (\hat{k} \times \hat{i}) \\
 &= \hat{j} \cdot \hat{j} \quad \dots\dots\dots (\because \hat{k} \times \hat{i} = \hat{j}) \\
 &= 1 \quad \dots\dots\dots (\because \hat{j} \cdot \hat{j} = 1) \\
 \therefore [\hat{j} \quad \hat{k} \quad \hat{i}] &= 1 \quad \dots\dots\dots \text{eq(2)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad [\hat{k} \quad \hat{i} \quad \hat{j}] &= \hat{k} \cdot (\hat{i} \times \hat{j}) \\
 &= \hat{k} \cdot \hat{k} \quad \dots\dots\dots (\because \hat{i} \times \hat{j} = \hat{k}) \\
 &= 1 \quad \dots\dots\dots (\because \hat{k} \cdot \hat{k} = 1) \\
 \therefore [\hat{k} \quad \hat{i} \quad \hat{j}] &= 1 \quad \dots\dots\dots \text{eq(3)}
 \end{aligned}$$

From eq(1), eq(2) and eq(3),

$$[\hat{i} \quad \hat{j} \quad \hat{k}] = [\hat{j} \quad \hat{k} \quad \hat{i}] = [\hat{k} \quad \hat{i} \quad \hat{j}] = 1$$

Hence Proved.

Notes :

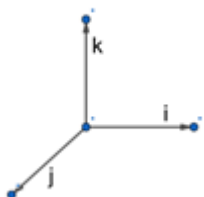
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = [\vec{b} \quad \vec{c} \quad \vec{a}] = [\vec{c} \quad \vec{a} \quad \vec{b}]$$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$$[\hat{i} \quad \hat{j} \quad \hat{k}] = 1$$

$$[\hat{k} \quad \hat{j} \quad \hat{i}] = -1$$



$$\text{ii. } [\hat{i} \quad \hat{k} \quad \hat{j}] = [\hat{k} \quad \hat{j} \quad \hat{i}] = [\hat{j} \quad \hat{i} \quad \hat{k}] = -1$$

Let,  $\hat{i}, \hat{j}, \hat{k}$  be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$$\text{Magnitude of } \hat{i} \text{ is } 1 \Rightarrow |\hat{i}| = 1$$

$$\text{Magnitude of } \hat{j} \text{ is } 1 \Rightarrow |\hat{j}| = 1$$

$$\text{Magnitude of } \hat{k} \text{ is } 1 \Rightarrow |\hat{k}| = 1$$

To Prove :

$$[\hat{i} \quad \hat{k} \quad \hat{j}] = [\hat{k} \quad \hat{j} \quad \hat{i}] = [\hat{j} \quad \hat{i} \quad \hat{k}] = -1$$

Formulae :

a) Dot Products :

$$\text{i) } \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\text{ii) } \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

b) Cross Products :

$$\text{i) } \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$\text{ii) } \hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$$

$$\text{iii) } \hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$$

c) Scalar Triple Product :

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$$

Answer :

$$\text{(i) } [\hat{i} \quad \hat{k} \quad \hat{j}] = \hat{i} \cdot (\hat{k} \times \hat{j})$$

$$= \hat{i} \cdot (-\hat{i}) \quad (\because \hat{k} \times \hat{j} = -\hat{i})$$

$$= -\hat{i} \cdot \hat{i}$$

$$= -1 \quad (\because \hat{i} \cdot \hat{i} = 1)$$

$$\therefore [\hat{i} \quad \hat{k} \quad \hat{j}] = -1 \quad \text{..... eq(1)}$$

$$\text{(ii) } [\hat{k} \quad \hat{j} \quad \hat{i}] = \hat{k} \cdot (\hat{j} \times \hat{i})$$

$$= \hat{k} \cdot (-\hat{k}) \dots\dots\dots (\because \hat{j} \times \hat{i} = -\hat{k})$$

$$= -\hat{k} \cdot \hat{k}$$

$$= -1 \dots\dots\dots (\because \hat{k} \cdot \hat{k} = 1)$$

$$\therefore [\hat{k} \quad \hat{j} \quad \hat{i}] = -1 \dots\dots\dots \text{eq(2)}$$

$$\text{(iii) } [\hat{j} \quad \hat{i} \quad \hat{k}] = \hat{j} \cdot (\hat{i} \times \hat{k})$$

$$= \hat{j} \cdot (-\hat{j}) \dots\dots\dots (\because \hat{i} \times \hat{k} = -\hat{j})$$

$$= -\hat{j} \cdot \hat{j}$$

$$= -1 \dots\dots\dots (\because \hat{j} \cdot \hat{j} = 1)$$

$$\therefore [\hat{j} \quad \hat{i} \quad \hat{k}] = -1 \dots\dots\dots \text{eq(3)}$$

From eq(1), eq(2) and eq(3),

$$[\hat{i} \quad \hat{k} \quad \hat{j}] = [\hat{k} \quad \hat{j} \quad \hat{i}] = [\hat{j} \quad \hat{i} \quad \hat{k}] = -1$$

Hence Proved.

Notes :

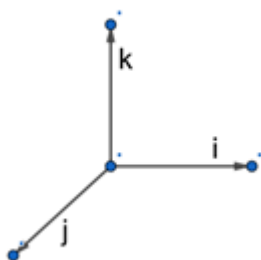
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = [\vec{b} \quad \vec{c} \quad \vec{a}] = [\vec{c} \quad \vec{a} \quad \vec{b}]$$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$$[\hat{i} \quad \hat{j} \quad \hat{k}] = 1$$

$$[\hat{k} \quad \hat{j} \quad \hat{i}] = -1$$



**Q. 2**

Find  $[\vec{a} \ \vec{b} \ \vec{c}]$ , when

- i.  $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$ .  
ii.  $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ ,  $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$  and  $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$   
iii.  $\vec{a} = 2\hat{i} - 3\hat{j}$ ,  $\vec{b} = \hat{i} + \hat{j} - \hat{k}$  and  $\vec{c} = 3\hat{i} - \hat{k}$

**Answer :**

i.  $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

Given Vectors :

1)  $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

2)  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

3)  $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

To Find :  $[\vec{a} \ \vec{b} \ \vec{c}]$

Formulae :

1) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

**Answer :**

For given vectors,

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$$

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$$= 2(2 \times 2 - 1 \times 1) - 1((-1) \times 2 - 3 \times 1) + 3((-1) \times 1 - 3 \times 2)$$

$$= 2(3) - 1(-5) + 3(-7)$$

$$= 6 + 5 - 21$$

$$= -10$$

$$\therefore [\vec{a} \quad \vec{b} \quad \vec{c}] = -10$$

ii.  $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ ,  $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$  and  $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

Given Vectors :

$$1) \vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$2) \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$$

$$3) \vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$$

To Find :  $[\vec{a} \quad \vec{b} \quad \vec{c}]$

Formulae :

1) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given vectors,

$$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= 2(2 \times 2 - (-1) \times (-1)) - (-3)(1 \times 2 - 3 \times (-1)) + 4(1 \times (-1) - 3 \times 2)$$

$$= 2(3) + 3(5) + 4(-7)$$

$$= 6 + 15 - 28$$

$$= -7$$

$$\therefore [\vec{a} \quad \vec{b} \quad \vec{c}] = -7$$

iii.  $\vec{a} = 2\hat{i} - 3\hat{j}$ ,  $\vec{b} = \hat{i} + \hat{j} - \hat{k}$  and  $\vec{c} = 3\hat{i} - \hat{k}$

Given Vectors :

1)  $\vec{a} = 2\hat{i} - 3\hat{j}$

2)  $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

3)  $\vec{c} = 3\hat{i} - \hat{k}$

To Find :  $[\vec{a} \quad \vec{b} \quad \vec{c}]$

Formulae :

1) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given vectors,

$$\vec{a} = 2\hat{i} - 3\hat{j} + 0\hat{k}$$

$$\vec{b} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{c} = 3\hat{i} + 0\hat{j} - \hat{k}$$

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

$$= 2(1 \times (-1) - (-1) \times 0) - (-3)(1 \times (-1) - 3 \times (-1)) + 0(1 \times 0 - 3 \times 1)$$

$$= 2(-1) + 3(2) + 0$$

$$= -2 + 6$$

$$= 4$$

$$\therefore [\vec{a} \quad \vec{b} \quad \vec{c}] = 4$$

### Q. 3

**Find the volume of the parallelepiped whose conterminous edges are represented by the vectors**

i.  $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

ii.  $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}, \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

iii.  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

iv.  $\vec{a} = 6\hat{i}, \vec{b} = 2\hat{j}, \vec{c} = 5\hat{k}$

**Answer :**

$$i. \quad \bar{a} = \hat{i} + \hat{j} + \hat{k}, \bar{b} = \hat{i} - \hat{j} + \hat{k}, \bar{c} = \hat{i} + 2\hat{j} - \hat{k}$$

Given :

Coterminous edges of parallelepiped are  $\bar{a}, \bar{b}, \bar{c}$  where,

$$\bar{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\bar{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\bar{c} = \hat{i} + 2\hat{j} - \hat{k}$$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If  $\bar{a}, \bar{b}, \bar{c}$  are coterminous edges of parallelepiped,

Where,

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

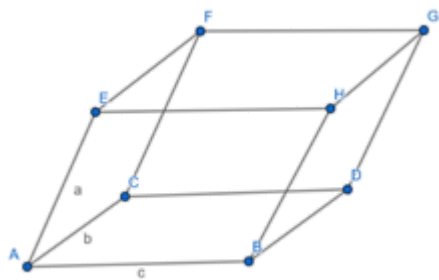
Answer :

Volume of parallelepiped with coterminous edges

$$\bar{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\bar{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\bar{c} = \hat{i} + 2\hat{j} - \hat{k}$$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 1((-1) \times (-1) - 2 \times 1) - 1(1 \times (-1) - 1 \times 1) + 1(1 \times 2 - 1 \times (-1))$$

$$= 1(-1) - 1(-2) + 1(3)$$

$$= -1 + 2 + 3$$

$$= 4$$

Therefore,

|                                         |
|-----------------------------------------|
| Volume of parallelepiped = 4 cubic unit |
|-----------------------------------------|

ii.  $\bar{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \bar{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}, \bar{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

Given :

Coterminous edges of parallelopiped are  $\bar{a}, \bar{b}, \bar{c}$  where,

$$\bar{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$$

$$\bar{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}$$

$$\bar{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If  $\vec{a}, \vec{b}, \vec{c}$  are coterminous edges of parallelepiped,

Where,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then, volume of parallelepiped V is given by,

$$V = [\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

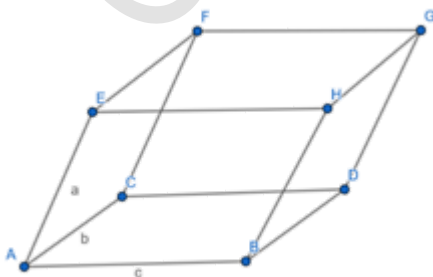
Answer :

Volume of parallelepiped with coterminous edges

$$\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$$

$$\vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}$$

$$\vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$

$$= -3(7 \times (-3) - (-5) \times (-3)) - 7((-5) \times (-3) - 7 \times (-3)) + 5((-5) \times (-5) - 7 \times 7)$$

$$= -3(-36) - 7(36) + 5(-24)$$

$$= 108 - 252 - 120$$

$$= -264$$

As volume is never negative

Therefore,

|                                           |
|-------------------------------------------|
| Volume of parallelepiped = 264 cubic unit |
|-------------------------------------------|

iii.  $\bar{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \bar{b} = 2\hat{i} + \hat{j} - \hat{k}, \bar{c} = \hat{j} + \hat{k}$

Given :

Coterminous edges of parallelepiped are  $\bar{a}, \bar{b}, \bar{c}$  where,

$$\bar{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\bar{b} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\bar{c} = \hat{j} + \hat{k}$$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If  $\bar{a}, \bar{b}, \bar{c}$  are coterminous edges of parallelepiped,

Where,

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

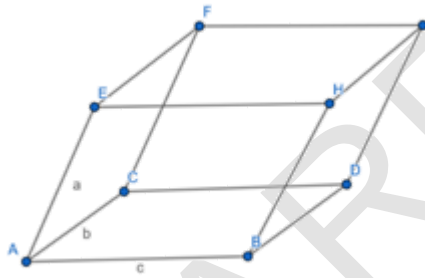
Answer :

Volume of parallelepiped with coterminous edges

$$\bar{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\bar{b} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\bar{c} = 0\hat{i} + \hat{j} + \hat{k}$$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 1(1 \times 1 - 1 \times (-1)) - (-2)(2 \times 1 - 0 \times (-1)) + 3(2 \times 1 - 0 \times 1)$$

$$= 1(2) + 2(2) + 3(2)$$

$$= 2 + 4 + 6$$

$$= 12$$

Therefore,

|                                                           |
|-----------------------------------------------------------|
| $\text{Volume of parallelepiped} = 12 \text{ cubic unit}$ |
|-----------------------------------------------------------|

$$\text{iv. } \bar{a} = 6\hat{i}, \bar{b} = 2\hat{j}, \bar{c} = 5\hat{k}$$

Given :

Coterminous edges of parallelopiped are  $\bar{a}, \bar{b}, \bar{c}$  where,

$$\bar{a} = 6\hat{i}$$

$$\bar{b} = 2\hat{j}$$

$$\bar{c} = 5\hat{k}$$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If  $\bar{a}, \bar{b}, \bar{c}$  are coterminous edges of parallelepiped,

Where,

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

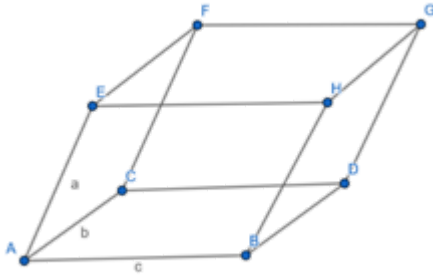
Answer :

Volume of parallelopiped with coterminous edges

$$\vec{a} = 6\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{b} = 0\hat{i} + 2\hat{j} + 0\hat{k}$$

$$\vec{c} = 0\hat{i} + 0\hat{j} + 5\hat{k}$$



$$V = [\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{vmatrix}$$

$$= 6(2 \times 5 - 0 \times 0) - 0(0 \times 5 - 0 \times 0) + 0(0 \times 0 - 0 \times 2)$$

$$= 6(10) + 0 + 0$$

$$= 60$$

Therefore,

Volume of parallelepiped = 60 cubic unit

**Q. 4**

Show that the vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar, when

i.  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$  and  $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

ii.  $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$  and  $\vec{c} = 7\hat{j} + 3\hat{k}$

iii.  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$  and  $\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

**Answer :**

i.  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$  and  $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

Given Vectors :

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$$

To Prove : Vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar.

i.e.  $[\vec{a} \ \vec{b} \ \vec{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given vectors,

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1(3 \times 5 - (-3) \times (-4)) - (-2)((-2) \times 5 - 1 \times (-4)) + 3((-2) \times (-3) - 3 \times 1)$$

$$= 1(3) + 2(-6) + 3(3)$$

$$= 3 - 12 + 9$$

$$= 0$$

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, the vectors  $\bar{a}, \bar{b}, \bar{c}$  are coplanar.

Note : For coplanar vectors  $\bar{a}, \bar{b}, \bar{c}$ ,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

$$\text{ii. } \bar{a} = \hat{i} + 3\hat{j} + \hat{k}, \bar{b} = 2\hat{i} - \hat{j} - \hat{k} \text{ and } \bar{c} = 7\hat{j} + 3\hat{k}$$

Given Vectors :

$$\bar{a} = \hat{i} + 3\hat{j} + \hat{k}$$

$$\bar{b} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\bar{c} = 7\hat{j} + 3\hat{k}$$

To Prove : Vectors  $\bar{a}, \bar{b}, \bar{c}$  are coplanar.

$$\text{i.e. } [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given vectors,

$$\bar{a} = \hat{i} + 3\hat{j} + \hat{k}$$

$$\bar{b} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\bar{c} = 7\hat{j} + 3\hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$$= 1((-1) \times 3 - 7 \times (-1)) - 3(2 \times 3 - 0 \times (-1)) + 1(2 \times 7 - 0 \times (-1))$$

$$= 1(4) - 3(6) + 1(14)$$

$$= 4 - 18 + 14$$

$$= 0$$

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, the vectors  $\bar{a}, \bar{b}, \bar{c}$  are coplanar.

Note : For coplanar vectors  $\bar{a}, \bar{b}, \bar{c}$ ,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

$$\text{iii. } \bar{a} = 2\hat{i} - \hat{j} + 2\hat{k}, \bar{b} = \hat{i} + 2\hat{j} - 3\hat{k} \text{ and } \bar{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$$

Given Vectors :

$$\bar{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\bar{b} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\bar{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$$

To Prove : Vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar.

$$\text{i.e. } [\vec{a} \ \vec{b} \ \vec{c}] = 0$$

Formulae :

1) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given vectors,

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 2 & -3 \\ 3 & -4 & 7 \end{vmatrix}$$

$$= 2(2 \times 7 - (-3) \times (-4)) - (-1)(1 \times 7 - 3 \times (-3)) + 2(1 \times (-4) - 3 \times 2)$$

$$= 2(2) + 1(16) + 2(-10)$$

$$= 4 + 16 - 20$$

$$= 0$$

$$\therefore [\vec{a} \ \vec{b} \ \vec{c}] = 0$$

Hence, the vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar.

Note : For coplanar vectors  $\vec{a}, \vec{b}, \vec{c}$ ,

$$[\vec{a} \ \vec{b} \ \vec{c}] = 0$$

**Q. 5**

Find the value of  $\lambda$  for which the vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar, when

i.  $\vec{a} = (2\hat{i} - \hat{j} + \hat{k}), \vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k})$  and  $\vec{c} = (3\hat{i} + \lambda\hat{j} + 5\hat{k})$

ii.  $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$  and  $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

iii.  $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$  and  $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

**Answer :**

i.  $\vec{a} = (2\hat{i} - \hat{j} + \hat{k}), \vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k})$  and  $\vec{c} = (3\hat{i} + \lambda\hat{j} + 5\hat{k})$

Given : Vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar.

Where,

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$$

To Find : value of  $\lambda$

Formulae :

1) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

As vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar

$$\therefore [\vec{a} \ \vec{b} \ \vec{c}] = 0 \dots\dots\dots \text{eq(1)}$$

For given vectors,

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & \lambda & 5 \end{vmatrix}$$

$$= 2(2 \times 5 - 3 \times \lambda) - (-1)(1 \times 5 - 3 \times 3) + 1(1 \times \lambda - 3 \times 2)$$

$$= 2(10 - 3\lambda) - 4 + 1(\lambda - 6)$$

$$= 20 - 6\lambda - 4 + \lambda - 6$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = 10 - 5\lambda \dots\dots\dots \text{eq(2)}$$

From eq(1) and eq(2),

$$10 - 5\lambda = 0$$

$$\therefore 5\lambda = 10$$

$\therefore \lambda = 2$

ii.  $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$  and  $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

Given : Vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar.

Where,

$$\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}$$

$$\vec{b} = -7\hat{i} - 5\hat{j}$$

$$\bar{c} = \hat{i} - 4\hat{j} - 3\hat{k}$$

To Find : value of  $\lambda$

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

As vectors  $\bar{a}, \bar{b}, \bar{c}$  are coplanar

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0 \dots\dots\dots \text{eq(1)}$$

For given vectors,

$$\bar{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}$$

$$\bar{b} = -7\hat{i} - 5\hat{j} + 0\hat{k}$$

$$\bar{c} = \hat{i} - 4\hat{j} - 3\hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} \lambda & -10 & -5 \\ -7 & -5 & 0 \\ 1 & -4 & -3 \end{vmatrix}$$

$$= \lambda((-5) \times (-3) - 0 \times (-4)) - (-10)((-7) \times (-3) - 0 \times 1) + (-5)((-7) \times (-4) - 1 \times (-5))$$

$$= \lambda(15) + 10(21) - 5(33)$$

$$= 15\lambda + 45$$

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = 15\lambda + 45 \dots\dots\dots \text{eq(2)}$$

From eq(1) and eq(2),

$$15\lambda + 45 = 0$$

$$\therefore 15\lambda = -45$$

$$\therefore \lambda = -3$$

$$\text{iii. } \vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k} \text{ and } \vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$$

Given : Vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar.

Where,

$$\vec{a} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$$

To Find : value of  $\lambda$

Formulae :

1) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

As vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar

$$\therefore [\vec{a} \ \vec{b} \ \vec{c}] = 0 \dots\dots\dots \text{eq(1)}$$

For given vectors,

$$\vec{a} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix}$$

$$= 1(1 \times \lambda - (-1) \times (-1)) - (-1)(2 \times \lambda - (-1) \times \lambda) + 1(2 \times (-1) - \lambda \times 1)$$

$$= 1(\lambda - 1) + 1(3\lambda) + 1(-\lambda - 2)$$

$$= \lambda - 1 + 3\lambda - 2 - \lambda$$

$$= 3\lambda - 3$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = 3\lambda - 3 \dots\dots\dots \text{eq(2)}$$

From eq(1) and eq(2),

$$3\lambda - 3 = 0$$

$$\therefore 3\lambda = 3$$

$\therefore \lambda = 1$

**Q. 6**

If  $\vec{a} = (2\hat{i} - \hat{j} + \hat{k})$ ,  $\vec{b} = (\hat{i} - 3\hat{j} - 5\hat{k})$  and  $\vec{c} = (3\hat{i} - 4\hat{j} - \hat{k})$ , find  $[\vec{a} \ \vec{b} \ \vec{c}]$  and interpret the result.

**Answer :**

Given Vectors :

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\bar{c} = 3\hat{i} - 4\hat{j} - \hat{k}$$

To Find :  $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given vectors,

$$\bar{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\bar{b} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\bar{c} = 3\hat{i} - 4\hat{j} - \hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -3 & -5 \\ 3 & -4 & -1 \end{vmatrix}$$

$$= 2((-3) \times (-1) - (-4) \times (-5)) - (-1)((-1) \times 1 - 3 \times (-5)) + 1((-4) \times 1 - 3 \times (-3))$$

$$= 2(-17) + 1(14) + 1(5)$$

$$= -34 + 14 + 5$$

$$= -15$$

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -15$$

**Q. 7**

The volume of the parallelepiped whose edges are  $(-12\hat{i} + \lambda\hat{k})$ ,  $(3\hat{j} - \hat{k})$  and  $(2\hat{i} + \hat{j} - 15\hat{k})$  is 546 cubic units. Find the value of  $\lambda$ .

**Answer :**

Given :

1) Coterminous edges of parallelepiped are

$$\bar{a} = -12\hat{i} + \lambda\hat{k}$$

$$\bar{b} = 3\hat{j} - \hat{k}$$

$$\bar{c} = 2\hat{i} + \hat{j} - 15\hat{k}$$

2) Volume of parallelepiped,

$$V = 546 \text{ cubic unit}$$

To Find : value of  $\lambda$

1) Volume of parallelepiped :

If  $\bar{a}, \bar{b}, \bar{c}$  are coterminous edges of parallelepiped,

Where,

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

**Answer :**

Given volume of parallelepiped,

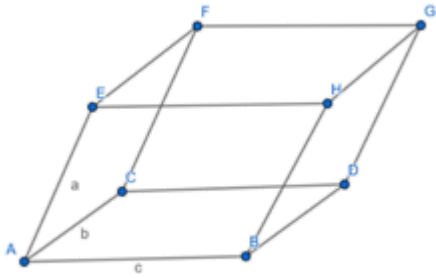
$$V = 546 \text{ cubic unit .....eq(1)}$$

Volume of parallelepiped with coterminous edges

$$\vec{a} = -12\hat{i} + \lambda\hat{k}$$

$$\vec{b} = 3\hat{j} - \hat{k}$$

$$\vec{c} = 2\hat{i} + \hat{j} - 15\hat{k}$$



$$V = [\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -12 & 0 & \lambda \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix}$$

$$= -12(3 \times (-15) - 1 \times (-1)) - 0 + \lambda(0 \times 1 - 3 \times 2)$$

$$= 528 - 0 - 6\lambda$$

$$= 528 - 6\lambda$$

$$\therefore V = (528 - 6\lambda) \text{ cubic unit .....eq(2)}$$

From eq(1) and eq(2)

$$528 - 6\lambda = 546$$

$$\therefore -6\lambda = 18$$

$$\therefore \lambda = -3$$

**Q. 8**

Show that the vectors  $\vec{a} = (\hat{i} + 3\hat{j} + \hat{k})$ ,  $\vec{b} = (2\hat{i} - \hat{j} - \hat{k})$  and  $\vec{c} = (7\hat{j} + 3\hat{k})$  are parallel to the same plane.

{HINT: Show that  $[\vec{a} \ \vec{b} \ \vec{c}] = 0$ }

**Answer :**

Given Vectors :

$$\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{c} = 7\hat{j} + 3\hat{k}$$

To Prove : Vectors  $\vec{a}, \vec{b}, \vec{c}$  are parallel to same plane.

Formulae :

1) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

**Answer :**

Vectors will be parallel to the same plane if they are coplanar.

For vectors  $\vec{a}, \vec{b}, \vec{c}$  to be coplanar,  $[\vec{a} \ \vec{b} \ \vec{c}] = 0$

Now, for given vectors,

$$\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$$

$$\bar{b} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\bar{c} = 7\hat{j} + 3\hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$$= 1(3 \times (-1) - 7 \times (-1)) - 3(2 \times 3 - 0 \times (-1)) + 1(2 \times 7 - 0 \times (-1))$$

$$= 1(4) - 3(6) + 1(14)$$

$$= 4 - 18 + 14$$

$$= 0$$

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, given vectors are parallel to the same plane.

**Q. 9**

If the vectors  $(a\hat{i} + a\hat{j} + c\hat{k}), (\hat{i} + \hat{k})$  and  $(c\hat{i} + c\hat{j} + b\hat{k})$  be coplanar, show that  $c^2 = ab$ .

**Answer :**

Given : vectors  $\bar{a}, \bar{b}, \bar{c}$  are coplanar. Where

$$\bar{a} = a\hat{i} + a\hat{j} + c\hat{k}$$

$$\bar{b} = \hat{i} + \hat{k}$$

$$\bar{c} = c\hat{i} + c\hat{j} + b\hat{k}$$

To Prove :  $c^2 = ab$

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

As vectors  $\bar{a}, \bar{b}, \bar{c}$  are coplanar

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0 \text{ .....eq(1)}$$

For given vectors,

$$\bar{a} = a\hat{i} + a\hat{j} + c\hat{k}$$

$$\bar{b} = \hat{i} + \hat{k}$$

$$\bar{c} = c\hat{i} + c\hat{j} + b\hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}$$

$$= a(0 \times b - c \times 1) - a(1 \times b - 1 \times c) + c(1 \times c - 0 \times c)$$

$$= a \cdot (-c) - a \cdot (b - c) + c(c)$$

$$= -ac - ab + ac + c^2$$

$$= -ab + c^2$$

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -ab + c^2 \text{ .....eq(2)}$$

From eq(1) and eq(2),

$$-ab + c^2 = 0$$

Therefore,

$$\boxed{c^2 = ab}$$

Hence proved.

Note : Three vectors  $\bar{a}, \bar{b}$  &  $\bar{c}$  are coplanar if and only if

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

**Q. 10**

Show that the four points with position vectors  $(4\hat{i} + 8\hat{j} + 12\hat{k})$ ,  $(2\hat{i} + 4\hat{j} + 6\hat{k})$ ,  $(3\hat{i} + 5\hat{j} + 4\hat{k})$  and  $(5\hat{i} + 8\hat{j} + 5\hat{k})$  are coplanar.

**Answer :**

Given :

Let A, B, C & D be four points with position vectors  $\vec{a}, \vec{b}, \vec{c}$  &  $\vec{d}$ .

Therefore,

$$\vec{a} = 4\hat{i} + 8\hat{j} + 12\hat{k}$$

$$\vec{b} = 2\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\vec{c} = 3\hat{i} + 5\hat{j} + 4\hat{k}$$

$$\vec{d} = 5\hat{i} + 8\hat{j} + 5\hat{k}$$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors  $\vec{a}$  &  $\vec{b}$ ,

Where,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then vector  $\vec{AB}$  is given by,

$$\vec{AB} = \vec{b} - \vec{a}$$

$$(b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

2) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given position vectors,

$$\vec{a} = 4\hat{i} + 8\hat{j} + 12\hat{k}$$

$$\vec{b} = 2\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\vec{c} = 3\hat{i} + 5\hat{j} + 4\hat{k}$$

$$\vec{d} = 5\hat{i} + 8\hat{j} + 5\hat{k}$$

Vectors  $\vec{BA}$ ,  $\vec{CA}$  &  $\vec{DA}$  are given by,

$$\vec{BA} = \vec{a} - \vec{b}$$

$$= (4 - 2)\hat{i} + (8 - 4)\hat{j} + (12 - 6)\hat{k}$$

$$\therefore \vec{BA} = 2\hat{i} + 4\hat{j} + 6\hat{k} \dots\dots\dots \text{eq(1)}$$

$$\vec{CA} = \vec{a} - \vec{c}$$

$$= (4 - 3)\hat{i} + (8 - 5)\hat{j} + (12 - 4)\hat{k}$$

$$\therefore \vec{CA} = \hat{i} + 3\hat{j} + 8\hat{k} \dots\dots\dots \text{eq(2)}$$

$$\vec{DA} = \vec{a} - \vec{d}$$

$$= (4 - 5)\hat{i} + (8 - 8)\hat{j} + (12 - 5)\hat{k}$$

$$\therefore \vec{DA} = -\hat{i} + 0\hat{j} + 7\hat{k} \dots\dots\dots \text{eq(3)}$$

Now, for vectors

$$\vec{BA} = 2\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\overrightarrow{CA} = \hat{i} + 3\hat{j} + 8\hat{k}$$

$$\overrightarrow{DA} = -\hat{i} + 0\hat{j} + 7\hat{k}$$

$$[\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 3 & 8 \\ -1 & 0 & 7 \end{vmatrix}$$

$$= 2(3 \times 7 - 0 \times 8) - 4(1 \times 7 - (-1) \times 8) + 6(1 \times 0 - (-1) \times 3)$$

$$= 2(21) - 4(15) + 6(3)$$

$$= 42 - 60 + 18$$

$$= 0$$

$$\therefore [\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = 0$$

Hence, vectors  $\overrightarrow{BA}, \overrightarrow{CA}$  &  $\overrightarrow{DA}$  are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if  $[\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = 0$

#### Q. 11

**Show that the four points with position vectors  $(6\hat{i} - 7\hat{j}), (16\hat{i} - 19\hat{j} - 4\hat{k}), (3\hat{j} - 6\hat{k})$  and  $(2\hat{i} - 5\hat{j} + 10\hat{k})$  are coplanar.**

**Answer :**

Given :

Let A, B, C & D be four points with position vectors  $\vec{a}, \vec{b}, \vec{c}$  &  $\vec{d}$ .

Therefore,

$$\vec{a} = 6\hat{i} - 7\hat{j}$$

$$\vec{b} = 16\hat{i} - 19\hat{j} - 4\hat{k}$$

$$\vec{c} = 3\hat{j} - 6\hat{k}$$

$$\vec{d} = 2\hat{i} - 5\hat{j} + 10\hat{k}$$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors  $\vec{a}$  &  $\vec{b}$ ,

Where,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then vector  $\overrightarrow{AB}$  is given by,

$$\overrightarrow{AB} = \vec{b} - \vec{a}$$

$$(b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

2) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

Then,

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given position vectors,

$$\vec{a} = 6\hat{i} - 7\hat{j}$$

$$\vec{b} = 16\hat{i} - 19\hat{j} - 4\hat{k}$$

$$\vec{c} = 3\hat{j} - 6\hat{k}$$

$$\vec{d} = 2\hat{i} - 5\hat{j} + 10\hat{k}$$

Vectors  $\overrightarrow{BA}$ ,  $\overrightarrow{CA}$  &  $\overrightarrow{DA}$  are given by,

$$\overrightarrow{BA} = \vec{a} - \vec{b}$$

$$= (6 - 16)\hat{i} + (-7 + 19)\hat{j} + (0 + 4)\hat{k}$$

$$\therefore \overrightarrow{BA} = -10\hat{i} + 12\hat{j} + 4\hat{k} \dots\dots\dots\text{eq(1)}$$

$$\overrightarrow{CA} = \vec{a} - \vec{c}$$

$$= (6 - 0)\hat{i} + (-7 - 3)\hat{j} + (0 + 6)\hat{k}$$

$$\therefore \overrightarrow{CA} = 6\hat{i} - 10\hat{j} + 6\hat{k} \dots\dots\dots\text{eq(2)}$$

$$\overrightarrow{DA} = \vec{a} - \vec{d}$$

$$= (6 - 2)\hat{i} + (-7 + 5)\hat{j} + (0 - 10)\hat{k}$$

$$\therefore \overrightarrow{DA} = 4\hat{i} - 2\hat{j} - 10\hat{k} \dots\dots\dots\text{eq(3)}$$

Now, for vectors

$$\overrightarrow{BA} = -10\hat{i} + 12\hat{j} + 4\hat{k}$$

$$\overrightarrow{CA} = 6\hat{i} - 10\hat{j} + 6\hat{k}$$

$$\overrightarrow{DA} = 4\hat{i} - 2\hat{j} - 10\hat{k}$$

$$\begin{aligned} [\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] &= \begin{vmatrix} -10 & 12 & 4 \\ 6 & -10 & 6 \\ 4 & -2 & -10 \end{vmatrix} \\ &= -10((-10) \times (-10) - (-2) \times 6) - 12(6 \times (-10) - 4 \times 6) \\ &\quad + 4(6 \times (-2) - (-10) \times 4) \\ &= -10(112) - 12(-84) + 4(28) \\ &= -1120 + 1008 + 112 \\ &= 0 \end{aligned}$$

$$\therefore [\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = 0$$

Hence, vectors  $\overrightarrow{BA}, \overrightarrow{CA}$  &  $\overrightarrow{DA}$  are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if  $[\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = 0$

## Q. 12

Find the value of  $\lambda$  for which the four points with position vectors  $(\hat{i} + 2\hat{j} + 3\hat{k})$ ,  $(3\hat{i} - \hat{j} + 2\hat{k})$ ,  $(-2\hat{i} + \lambda\hat{j} + \hat{k})$  and  $(6\hat{i} - 4\hat{j} + 2\hat{k})$  are coplanar.

**Ans.  $\lambda = 3$**

**Answer :**

Given :

Let, A, B, C & D be four points with given position vectors

$$\vec{a} = 1\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{c} = -2\hat{i} + \lambda\hat{j} + \hat{k}$$

$$\vec{d} = 6\hat{i} - 4\hat{j} + 2\hat{k}$$

To Find : value of  $\lambda$

Formulae :

1) Vectors :

If A & B are two points with position vectors  $\vec{a}$  &  $\vec{b}$ ,

Where,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then vector  $\vec{AB}$  is given by,

$$\vec{AB} = \vec{b} - \vec{a}$$

$$(b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

2) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$$\vec{a} = 1\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{c} = -2\hat{i} + \lambda\hat{j} + \hat{k}$$

$$\vec{d} = 6\hat{i} - 4\hat{j} + 2\hat{k}$$

Vectors  $\vec{BA}$ ,  $\vec{CA}$  &  $\vec{DA}$  are given by,

$$\vec{BA} = \vec{a} - \vec{b}$$

$$= (1 - 3)\hat{i} + (2 + 1)\hat{j} + (3 - 2)\hat{k}$$

$$\therefore \vec{BA} = -2\hat{i} + 3\hat{j} + \hat{k} \dots\dots\dots\text{eq(1)}$$

$$\vec{CA} = \vec{a} - \vec{c}$$

$$= (1 + 2)\hat{i} + (2 - \lambda)\hat{j} + (3 - 1)\hat{k}$$

$$\therefore \vec{CA} = 3\hat{i} + (2 - \lambda)\hat{j} + 2\hat{k} \dots\dots\dots\text{eq(2)}$$

$$\vec{DA} = \vec{a} - \vec{d}$$

$$= (1 - 6)\hat{i} + (2 + 4)\hat{j} + (3 - 2)\hat{k}$$

$$\therefore \vec{DA} = -5\hat{i} + 6\hat{j} + \hat{k} \dots\dots\dots\text{eq(3)}$$

Now, for vectors

$$\vec{BA} = -2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{CA} = 3\hat{i} + (2 - \lambda)\hat{j} + 2\hat{k}$$

$$\vec{DA} = -5\hat{i} + 6\hat{j} + \hat{k}$$

$$[\vec{BA} \quad \vec{CA} \quad \vec{DA}] = \begin{vmatrix} -2 & 3 & 1 \\ 3 & (2 - \lambda) & 2 \\ -5 & 6 & 1 \end{vmatrix}$$

$$\begin{aligned}
 &= -2((2 - \lambda) \times 1 - 2 \times 6) - 3(3 \times 1 - 2 \times (-5)) \\
 &\quad + 1(6 \times 3 - (2 - \lambda) \times (-5)) \\
 &= -2(-\lambda - 10) - 3(13) + 1(28 - 5\lambda) \\
 &= 2\lambda + 20 - 39 + 28 - 5\lambda \\
 &= 9 - 3\lambda
 \end{aligned}$$

$$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 9 - 3\lambda \dots\dots\dots \text{eq(4)}$$

Four points A, B, C & D are coplanar if and only if

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0 \dots\dots\dots \text{eq(5)}$$

From eq(4) and eq(5)

$$9 - 3\lambda = 0$$

$$3\lambda = 9$$

$$\boxed{\lambda = 3}$$

**Q. 13**

**Find the value of  $\lambda$  for which the four points with position vectors  $(-\hat{j} + \hat{k})$ ,  $(2\hat{i} - \hat{j} - \hat{k})$ ,  $(\hat{i} + \lambda\hat{j} + \hat{k})$  and  $(3\hat{j} + 3\hat{k})$  are coplanar.**

**Answer :**

Given :

Let, A, B, C & D be four points with given position vectors

$$\vec{a} = -\hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{c} = \hat{i} + \lambda\hat{j} + \hat{k}$$

$$\vec{d} = 3\hat{j} + 3\hat{k}$$

To Find : value of  $\lambda$

Formulae :

1) Vectors :

If A & B are two points with position vectors  $\vec{a}$  &  $\vec{b}$ ,

Where,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then vector  $\overline{AB}$  is given by,

$$\overline{AB} = \vec{b} - \vec{a}$$

$$(b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

2) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given position vectors,

$$\vec{a} = -\hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{c} = \hat{i} + \lambda\hat{j} + \hat{k}$$

$$\vec{d} = 3\hat{j} + 3\hat{k}$$

Vectors  $\overline{BA}$ ,  $\overline{CA}$  &  $\overline{DA}$  are given by,

$$\overline{BA} = \vec{a} - \vec{b}$$

$$= (0 - 2)\hat{i} + (-1 + 1)\hat{j} + (1 + 1)\hat{k}$$

$$\therefore \overrightarrow{BA} = -2\hat{i} + 0\hat{j} + 2\hat{k} \dots\dots\dots\text{eq(1)}$$

$$\overrightarrow{CA} = \vec{a} - \vec{c}$$

$$= (0 - 1)\hat{i} + (-1 - \lambda)\hat{j} + (1 - 1)\hat{k}$$

$$\therefore \overrightarrow{CA} = -\hat{i} + (-1 - \lambda)\hat{j} + 0\hat{k} \dots\dots\dots\text{eq(2)}$$

$$\overrightarrow{DA} = \vec{a} - \vec{d}$$

$$= (0 - 0)\hat{i} + (-1 - 3)\hat{j} + (1 - 3)\hat{k}$$

$$\therefore \overrightarrow{DA} = 0\hat{i} - 4\hat{j} - 2\hat{k} \dots\dots\dots\text{eq(3)}$$

Now, for vectors

$$\overrightarrow{BA} = -2\hat{i} + 0\hat{j} + 2\hat{k}$$

$$\overrightarrow{CA} = -\hat{i} + (-1 - \lambda)\hat{j} + 0\hat{k}$$

$$\overrightarrow{DA} = 0\hat{i} - 4\hat{j} - 2\hat{k}$$

$$\begin{aligned} [\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] &= \begin{vmatrix} -2 & 0 & 2 \\ -1 & (-1 - \lambda) & 0 \\ 0 & -4 & -2 \end{vmatrix} \\ &= -2((-1 - \lambda) \times (-2) - (-4) \times 0) - 0((-1) \times (-2) - 0 \times 0) \\ &\quad + 2((-1) \times (-4) - (-1 - \lambda) \times 0) \\ &= -2(2 + 2\lambda) - 0 + 2(4) \\ &= -4 - 4\lambda + 8 \\ &= 4 - 4\lambda \end{aligned}$$

$$\therefore [\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = 4 - 4\lambda \dots\dots\dots\text{eq(4)}$$

Four points A, B, C & D are coplanar if and only if

$$[\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = 0 \dots\dots\dots\text{eq(5)}$$

From eq(4) and eq(5)

$$4 - 4\lambda = 0$$

$$4\lambda = 4$$

$$\boxed{\lambda = 1}$$

**Using vector method, show that the points A(4, 5, 1), B(0, -1, -1), C(3, 9, 4) and D(-4, 4, 4) are coplanar.**

**Answer :**

Given Points :

$$A \equiv (4, 5, 1)$$

$$B \equiv (0, -1, -1)$$

$$C \equiv (3, 9, 4)$$

$$D \equiv (-4, 4, 4)$$

To Prove : Points A, B, C & D are coplanar.

Formulae :

4) Position Vectors :

If A is a point with co-ordinates  $(a_1, a_2, a_3)$

then its position vector is given by,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

5) Vectors :

If A & B are two points with position vectors  $\vec{a}$  &  $\vec{b}$ ,

Where,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then vector  $\vec{AB}$  is given by,

$$\vec{AB} = \vec{b} - \vec{a}$$

$$(b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

6) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

7) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given points,

$$A \equiv (4, 5, 1)$$

$$B \equiv (0, -1, -1)$$

$$C \equiv (3, 9, 4)$$

$$D \equiv (-4, 4, 4)$$

Position vectors of above points are,

$$\bar{a} = 4\hat{i} + 5\hat{j} + \hat{k}$$

$$\bar{b} = 0\hat{i} - \hat{j} - \hat{k}$$

$$\bar{c} = 3\hat{i} + 9\hat{j} + 4\hat{k}$$

$$\bar{d} = -4\hat{i} + 4\hat{j} + 4\hat{k}$$

Vectors  $\overline{BA}$ ,  $\overline{CA}$  &  $\overline{DA}$  are given by,

$$\overline{BA} = \bar{a} - \bar{b}$$

$$= (4 - 0)\hat{i} + (5 + 1)\hat{j} + (1 + 1)\hat{k}$$

$$\therefore \overline{BA} = 4\hat{i} + 6\hat{j} + 2\hat{k} \dots\dots\dots \text{eq(1)}$$

$$\overline{CA} = \bar{a} - \bar{c}$$

$$= (4 - 3)\hat{i} + (5 - 9)\hat{j} + (1 - 4)\hat{k}$$

$$\therefore \overline{CA} = \hat{i} - 4\hat{j} - 3\hat{k} \dots\dots\dots \text{eq(2)}$$

$$\overline{DA} = \bar{a} - \bar{d}$$

$$= (4 + 4)\hat{i} + (5 - 4)\hat{j} + (1 - 4)\hat{k}$$

$$\therefore \overline{DA} = 8\hat{i} + 1\hat{j} - 3\hat{k} \dots\dots\dots \text{eq(3)}$$

Now, for vectors

$$\overrightarrow{BA} = 4\hat{i} + 6\hat{j} + 2\hat{k}$$

$$\overrightarrow{CA} = \hat{i} - 4\hat{j} - 3\hat{k}$$

$$\overrightarrow{DA} = 8\hat{i} + 1\hat{j} - 3\hat{k}$$

$$[\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = \begin{vmatrix} 4 & 6 & 2 \\ 1 & -4 & -3 \\ 8 & 1 & -3 \end{vmatrix}$$

$$= 4((-4) \times (-3) - 1 \times (-3)) - 6(1 \times (-3) - (-3) \times 8) + 2(1 \times 1 - (-4) \times 8)$$

$$= 4(15) - 6(21) + 2(33)$$

$$= 60 - 126 + 66$$

$$= 0$$

$$\therefore [\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = 0$$

Hence, vectors  $\overrightarrow{BA}, \overrightarrow{CA}$  &  $\overrightarrow{DA}$  are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if  $[\overrightarrow{BA} \quad \overrightarrow{CA} \quad \overrightarrow{DA}] = 0$

### Q. 15

**Find the value of  $\lambda$  for which the points A(3, 2, 1), B(4,  $\lambda$ , 5), C(4, 2, -2) and D(6, 5, -1) are coplanar.**

**Ans.  $\lambda = 5$**

Answer :

Given :

Points A, B, C & D are coplanar where,

$$A \equiv (3, 2, 1)$$

$$B \equiv (4, \lambda, 5)$$

$$C \equiv (4, 2, -2)$$

$$D \equiv (6, 5, -1)$$

To Find : value of  $\lambda$

Formulae :

1) Position Vectors :

If A is a point with co-ordinates  $(a_1, a_2, a_3)$

then its position vector is given by,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

2) Vectors :

If A & B are two points with position vectors  $\vec{a}$  &  $\vec{b}$  ,

Where,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then vector  $\vec{AB}$  is given by,

$$\vec{AB} = \vec{b} - \vec{a}$$

$$(b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

3) Scalar Triple Product:

If

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Then,

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

4) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 \cdot c_3 - c_2 \cdot b_3) - a_2(b_1 \cdot c_3 - c_1 \cdot b_3) + a_3(b_1 \cdot c_2 - c_1 \cdot b_2)$$

Answer :

For given points,

$$A \equiv (3, 2, 1)$$

$$B \equiv (4, \lambda, 5)$$

$$\equiv (4, 2, -2)$$

$$D \equiv (6, 5, -1)$$

Position vectors of above points are,

$$\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{b} = 4\hat{i} + \lambda\hat{j} + 5\hat{k}$$

$$\vec{c} = 4\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{d} = 6\hat{i} + 5\hat{j} - \hat{k}$$

Vectors  $\vec{BA}$ ,  $\vec{CA}$  &  $\vec{DA}$  are given by,

$$\vec{BA} = \vec{a} - \vec{b}$$

$$= (3 - 4)\hat{i} + (2 - \lambda)\hat{j} + (1 - 5)\hat{k}$$

$$\therefore \vec{BA} = -\hat{i} + (2 - \lambda)\hat{j} - 4\hat{k} \dots\dots\dots\text{eq(1)}$$

$$\vec{CA} = \vec{a} - \vec{c}$$

$$= (3 - 4)\hat{i} + (2 - 2)\hat{j} + (1 + 2)\hat{k}$$

$$\therefore \vec{CA} = -\hat{i} + 0\hat{j} + 3\hat{k} \dots\dots\dots\text{eq(2)}$$

$$\vec{DA} = \vec{a} - \vec{d}$$

$$= (3 - 6)\hat{i} + (2 - 5)\hat{j} + (1 + 1)\hat{k}$$

$$\therefore \vec{DA} = -3\hat{i} - 3\hat{j} + 2\hat{k} \dots\dots\dots\text{eq(3)}$$

Now, for vectors

$$\vec{BA} = -\hat{i} + (2 - \lambda)\hat{j} - 4\hat{k}$$

$$\vec{CA} = -\hat{i} + 0\hat{j} + 3\hat{k}$$

$$\vec{DA} = -3\hat{i} - 3\hat{j} + 2\hat{k}$$

$$[\vec{BA} \quad \vec{CA} \quad \vec{DA}] = \begin{vmatrix} -1 & (2 - \lambda) & -4 \\ -1 & 0 & 3 \\ -3 & -3 & 2 \end{vmatrix}$$

$$= -1(0 \times 2 - 3 \times (-3)) - (2 - \lambda)(2 \times (-1) - (-3) \times 3) - 4((-1) \times (-3) - (-3) \times 0)$$

$$= -1(9) - (2 - \lambda).(7) - 4(3)$$

$$= -9 - 14 + 7\lambda - 12$$

$$= 7\lambda - 35$$

$$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 7\lambda - 35 \dots\dots\dots \text{eq(4)}$$

But points A, B, C & D are coplanar if and only if

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0 \dots\dots\dots \text{eq(5)}$$

From eq(4) and eq(5)

$$7\lambda - 35 = 0$$

$$\therefore 7\lambda = 35$$

### Exercise 25B

#### Q. 1

If  $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$  and  $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$  are two equal vectors the  $x + y + z = ?$

**Answer :**

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$$

$$\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$$

$$\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$$

Since, these two vectors are equal, therefore comparing these two vectors we get,

$$x = 3, -y = 2, -z = 1$$

$$\Rightarrow x = 3, y = -2, z = -1$$

$$\therefore x + y + z = 3 + (-2) + (-1) = 3 - 2 - 1 = 0$$

$$\text{Ans: } x + y + z = 0$$

#### Q. 2

Write a unit vector in the direction of the sum of the vectors  $\vec{a} = (2\hat{i} + 2\hat{j} - 5\hat{k})$  and  $\vec{b} = (2\hat{i} + \hat{j} - 7\hat{k})$ .

**Answer :**

Let  $\vec{s}$  be the sum of the vectors  $\vec{a}$  and  $\vec{b}$

$$\Rightarrow \vec{s} = \vec{a} + \vec{b}$$

$$\Rightarrow \vec{s} = 2\hat{i} + 2\hat{j} - 5\hat{k} + 2\hat{i} + \hat{j} - 7\hat{k}$$

$$\Rightarrow \vec{s} = 4\hat{i} + 3\hat{j} - 12\hat{k}$$

$$|\vec{s}| = (4^2 + 3^2 + (-12)^2)^{1/2}$$

$$\Rightarrow |\vec{s}| = (16 + 9 + 144)^{1/2} = (169)^{1/2} = 13$$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{13}$$

Ans:  $\hat{s} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{13}$

**Q. 3**

Write the value of  $\lambda$  so that the vectors  $\vec{a} = (2\hat{i} + \lambda\hat{j} + \hat{k})$  and  $\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k})$  are perpendicular to each other.

**Answer :**

$$\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$$

Since these two vectors are perpendicular the dot product of these two vectors is zero.

$$\text{i.e.: } \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow (2\hat{i} + \lambda\hat{j} + \hat{k}) \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow 2 + \lambda \times (-2) + 3 = 0$$

$$\Rightarrow 5 = 2\lambda$$

$$\Rightarrow \lambda = 5/2$$

Ans:  $\lambda = 5/2$

**Q. 4**

Find the value of  $p$  for which the vectors  $\vec{a} = (3\hat{i} + 2\hat{j} + 9\hat{k})$  and  $\vec{b} = (\hat{i} - 2p\hat{j} + 3\hat{k})$  are parallel.

**Answer :**

$$\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$$

$$\vec{b} = \hat{i} - 2p\hat{j} + 3\hat{k}$$

Since these two vectors are parallel

$$\therefore \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$$

$$\Rightarrow \frac{3}{1} = \frac{1}{-p}$$

$$\Rightarrow p = \frac{-1}{3}$$

Ans:  $p = \frac{-1}{3}$

**Q. 5**

Find the value of  $\lambda$  when the projection of  $\vec{a} = (\lambda\hat{i} + \hat{j} + 4\hat{k})$  on  $\vec{b} = (2\hat{i} + 6\hat{j} + 3\hat{k})$  is 4 units.

**Answer :**

$$\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

projection of a on b is given by:  $\vec{a} \cdot \hat{b}$

$$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$$

$$\Rightarrow |\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{i} + 6\hat{j} + 3\hat{k}}{7}$$

Now it is given that:  $\vec{a} \cdot \hat{b} = 4$

$$\Rightarrow (\lambda\hat{i} + \hat{j} + 4\hat{k}) \cdot \left(\frac{2\hat{i} + 6\hat{j} + 3\hat{k}}{7}\right) = 4$$

$$\Rightarrow 2\lambda + 6 + (3 \times 4) = 28$$

$$\Rightarrow \lambda = (28 - 12 - 6)/2$$

$$\Rightarrow \lambda = 10/2 = 5$$

Ans:  $\lambda = 5$

**Q. 6**

If  $\vec{a}$  and  $\vec{b}$  are perpendicular vectors such that  $|\vec{a} + \vec{b}| = 13$  and  $|\vec{a}| = 5$ , find the value of  $|\vec{b}|$ .

**Answer :**

Since a and b vectors are perpendicular .

$$\Rightarrow \theta = \frac{\pi}{2}$$

Now,

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos \theta$$

$$\Rightarrow 13^2 = 5^2 + |\vec{b}|^2 + 0 \dots (\cos \theta = \cos \frac{\pi}{2} = 0)$$

$$\Rightarrow |\vec{b}|^2 = 169 - 25 = 144$$

$$\Rightarrow |\vec{b}| = 12$$

$$\text{Ans: } |\vec{b}| = 12$$

**Q. 7**

If  $\vec{a}$  is a unit vector such that  $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$ , find  $|\vec{x}|$ .

**Answer :**

$$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$$

$$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 15$$

$$\Rightarrow |\vec{x}|^2 = |\vec{a}|^2 + 15$$

Now , a is a unit vector,

$$\Rightarrow |\vec{a}| = 1$$

$$\Rightarrow |\vec{x}|^2 = 1^2 + 15$$

$$\Rightarrow |\vec{x}|^2 = 16$$

$$\Rightarrow |\vec{x}| = 4$$

$$\text{Ans: } |\vec{x}| = 4$$

**Q. 8**

**Find the sum of the vectors**  $\vec{a} = (\hat{i} - 3\hat{k})$ ,  $\vec{b} = (2\hat{j} - \hat{k})$  **and**  $\vec{c} = (2\hat{i} - 3\hat{j} + 2\hat{k})$ .

**Answer :**

$$\vec{a} = \hat{i} - 3\hat{k}$$

$$\vec{b} = 2\hat{j} - \hat{k}$$

$$\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$$

Now ,

$$\vec{a} + \vec{b} + \vec{c} = \hat{i} - 3\hat{j} + 2\hat{j} - \hat{k} + 2\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$$

$$\text{Ans: } \vec{a} + \vec{b} + \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$$

**Q. 9**

**Find the sum of the vectors**  $\vec{a} = (\hat{i} - 2\hat{j})$ ,  $\vec{b} = (2\hat{i} - 3\hat{j})$  **and**  $\vec{c} = (2\hat{i} + 3\hat{k})$ .

**Answer :**

$$\vec{a} = \hat{i} - 2\hat{j}$$

$$\vec{b} = 2\hat{i} - 3\hat{j}$$

$$\vec{c} = 2\hat{i} + 3\hat{k}$$

Now ,

$$\vec{a} + \vec{b} + \vec{c} = \hat{i} - 2\hat{j} + 2\hat{i} - 3\hat{j} + 2\hat{i} + 3\hat{k}$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 5\hat{i} - 5\hat{j} + 3\hat{k}$$

$$\text{Ans: } \vec{a} + \vec{b} + \vec{c} = 5\hat{i} - 5\hat{j} + 3\hat{k}$$

**Q. 10**

**Write the projection of the vector**  $(\hat{i} + \hat{j} + \hat{k})$  **along the vector**  $\hat{j}$ .

**Answer :**

projection of a on b is given by:  $\vec{a} \cdot \hat{b}$

∴ the projection of the vector  $(\hat{i} + \hat{j} + \hat{k})$  along the vector  $\hat{j}$  is :

$$(\hat{i} + \hat{j} + \hat{k}) \cdot \hat{j} = 0 + 1 + 0 = 1$$

Ans: the projection of the vector  $(\hat{i} + \hat{j} + \hat{k})$  along the vector  $\hat{j}$  is: 1

### Q. 11

Write the projection of the vector  $(7\hat{i} + \hat{j} - 4\hat{k})$  on the vector  $(2\hat{i} + 6\hat{j} + 3\hat{k})$ .

**Answer :**

$$\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$$

$$\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

projection of a on b is given by:  $\vec{a} \cdot \hat{b}$

$$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$$

$$\Rightarrow |\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{i} + 6\hat{j} + 3\hat{k}}{7}$$

$$\begin{aligned} \vec{a} \cdot \hat{b} &= (7\hat{i} + \hat{j} - 4\hat{k}) \cdot \left( \frac{2\hat{i} + 6\hat{j} + 3\hat{k}}{7} \right) = \frac{(7 \times 2) + (1 \times 6) - (4 \times 3)}{7} \\ &= \frac{14 + 6 - 12}{7} = \frac{8}{7} \end{aligned}$$

Ans: the projection of the vector  $(7\hat{i} + \hat{j} - 4\hat{k})$  on the vector  $(2\hat{i} + 6\hat{j} + 3\hat{k})$ .

### Q. 12

Find  $\vec{a} \cdot (\vec{b} \times \vec{c})$  when  $\vec{a} = (2\hat{i} + \hat{j} + 3\hat{k})$ ,  $\vec{b} = (-\hat{i} + 2\hat{j} + \hat{k})$  and  $\vec{c} = (3\hat{i} + \hat{j} + 2\hat{k})$ .

**Answer :**

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{b} \times \vec{c} = (-\hat{i} + 2\hat{j} + \hat{k}) \times (3\hat{i} + \hat{j} + 2\hat{k}) = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \hat{i}(4-1) - \hat{j}(-2-3) + \hat{k}(-1-6) = 3\hat{i} + 5\hat{j} - 7\hat{k}$$

$$\therefore \vec{b} \times \vec{c} = 3\hat{i} + 5\hat{j} - 7\hat{k}$$

$$\therefore \vec{a} \cdot (\vec{b} \times \vec{c}) = (2\hat{i} + \hat{j} + 3\hat{k}) \cdot (3\hat{i} + 5\hat{j} - 7\hat{k}) = (2 \times 3) + (1 \times 5) + (3 \times -7)$$

$$= 6 + 5 - 21 = -10$$

Ans: -10

**Q. 13**

Find a vector in the direction of  $(2\hat{i} - 3\hat{j} + 6\hat{k})$  which has magnitude 21 units.

**Answer :**

$$\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$|\vec{a}| = (2^2 + (-3)^2 + 6^2)^{1/2}$$

$$\Rightarrow |\vec{a}| = (4 + 9 + 36)^{1/2} = (49)^{1/2} = 7$$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$$

a vector in the direction of  $(2\hat{i} - 3\hat{j} + 6\hat{k})$  which has magnitude 21 units.

$$= 21\hat{a} = 21 \times \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7} = 3(2\hat{i} - 3\hat{j} + 6\hat{k}) = 6\hat{i} - 9\hat{j} + 18\hat{k}$$

Ans:  $6\hat{i} - 9\hat{j} + 18\hat{k}$

**Q. 14**

If  $\vec{a} = (2\hat{i} + 2\hat{j} + 3\hat{k})$ ,  $\vec{b} = (-\hat{i} + 2\hat{j} + \hat{k})$  and  $\vec{c} = (3\hat{i} + \hat{j})$  are such that  $(\vec{a} + \lambda\vec{b})$  is perpendicular to  $\vec{c}$  then find the value of  $\lambda$ .

**Answer :**

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{c} = 3\hat{i} + \hat{j}$$

$$\vec{a} + \lambda\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-\hat{i} + 2\hat{j} + \hat{k})$$

$$\Rightarrow \vec{a} + \lambda\vec{b} = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$$

Since  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$

$$\Rightarrow (\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$$

$$\Rightarrow ((2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}) \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow (2 - \lambda) \times 3 + (2 + 2\lambda) \times 1 = 0$$

$$\Rightarrow 6 + 2 - 3\lambda + 2\lambda = 0$$

$$\Rightarrow \lambda = 8$$

Ans:  $\lambda = 8$

**Q. 15**

Write the vector of magnitude 15 units in the direction of vector  $(\hat{i} - 2\hat{j} + 2\hat{k})$ .

**Answer :**

$$\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$|\vec{a}| = (1^2 + (-2)^2 + 2^2)^{1/2}$$

$$\Rightarrow |\vec{a}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

a vector in the direction of  $(\hat{i} - 2\hat{j} + 2\hat{k})$  which has magnitude 15 units.

$$= 15\hat{a} = 15 \times \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} = 5(\hat{i} - 2\hat{j} + 2\hat{k}) = 5\hat{i} - 10\hat{j} + 10\hat{k}.$$

Ans:  $5\hat{i} - 10\hat{j} + 10\hat{k}$ .

#### Q. 16

If  $\vec{a} = (\hat{i} + \hat{j} + \hat{k})$ ,  $\vec{b} = (4\hat{i} - 2\hat{j} + 3\hat{k})$  and  $\vec{c} = (\hat{i} - 2\hat{j} + \hat{k})$ , find a vector of magnitude 6 units which is parallel to the vector  $(2\vec{a} - \vec{b} + 3\vec{c})$ .

**Answer :**

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\therefore (2\vec{a} - \vec{b} + 3\vec{c}) = 2(\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$$

$$\Rightarrow (2\vec{a} - \vec{b} + 3\vec{c}) = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\text{LET, } (2\vec{a} - \vec{b} + 3\vec{c}) = \vec{s}$$

$$\vec{s} = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$|\vec{s}| = (1^2 + (-2)^2 + 2^2)^{1/2}$$

$$\Rightarrow |\vec{s}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

a vector of magnitude 6 units which is parallel to the vector  $(2\vec{a} - \vec{b} + 3\vec{c})$  is:

$$6\hat{s} = 6 \times \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} = 2(\hat{i} - 2\hat{j} + 2\hat{k}) = 2\hat{i} - 4\hat{j} + 4\hat{k}.$$

Ans:  $2\hat{i} - 4\hat{j} + 4\hat{k}$

#### Q. 17

**Write the projection of the vector  $(\hat{i} - \hat{j})$  on the vector  $(\hat{i} + \hat{j})$ .**

**Answer :**

$$\vec{a} = \hat{i} - \hat{j}$$

$$\vec{b} = \hat{i} + \hat{j}$$

projection of a on b is given by:  $\vec{a} \cdot \hat{b}$

$$|\vec{b}| = (1^2 + 1^2 + 0^2)^{1/2}$$

$$\Rightarrow |\vec{b}| = (1 + 1)^{1/2} = (2)^{1/2}$$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

$$\vec{a} \cdot \hat{b} = (\hat{i} - \hat{j}) \cdot \left( \frac{\hat{i} + \hat{j}}{\sqrt{2}} \right) = \frac{(1 \times 1) + (-1 \times 1)}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$$

Ans: the projection of the vector  $(7\hat{i} + \hat{j} - 4\hat{k})$  on the vector  $(2\hat{i} + 6\hat{j} + 3\hat{k})$ .

**Q. 18**

**Write the angle between two vectors  $\vec{a}$  and  $\vec{b}$  with magnitudes  $\sqrt{3}$  and 2 respectively having  $\vec{a} \cdot \vec{b} = \sqrt{6}$ .**

**Answer :**

$$|\vec{a}| = \sqrt{3}$$

$$|\vec{b}| = 2$$

$$\text{Since, } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

Substituting the given values we get:

$$\Rightarrow \sqrt{6} = \sqrt{3} \times 2 \times \cos \theta$$

$$\Rightarrow \cos \theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \cos^{-1} \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = 45^\circ = \frac{\pi}{4}$$

$$\text{Ans: } \theta = 45^\circ = \frac{\pi}{4}$$

**Q. 19**

If  $\vec{a} = (\hat{i} - 7\hat{j} + 7\hat{k})$  and  $\vec{b} = (3\hat{i} - 2\hat{j} + 2\hat{k})$  then find  $|\vec{a} \times \vec{b}|$ .

**Answer :**

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$$

$$\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a} \times \vec{b} = (\hat{i} - 7\hat{j} + 7\hat{k}) \times (3\hat{i} - 2\hat{j} + 2\hat{k}) = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix} = \hat{i}(-14 - (-14)) - \hat{j}(2 - 21) + \hat{k}(-2 - (-21))$$

$$= 0\hat{i} + 19\hat{j} + 19\hat{k}$$

$$\therefore \vec{a} \times \vec{b} = 0\hat{i} + 19\hat{j} + 19\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = (0^2 + 19^2 + 19^2)^{1/2} = (2 \times 19^2)^{1/2} = 19\sqrt{2}$$

$$\text{Ans: } \therefore |\vec{a} \times \vec{b}| = 19\sqrt{2}$$

**Q. 20**

Find the angle between two vectors  $\vec{a}$  and  $\vec{b}$  with magnitudes 1 and 2 respectively, when  $|\vec{a} \times \vec{b}| = \sqrt{3}$ .

**Answer :**

$$|\vec{a}| = 1$$

$$|\vec{b}| = 2$$

$$\text{Since, } |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

Substituting the given values we get:

$$\Rightarrow \sqrt{3} = 1 \times 2 \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \sin^{-1} \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = 60^\circ = \frac{\pi}{3}$$

$$\text{Ans: } \theta = 60^\circ = \frac{\pi}{3}$$

### Q. 21

What conclusion can you draw about vectors  $\vec{a}$  and  $\vec{b}$  when  $\vec{a} \times \vec{b} = \vec{0}$  and  $\vec{a} \cdot \vec{b} = 0$ ?

**Answer :**

It is given that:

$$\vec{a} \times \vec{b} = \vec{0} \text{ and } \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow |\vec{a}| |\vec{b}| \sin \theta = |\vec{a}| |\vec{b}| \cos \theta = 0$$

Since  $\sin \theta$  and  $\cos \theta$  cannot be 0 simultaneously  $\therefore |\vec{a}| = |\vec{b}| = 0$

Conclusion: when  $\vec{a} \times \vec{b} = \vec{0}$  and  $\vec{a} \cdot \vec{b} = 0$

Then  $|\vec{a}| = |\vec{b}| = 0$

### Q. 22

Find the value of  $\lambda$  when the vectors  $\vec{a} = (\hat{i} + \lambda \hat{j} + 3\hat{k})$  and  $\vec{b} = (3\hat{i} + 2\hat{j} + 9\hat{k})$  are parallel.

**Answer :**

$$\vec{a} = \hat{i} + \lambda \hat{j} + 3\hat{k}$$

$$\vec{b} = 3\hat{i} + 2\hat{j} + 9\hat{k}$$

It is given that  $\vec{a} \parallel \vec{b}$

$$\Rightarrow \frac{1}{3} = \frac{\lambda}{2} = \frac{3}{9}$$

$$\Rightarrow \frac{1}{3} = \frac{\lambda}{2}$$

$$\Rightarrow \lambda = 2 \times \frac{1}{3} = \frac{2}{3}$$

Ans:  $\lambda = 2/3$

### Q. 23

Write the value of

$$\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j}).$$

**Answer :**

We know that:

$$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j},$$

$$\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\therefore \hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j}) = \hat{i} \cdot \hat{i} + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot \hat{k} = 1 - 1 + 1 = 1$$

$$\text{Ans: } \hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j}) = 1$$

### Q. 24

Find the volume of the parallelepiped whose edges are represented by the vectors

$$\vec{a} = (2\hat{i} - 3\hat{j} + 4\hat{k}), \vec{b} = (\hat{i} + 2\hat{j} - \hat{k}) \quad \text{and} \quad \vec{c} = (3\hat{i} - 2\hat{j} + 2\hat{k}).$$

**Answer :**

Scalar triple product geometrically represents the volume of the parallelepiped whose three coterminous edges are represented by  $\vec{a}, \vec{b}, \vec{c}$ . i.e.  $V = [\vec{a} \vec{b} \vec{c}]$

$$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{c} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\therefore V = [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -2 & 2 \end{vmatrix} = 2(4 - 2) - (-3)(2 - (-3)) + 4(-2 - 6) = 4 + 15 - 32 = -13 =$$

13 cubic units.

Ans: 13 cubic units.

### Q. 25

If  $\vec{a} = (-2\hat{i} - 2\hat{j} + 4\hat{k})$ ,  $\vec{b} = (-2\hat{j} + 4\hat{j} - 2\hat{k})$  and  $\vec{c} = (4\hat{i} - 2\hat{j} - 2\hat{k})$  then prove that  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are coplanar.

**Answer :**

$$\vec{a} = -2\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\vec{c} = 4\hat{i} - 2\hat{j} - 2\hat{k}$$

If  $\vec{a}, \vec{b}, \vec{c}$  are coplanar then  $[\vec{a}\vec{b}\vec{c}] = 0$

$$\text{L.H.S} = \begin{vmatrix} -2 & -2 & 4 \\ -2 & 4 & -2 \\ 4 & -2 & -2 \end{vmatrix} = -2(-8 - 4) + 2(4 + 8) + 4(4 - 16) = 24 + 24 - 48 = 0 = \text{R.H.S}$$

$\therefore \text{L.H.S} = \text{R.H.S}$

Hence proved that the vectors  $\vec{a} = -2\hat{i} - 2\hat{j} + 4\hat{k}$

$$\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\vec{c} = 4\hat{i} - 2\hat{j} - 2\hat{k}$$

Are coplanar.

### Q. 26

If  $\vec{a} = (2\hat{i} + 6\hat{j} + 27\hat{k})$  and  $\vec{b} = (\hat{i} + \lambda\hat{j} + \mu\hat{k})$  are such that  $\vec{a} \times \vec{b} = \vec{0}$  then find the values of  $\lambda$  and  $\mu$ .

**Answer :**

$$\vec{a} = 2\hat{i} + 6\hat{j} + 27\hat{k}$$

$$\vec{b} = \hat{i} + \lambda\hat{j} + \mu\hat{k}$$

It is given that  $\vec{a} \times \vec{b} = \vec{0}$

$$\Rightarrow (2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$$

$$\Rightarrow \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{bmatrix} = 0 = \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6)$$

$$\Rightarrow 2\lambda - 6 = 0$$

$$\Rightarrow \lambda = 6/2 = 3$$

$$\Rightarrow 2\mu - 27 = 0$$

$$\Rightarrow \mu = 27/2$$

Ans:  $\lambda = 3$  ,  $\mu = 27/2$

### Q. 27

If  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ , and  $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$  then what is the value of  $\theta$ ?

**Answer :**

It is given that:

$$|\vec{a} \times \vec{b}| = |\vec{a} \cdot \vec{b}|$$

$$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = |\vec{a}||\vec{b}|\cos\theta$$

$$\Rightarrow \sin\theta = \cos\theta$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \tan^{-1} 1 = \frac{\pi}{4}$$

Ans:  $\theta = \frac{\pi}{4}$

### Q. 28

When does  $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$  hold?

**Answer :**

When the two vectors are parallel or collinear, they can be added in a scalar way because the angle between them is zero degrees, they are in the same or opposite direction.

Therefore when two vectors  $\vec{a}$  and  $\vec{b}$  are either parallel or collinear then

$$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$$

**Q. 29**

**Find the direction cosines of a vector which is equally inclined to the x - axis, y - axis and z - axis.**

**Answer :**

Let the inclination with:

$$x - \text{axis} = \alpha$$

$$y - \text{axis} = \beta$$

$$z - \text{axis} = \gamma$$

The vector is equally inclined to the three axes.

$$\Rightarrow \alpha = \beta = \gamma$$

Direction cosines:  $\cos\alpha, \cos\beta, \cos\gamma$

$$\text{We know that: } \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1 \dots (\alpha = \beta = \gamma)$$

$$\Rightarrow 3 \cos^2 \alpha = 1$$

$$\cos \alpha = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

$$\cos \beta = \frac{1}{\sqrt{3}}$$

$$\cos \gamma = \frac{1}{\sqrt{3}}$$

$$\text{Ans: } \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

**Q. 30**

**If  $P(1, 5, 4)$  and  $Q(4, 1, -2)$  be the position vectors of two points P and Q, find the direction ratios of  $\overline{PQ}$ .**

**Answer :**

Let  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  be the two points then Direction ratios of line joining P and Q i.e. PQ are  $x_2 - x_1, y_2 - y_1, z_2 - z_1$

Here, P is (1, 5, 4) and Q is (4, 1, - 2)

Direction ratios of PQ are: (4 - 1), (1 - 5), (- 2 - 4) = 3, - 4, - 6

Ans: the direction ratios of  $\overrightarrow{PQ}$  are: 3, - 4, - 6

### Q. 31

**Find the direction cosines of the vector**  $\vec{a} = (\hat{i} + 2\hat{j} + 3\hat{k})$

**Answer :**

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Let the inclination with:

x - axis =  $\alpha$

y - axis =  $\beta$

z - axis =  $\gamma$

Direction cosines:  $\cos\alpha, \cos\beta, \cos\gamma = l, m, n$

For a vector  $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$\therefore l = \frac{1}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{1}{\sqrt{1 + 4 + 9}} = \frac{1}{\sqrt{14}}$$

$$\therefore m = \frac{2}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{1 + 4 + 9}} = \frac{2}{\sqrt{14}}$$

$$\therefore n = \frac{3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{3}{\sqrt{1 + 4 + 9}} = \frac{3}{\sqrt{14}}$$

$$\text{Ans: } \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$$

### Q. 32

If  $\hat{a}$  and  $\hat{b}$  are unit vectors such that  $(\hat{a} + \hat{b})$  is a unit vector, what is the angle between  $\hat{a}$  and  $\hat{b}$ ?

**Answer :**

It is given that  $\hat{a}$  and  $\hat{b}$  are unit vectors, as well as  $(\hat{a} + \hat{b})$  is also a unit vector

$$\Rightarrow |\hat{a}| = |\hat{b}| = |\hat{a} + \hat{b}| = 1$$

Since the modulus of a unit vector is unity.

Now,

$$|\hat{a} + \hat{b}|^2 = |\hat{a}|^2 + |\hat{b}|^2 + 2|\hat{a}||\hat{b}|\cos\theta$$

$$\Rightarrow 1^2 = 1^2 + 1^2 + 2 \times 1 \times 1 \times \cos\theta$$

$$\Rightarrow \cos\theta = (1 - 1 - 1)/2$$

$$\Rightarrow \cos\theta = \frac{-1}{2}$$

$$\Rightarrow \theta = \cos^{-1} \frac{-1}{2} = \frac{2\pi}{3}$$

$$\text{Ans: } \frac{2\pi}{3}$$

## Objective Questions

### Q. 1

Mark (✓) against the correct answer in each of the following:

A unit vector in the direction of the vector  $\vec{a} = (2\hat{i} - 3\hat{j} + 6\hat{k})$  is

A.  $\left(\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$

B.  $\left(\frac{2}{5}\hat{i} - \frac{3}{5}\hat{j} + \frac{6}{5}\hat{k}\right)$

C.  $\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)$

D. none of these

**Answer :**

Tip – A vector in the direction of another vector  $a\hat{i} + b\hat{j} + c\hat{k}$  is given by  $\lambda(a\hat{i} + b\hat{j} + c\hat{k})$  and the unit vector is given by  $\frac{\lambda(a\hat{i} + b\hat{j} + c\hat{k})}{\sqrt{(a\lambda)^2 + (b\lambda)^2 + (c\lambda)^2}}$

So, a vector parallel to  $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$  is given by  $\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$  where  $\lambda$  is an arbitrary constant.

$$\text{Now, } |\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = 7$$

Hence, the required unit vector

$$= \frac{\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})}{\sqrt{(2\lambda)^2 + (3\lambda)^2 + (6\lambda)^2}}$$

$$= \frac{\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})}{\lambda\sqrt{2^2 + 3^2 + 6^2}}$$

$$= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

**Q. 2**

**Mark (✓) against the correct answer in each of the following:**

The direction cosines of the vector  $\vec{a} = (-2\hat{i} + \hat{j} - 5\hat{k})$  are

A. -2, 1, -5

B.  $\frac{1}{3}, \frac{-1}{6}, \frac{-5}{6}$

C.  $\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}$

D.  $\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

**Answer :**

Formula to be used – The direction cosines of a vector  $a\hat{i} + b\hat{j} + c\hat{k}$  is given by

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}.$$

Hence, the direction cosines of the vector  $-2\hat{i} + \hat{j} - 5\hat{k}$  is given by

$$\left( \frac{-2}{\sqrt{2^2 + 1^2 + 5^2}}, \frac{1}{\sqrt{2^2 + 1^2 + 5^2}}, \frac{-5}{\sqrt{2^2 + 1^2 + 5^2}} \right)$$

$$= \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$$

**Q. 3**

**Mark (✓) against the correct answer in each of the following:**

If A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector  $\overrightarrow{AB}$  then the direction cosines of  $\overrightarrow{AB}$  are

A. -2, -4, 4

B.  $\frac{-1}{2}, -1, 1$

C.  $\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

D. none of these

**Answer :**

Given - A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector  $\overrightarrow{AB}$

Tip - If P(a<sub>1</sub>, b<sub>1</sub>, c<sub>1</sub>) and Q(a<sub>2</sub>, b<sub>2</sub>, c<sub>2</sub>) be two points then the vector  $\overrightarrow{PQ}$  is represented by  $(a_2 - a_1)\hat{i} + (b_2 - b_1)\hat{j} + (c_2 - c_1)\hat{k}$

Hence,  $\overrightarrow{AB} = (-1 - 1)\hat{i} + (-2 - 2)\hat{j} + (1 + 3)\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}$

Formula to be used - The direction cosines of a vector  $a\hat{i} + b\hat{j} + c\hat{k}$  is given by

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}.$$

Hence, the direction cosines of the vector  $-2\hat{i} - 4\hat{j} + 4\hat{k}$  is given by

$$\left( \frac{-2}{\sqrt{2^2 + 4^2 + 4^2}}, \frac{-4}{\sqrt{2^2 + 4^2 + 4^2}}, \frac{4}{\sqrt{2^2 + 4^2 + 4^2}} \right)$$

$$= \left( \frac{-2}{6}, \frac{-4}{6}, \frac{4}{6} \right)$$

$$= \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$$

**Q. 4**

**Mark (✓) against the correct answer in each of the following:**

**If a vector makes angle  $\alpha$ ,  $\beta$  and  $\gamma$  with the x-axis, y-axis and z-axis respectively then the value of  $(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$  is**

**A. 1**

**B. 2**

**C. 0**

**D. 3**

**Answer :**

Given - A vector makes angle  $\alpha$ ,  $\beta$  and  $\gamma$  with the x-axis, y-axis and z-axis respectively.

To Find -  $(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$

Formula to be used -  $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Hence,

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$$

$$= (1 - \cos^2\alpha) + (1 - \cos^2\beta) + (1 - \cos^2\gamma)$$

$$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

$$= 3 - 1$$

$$= 2$$

**Q. 5**

**Mark (✓) against the correct answer in each of the following:**

**The vector  $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\cos\beta)\hat{j} + (\sin\alpha)\hat{k}$  is a**

**A. null vector**

**B. unit vector**

**C. a constant vector**

**D. none of these**

**Answer :**

Tip - Magnitude of a vector  $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

A unit vector is a vector whose magnitude = 1.

Formula to be used -  $\sin^2 \theta + \cos^2 \theta = 1$

Hence, magnitude of  $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\sin\beta)\hat{j} + (\sin\alpha)\hat{k}$  will be given by

$$\sqrt{(\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta)^2 + (\sin\alpha)^2}$$

$$= \sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$$

$$= \sqrt{\cos^2\alpha + \sin^2\alpha}$$

= 1 i.e a unit vector

**Q. 6**

**Mark (✓) against the correct answer in each of the following:**

**What is the angle which the vector  $(\hat{i} + \hat{j} + \sqrt{2}\hat{k})$  makes with the z-axis?**

**A.**  $\frac{\pi}{4}$

**B.**  $\frac{\pi}{3}$

**C.**  $\frac{\pi}{6}$

**D.**  $\frac{2\pi}{3}$

**Answer :**

Formula to be used – The direction cosines of a vector  $a\hat{i} + b\hat{j} + c\hat{k}$  is given by

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}.$$

Hence, the direction cosines of the vector  $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$  is given by

$$\left( \frac{1}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}, \frac{1}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}, \frac{\sqrt{2}}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}} \right)$$

$$= \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}$$

$$= \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$$

The direction cosine of z-axis =  $\frac{1}{\sqrt{2}}$  i.e.  $\cos \theta = \frac{1}{\sqrt{2}}$  where  $\theta$  is the angle the vector makes with the z-axis.

$$\therefore \theta = \cos^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

**Q. 7**

**Mark (✓) against the correct answer in each of the following:**

If  $\vec{a}$  and  $\vec{b}$  are vectors such that  $|\vec{a}| = \sqrt{3}$ ,  $|\vec{b}| = 2$  and  $\vec{a} \cdot \vec{b} = \sqrt{6}$  then the angle between  $\vec{a}$  and  $\vec{b}$  is

A.  $\frac{\pi}{6}$

B.  $\frac{\pi}{3}$

C.  $\frac{\pi}{4}$

D.  $\frac{2\pi}{3}$

**Answer :**

Given -  $\vec{a}$  and  $\vec{b}$  are vectors such that  $|\vec{a}| = \sqrt{3}$  and  $|\vec{b}| = 2$  and  $\vec{a} \cdot \vec{b} = \sqrt{6}$

To find - Angle between  $\vec{a}$  and  $\vec{b}$ .

Formula to be used -  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$

Hence,  $\sqrt{6} = 2\sqrt{3} \cos \theta$  i.e.  $\cos \theta = \frac{1}{\sqrt{2}} \therefore \theta = \frac{\pi}{4}$

**Q. 8**

**Mark (✓) against the correct answer in each of the following:**

If  $\vec{a}$  and  $\vec{b}$  are two vectors such that  $|\vec{a}| = |\vec{b}| = \sqrt{2}$  and  $\vec{a} \cdot \vec{b} = -1$  then the angle between

$\vec{a}$  and  $\vec{b}$  is

- A.  $\frac{\pi}{6}$
- B.  $\frac{\pi}{4}$
- C.  $\frac{\pi}{3}$
- D.  $\frac{2\pi}{3}$

**Answer :**

Given -  $\vec{a}$  and  $\vec{b}$  are vectors such that  $|\vec{a}| = |\vec{b}| = \sqrt{2}$  and  $\vec{a} \cdot \vec{b} = -1$

To find - Angle between  $\vec{a}$  and  $\vec{b}$ .

Formula to be used -  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$

Hence,  $-1 = \sqrt{2}\sqrt{2} \cos \theta$  i.e.  $\cos \theta = \frac{-1}{2} \therefore \theta = \frac{2\pi}{3}$

**Q. 9**

**Mark (✓) against the correct answer in each of the following:**

**The angle between the vectors  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$  is**

- A.  $\cos^{-1} \frac{5}{7}$
- B.  $\cos^{-1} \frac{3}{5}$
- C.  $\cos^{-1} \frac{3}{\sqrt{14}}$
- D. none of these

**Answer :**

Given -  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

To find - Angle between  $\vec{a}$  and  $\vec{b}$ .

Formula to be used -  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Tip - Magnitude of a vector  $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here,  $\vec{a} \cdot \vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} - 2\hat{j} + \hat{k}) = 3 + 4 + 3 = 10$

$$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$$

Hence,  $10 = \sqrt{14}\sqrt{14}\cos\theta$  i.e.  $\cos\theta = \frac{10}{14} = \frac{5}{7}$

$$\therefore \theta = \cos^{-1} \frac{5}{7}$$

#### Q. 10

Mark (✓) against the correct answer in each of the following:

If  $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$  and  $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$  then the angle between  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$  is

A.  $\frac{\pi}{3}$

B.  $\frac{\pi}{4}$

C.  $\frac{\pi}{2}$

D.  $\frac{2\pi}{3}$

**Answer :**

Given -  $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$  and  $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

To find - Angle between  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ .

Formula to be used -  $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}|\cos\theta$  where  $\vec{p}$  and  $\vec{q}$  are two vectors

Tip – Magnitude of a vector  $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

$$\text{Here, } \vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 4\hat{i} + \hat{j} - \hat{k}$$

$$\text{and } \vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\therefore (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = (4\hat{i} + \hat{j} - \hat{k}) \cdot (-2\hat{i} + 3\hat{j} - 5\hat{k}) = -8 + 3 + 5 = 0$$

$$|\vec{a} + \vec{b}| = \sqrt{4^2 + 1^2 + 1^2} = \sqrt{18}$$

$$|\vec{a} - \vec{b}| = \sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$$

$$\text{Hence, } 0 = \sqrt{18}\sqrt{38} \cos \theta \text{ i.e. } \cos \theta = 0$$

$$\therefore \theta = \frac{\pi}{2}$$

### Q. 11

Mark (✓) against the correct answer in each of the following:

If  $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$  and  $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$  then the angle between  $(2\vec{a} + \vec{b})$  and  $(\vec{a} + 2\vec{b})$  is

A.  $\cos^{-1}\left(\frac{21}{40}\right)$

B.  $\cos^{-1}\left(\frac{31}{50}\right)$

C.  $\cos^{-1}\left(\frac{11}{30}\right)$

D. none of these

Answer :

$$\text{Given - } \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k} \text{ and } \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$$

To find – Angle between  $2\vec{a} + \vec{b}$  and  $\vec{a} + 2\vec{b}$ .

Formula to be used -  $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}| \cos \theta$  where  $\vec{p}$  and  $\vec{q}$  are two vectors

Tip – Magnitude of a vector  $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

$$\text{Here, } 2\vec{a} + \vec{b} = 2(\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 5\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\text{and } \vec{a} + 2\vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + 2(3\hat{i} - \hat{j} + 2\hat{k}) = 7\hat{i} + \hat{k}$$

$$\therefore (2\vec{a} + \vec{b}) \cdot (\vec{a} - 2\vec{b}) = (5\hat{i} + 3\hat{j} - 4\hat{k}) \cdot (7\hat{i} + \hat{k}) = 35 - 4 = 31$$

$$|2\vec{a} + \vec{b}| = \sqrt{5^2 + 3^2 + 4^2} = \sqrt{50}$$

$$|\vec{a} - 2\vec{b}| = \sqrt{7^2 + 1^2} = \sqrt{50}$$

$$\text{Hence, } 31 = \sqrt{50}\sqrt{50} \cos \theta \text{ i.e. } \cos \theta = \frac{31}{50}$$

$$\therefore \theta = \cos^{-1} \frac{31}{50}$$

### Q. 12

Mark (✓) against the correct answer in each of the following:

If  $\vec{a} = (2\hat{i} + 4\hat{j} - \hat{k})$  and  $\vec{b} = (3\hat{i} - 2\hat{j} + \lambda\hat{k})$  be such that  $\vec{a} \perp \vec{b}$  then  $\lambda = ?$

A. 2

B. -2

C. 3

D. -3

Answer :

Given -  $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$  and  $\vec{a} \perp \vec{b}$

To find - Value of  $\lambda$

Formula to be used -  $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}| \cos \theta$  where  $\vec{p}$  and  $\vec{q}$  are two vectors

Tip - For perpendicular vectors,  $\theta = \frac{\pi}{2}$  i.e.  $\cos \theta = 0$  i.e. the dot product = 0

Hence,  $\vec{a} \cdot \vec{b} = 0$

$$\therefore (2\hat{i} + 4\hat{j} - \hat{k}) \cdot (3\hat{i} - 2\hat{j} + \lambda\hat{k}) = 0$$

$$\Rightarrow 6 - 8 - \lambda = 0$$

$$\text{i.e. } \lambda = -2$$

### Q. 13

Mark (✓) against the correct answer in each of the following:

What is the projection of  $\vec{a} = (2\hat{i} - \hat{j} + \hat{k})$  on  $\vec{b} = (\hat{i} - 2\hat{j} + \hat{k})$  ?

A.  $\frac{2}{\sqrt{3}}$

B.  $\frac{4}{\sqrt{5}}$

C.  $\frac{5}{\sqrt{6}}$

D. none of these

**Answer :**

Given -  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

To find - Projection of  $\vec{a}$  on  $\vec{b}$  i.e.  $\vec{a} \cos \theta$

Formula to be used -  $\vec{p} \cdot \vec{q} = |\vec{p}| |\vec{q}| \cos \theta$  where  $\vec{p}$  and  $\vec{q}$  are two vectors

Tip - If  $\vec{p}$  and  $\vec{q}$  are two vectors, then the projection of  $\vec{p}$  on  $\vec{q}$  is defined as  $\vec{p} \cos \theta$

Magnitude of a vector  $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\Rightarrow (2\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} - 2\hat{j} + \hat{k}) = \sqrt{1^2 + 2^2 + 1^2} |\vec{a}| \cos \theta$$

$$\Rightarrow |\vec{a}| \cos \theta = \frac{2 + 2 + 1}{\sqrt{6}}$$

$$\Rightarrow |\vec{a}| \cos \theta = \frac{5}{\sqrt{6}}$$

**Q. 14**

**Mark (✓) against the correct answer in each of the following:**

If  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ , then

A.  $|\vec{a}| = |\vec{b}|$

B.  $\vec{a} \parallel \vec{b}$

C.  $\vec{a} \perp \vec{b}$

D. none of these

**Answer :**

Given -  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

Tip - If  $\vec{a}$  and  $\vec{b}$  are two vectors then  $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

Hence,

$$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$$

$$\Rightarrow \sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

$$\Rightarrow a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$$

$$\Rightarrow 4ab\cos\theta = 0$$

$$\Rightarrow \cos\theta = 0$$

i.e.  $\theta = \frac{\pi}{2}$

So,  $\vec{a} \perp \vec{b}$

**Q. 15**

**Mark (✓) against the correct answer in each of the following:**

If  $\vec{a}$  and  $\vec{b}$  are mutually perpendicular unit vectors then  $(3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b}) = ?$

A. 3

B. 5

C. 6

D. 12

**Answer :**

Given -  $\vec{a}$  and  $\vec{b}$  are two mutually perpendicular unit vectors i.e.  $|\vec{a}| = |\vec{b}| = 1$

To Find -  $(3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b})$

Formula to be used -  $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}|\cos\theta$  where  $\vec{p}$  and  $\vec{q}$  are two vectors

Tip -  $\vec{a} \perp \vec{b}$

$$\therefore |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\cos\frac{\pi}{2} = 0$$

$$\therefore \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = 0$$

Hence,

$$(3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b})$$

$$= 15|\vec{a}|^2 + 10\vec{b} \cdot \vec{a} - 18\vec{a} \cdot \vec{b} - 12|\vec{b}|^2$$

$$= 15 - 12$$

$$= 3$$

**Q. 16**

**Mark (✓) against the correct answer in each of the following:**

If the vectors  $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$  and  $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$  are perpendicular to each other then  $\lambda =$  ?

A. -3

B. -6

C. -9

D. -1

**Answer :**

Given -  $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ ,  $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$  and  $\vec{a} \perp \vec{b}$

To find - Value of  $\lambda$

Formula to be used -  $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}| \cos \theta$  where  $\vec{p}$  and  $\vec{q}$  are two vectors

Tip - For perpendicular vectors,  $\theta = \frac{\pi}{2}$  i.e.  $\cos \theta = 0$  i.e. the dot product = 0

Hence,  $\vec{a} \cdot \vec{b} = 0$

$$\therefore (3\hat{i} + \hat{j} - 2\hat{k}) \cdot (\hat{i} + \lambda\hat{j} - 3\hat{k}) = 0$$

$$\Rightarrow 3 + \lambda + 6 = 0$$

$$\text{i.e. } \lambda = -9$$

**Q. 17**

**Mark (✓) against the correct answer in each of the following:**

If  $\theta$  is the angle between two unit vectors  $\hat{a}$  and  $\hat{b}$  then  $\frac{1}{2}|\hat{a} - \hat{b}| = ?$

A.  $\cos \frac{\theta}{2}$

B.  $\sin \frac{\theta}{2}$

C.  $\tan \frac{\theta}{2}$

D. none of these

**Answer :**

Given -  $\hat{a}$  and  $\hat{b}$  are two unit vectors with an angle  $\theta$  between them

To find -  $\frac{1}{2}|\hat{a} - \hat{b}|$

Formula used - If  $\vec{a}$  and  $\vec{b}$  are two vectors then  $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

Tip -  $|\hat{a}|^2 = |\hat{b}|^2 = 1$  &  $\hat{a} \cdot \hat{b} = \cos \theta$

Hence,

$$\frac{1}{2}|\hat{a} - \hat{b}|$$

$$= \frac{1}{2} \sqrt{|\hat{a}|^2 + |\hat{b}|^2 - 2\hat{a} \cdot \hat{b}}$$

$$= \frac{1}{2} \sqrt{2 - 2\cos \theta}$$

$$= \frac{1}{\sqrt{2}} \sqrt{1 - \cos \theta}$$

$$= \frac{1}{\sqrt{2}} \times \sqrt{2\sin^2 \frac{\theta}{2}}$$

$$= \sin \frac{\theta}{2}$$

**Q. 18**

**Mark (✓) against the correct answer in each of the following:**

If  $\vec{a} = (\hat{i} - \hat{j} + 2\hat{k})$  and  $\vec{b} = (2\hat{i} + 3\hat{j} - 4\hat{k})$  then  $|\vec{a} \times \vec{b}| = ?$

A.  $\sqrt{174}$

B.  $\sqrt{87}$

C.  $\sqrt{93}$

D. none of these

**Answer :**

Given -  $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$  and  $\vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$  are two vectors.

To find -  $|\vec{a} \times \vec{b}|$

Formula to be used -  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$  where  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - Magnitude of a vector  $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$$\vec{a} \times \vec{b}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= \hat{i}(4 - 6) + \hat{j}(4 + 4) + \hat{k}(3 + 2)$$

$$= -2\hat{i} + 8\hat{j} + 5\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{2^2 + 8^2 + 5^2} = \sqrt{93}$$

**Q. 19**

**Mark (✓) against the correct answer in each of the following:**

If  $\vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$  and  $\vec{b} = (\hat{i} - 3\hat{k})$  then  $|\vec{b} \times 2\vec{a}| = ?$

A.  $10\sqrt{3}$

B.  $5\sqrt{17}$

C.  $4\sqrt{19}$

D.  $2\sqrt{23}$

**Answer :**

Given -  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  and  $\vec{b} = \hat{i} - 3\hat{k}$  are two vectors.

To find -  $|\vec{b} \times 2\vec{a}|$

Formula to be used -  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$  where  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - Magnitude of a vector  $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$$\vec{b} \times 2\vec{a}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 2 & -4 & 6 \end{vmatrix}$$

$$= \hat{i}(12) + \hat{j}(-6 - 6) + \hat{k}(-4)$$

$$= 12\hat{i} - 12\hat{j} - 4\hat{k}$$

$$\therefore |\vec{b} \times 2\vec{a}| = \sqrt{12^2 + 12^2 + 4^2} = \sqrt{304} = 4\sqrt{19}$$

**Q. 20**

**Mark (✓) against the correct answer in each of the following:**

If  $|\vec{a}| = 2, |\vec{b}| = 7$  and  $(\vec{a} \times \vec{b}) = (3\hat{i} + 2\hat{j} + 6\hat{k})$  then the angle between  $\vec{a}$  and  $\vec{b}$  is

A.  $\frac{\pi}{6}$

B.  $\frac{\pi}{3}$

C.  $\frac{2\pi}{3}$

D.  $\frac{3\pi}{4}$

**Answer :**

Given -  $|\vec{a}| = 2, |\vec{b}| = 7$  and  $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

To find - Angle between  $\vec{a}$  and  $\vec{b}$

Formula to be used -  $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$

Tip -  $|\vec{p} \times \vec{q}| = ||\vec{p}||\vec{q}|\sin\theta\hat{n}| = |\vec{p}||\vec{q}|\sin\theta$  & magnitude of a vector  $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,  $|\vec{a} \times \vec{b}| = |3\hat{i} + 2\hat{j} + 6\hat{k}| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$$\therefore 7 = 2 \times 7 \sin\theta$$

$$\Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

**Q. 21**

**Mark (✓) against the correct answer in each of the following:**

If  $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$  and  $|\vec{a} \times \vec{b}| = 35$  then  $\vec{a} \cdot \vec{b} = ?$

**A. 5**

**B. 7**

**C. 13**

**D. 12**

**Answer :**

Given -  $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$  and  $|\vec{a} \times \vec{b}| = 35$

To find -  $\vec{a} \cdot \vec{b}$

Formula to be used -  $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$  &  $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}|\cos\theta$  where  $\vec{p}$  &  $\vec{q}$  are any two vectors

Tip -  $|\vec{p} \times \vec{q}| = ||\vec{p}||\vec{q}|\sin\theta\hat{n}| = |\vec{p}||\vec{q}|\sin\theta$

So,

$$|\vec{a} \times \vec{b}| = 35$$

$$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 35$$

$$\Rightarrow \sin\theta = \frac{35}{7\sqrt{26}} = \frac{5}{\sqrt{26}}$$

$$\therefore \cos\theta = \sqrt{1 - \left(\frac{5}{\sqrt{26}}\right)^2} = \frac{1}{\sqrt{26}}$$

$$\therefore \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos\theta = \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 7$$

**Q. 22**

**Mark (✓) against the correct answer in each of the following:**

Two adjacent sides of a || gm are represented by the vectors  $\vec{a} = (3\hat{i} + \hat{j} + 4\hat{k})$  and  $\vec{b} = (\hat{i} - \hat{j} + \hat{k})$ . The area of the || gm is

A.  $\sqrt{42}$  sq units

B. 6 sq units

C.  $\sqrt{35}$  sq units

D. none of these

**Answer :**

Given - Two adjacent sides of a || gm are represented by the vectors  $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$  and  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

To find - Area of the parallelogram

Formula to be used -  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$  where  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - Area of ||gm =  $|\vec{a} \times \vec{b}|$  and magnitude of a vector  $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$$\vec{a} \times \vec{b}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(-4 - 1) + \hat{j}(4 - 3) + \hat{k}(-3 - 1)$$

$$= -5\hat{i} + \hat{j} - 4\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + 4^2} = \sqrt{42}$$

i.e. the area of the parallelogram =  $\sqrt{42}$  sq. units

### Q. 23

Mark (✓) against the correct answer in each of the following:

The diagonals of a || gm are represented by the vectors  $\vec{d}_1 = (3\hat{i} + \hat{j} - 2\hat{k})$  and  $\vec{d}_2 = (\hat{i} - 3\hat{j} + 4\hat{k})$ . The area of the || gm is

A.  $7\sqrt{3}$  sq units

B.  $5\sqrt{3}$  sq units

C.  $3\sqrt{5}$  sq units

D. none of these

Answer :

Given - Two diagonals of a || gm are represented by the vectors  $\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$  and  $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$

To find - Area of the parallelogram

Formula to be used -  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$  where  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - Area of ||gm =  $\frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$  and magnitude of a vector  $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$$\vec{d}_1 \times \vec{d}_2$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}(4 - 6) + \hat{j}(-2 - 12) + \hat{k}(-9 - 1)$$

$$= -2\hat{i} - 14\hat{j} - 10\hat{k}$$

$$\therefore |\vec{d}_1 \times \vec{d}_2| = \sqrt{2^2 + 14^2 + 10^2} = \sqrt{300}$$

$$\text{i.e. the area of the parallelogram} = \frac{1}{2} \times \sqrt{300} = 5\sqrt{3} \text{ sq. units}$$

**Q. 24**

**Mark (✓) against the correct answer in each of the following:**

**Two adjacent sides of a triangle are represented by the vectors  $\vec{a} = 3\hat{i} + 4\hat{j}$  and  $\vec{b} = -5\hat{i} + 7\hat{j}$ . The area of the triangle is**

**A. 41 sq units**

**B. 37 sq units**

**C.  $\frac{41}{2}$  sq units**

**D. none of these**

**Answer :**

Given - Two adjacent sides of a triangle are represented by the vectors  $\vec{a} = 3\hat{i} + 4\hat{j}$  and  $\vec{b} = -5\hat{i} + 7\hat{j}$

To find - Area of the triangle

Formula to be used -  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$  where  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - Area of triangle  $= \frac{1}{2} |\vec{a} \times \vec{b}|$  and magnitude of a vector  $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$  is given by  $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$$\vec{a} \times \vec{b}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$$

$$= \hat{k}(21 + 20)$$

$$= 41\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{41^2} = 41$$

i.e. the area of the parallelogram =  $\frac{41}{2}$  sq. units

### Q. 25

**Mark (✓) against the correct answer in each of the following:**

The unit vector normal to the plane containing  $\vec{a} = (\hat{i} - \hat{j} - \hat{k})$  and  $\vec{b} = (\hat{i} + \hat{j} + \hat{k})$  is

- A.  $(\hat{j} - \hat{k})$
- B.  $(-\hat{j} + \hat{k})$
- C.  $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$
- D.  $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$

**Answer :**

Given -  $\vec{a} = \hat{i} - \hat{j} - \hat{k}$  &  $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

To find - A unit vector perpendicular to the two given vectors.

Formula to be used -  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$  where  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - A vector perpendicular to two given vectors is their cross product.

The unit vector of any vector  $a\hat{i} + b\hat{j} + c\hat{k}$  is given by  $\frac{(a\hat{i} + b\hat{j} + c\hat{k})}{\sqrt{a^2 + b^2 + c^2}}$

Hence,

$$\vec{a} \times \vec{b}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= -2\hat{j} + 2\hat{k}$ , which the vector perpendicular to the two given vectors.

The required unit vector  $= \frac{-2\hat{j} + 2\hat{k}}{\sqrt{2^2 + 2^2}} = \frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

**Q. 26**

Mark (✓) against the correct answer in each of the following:

If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  then  $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = ?$

A.  $\frac{1}{2}$

B.  $\frac{-1}{2}$

C.  $\frac{3}{2}$

D.  $\frac{-3}{2}$

**Answer :**

Given -  $\vec{a}, \vec{b}, \vec{c}$  are three unit vectors and  $(\vec{a} + \vec{b} + \vec{c}) = \vec{0}$

To find -  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

Tip -  $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

So,

$$(\vec{a} + \vec{b} + \vec{c})^2 = 0$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = \frac{-3}{2}$$

**Q. 27**

Mark (✓) against the correct answer in each of the following:

If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are mutually perpendicular unit vectors then  $[\vec{a} + \vec{b} + \vec{c}] = ?$

A. 1

B.  $\sqrt{2}$

C.  $\sqrt{3}$

D. 2

**Answer :**

Given -  $\vec{a}, \vec{b}, \vec{c}$  are three mutually perpendicular unit vectors

To find -  $[\vec{a} + \vec{b} + \vec{c}]$

Tip -  $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$  &  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

So,

$$(\vec{a} + \vec{b} + \vec{c})^2$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$= 3$$

$$\therefore [\vec{a} + \vec{b} + \vec{c}] = \sqrt{3}$$

**Q. 28**

**Mark (✓) against the correct answer in each of the following:**

$$[\hat{i} \hat{j} \hat{k}] = ?$$

A. 0

B. 1

C. 2

D. 3

**Answer :**

To find -  $[\hat{i} \hat{j} \hat{k}]$

Formula to be used -  $[\hat{a} \hat{b} \hat{c}] = \hat{a} \cdot (\hat{b} \times \hat{c})$

$$\therefore [\hat{i} \hat{j} \hat{k}]$$

$$= \hat{i} \cdot (\hat{j} \times \hat{k})$$

$$= \hat{i} \cdot \hat{i}$$

$$= |\hat{i}|^2$$

$$= 1$$

**Q. 29**

Mark (✓) against the correct answer in each of the following:

If  $\vec{a} = (2\hat{i} - 3\hat{j} + 4\hat{k})$ ,  $\vec{b} = (\hat{i} + 2\hat{j} - \hat{k})$  and  $\vec{c} = (3\hat{i} - \hat{j} - 2\hat{k})$  be the coterminous edges of a parallelepiped then its volume is

A. 21 cubic units

B. 14 cubic units

C. 7 cubic units

D. none of these

**Answer :**

Given - The three coterminous edges of a parallelepiped are  $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ ,

$$\vec{b} = \hat{i} + 2\hat{j} - \hat{k} \text{ \& } \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$$

To find - The volume of the parallelepiped

Formula to be used -  $[\hat{a} \ \hat{b} \ \hat{c}] = \hat{a} \cdot (\hat{b} \times \hat{c})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \text{ where } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \text{ and } \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

Tip - The volume of the parallelepiped =  $|[\hat{a} \ \hat{b} \ \hat{c}]|$

Hence,

$$[\hat{a} \ \hat{b} \ \hat{c}]$$

$$= \hat{a} \cdot (\hat{b} \times \hat{c})$$

$$= (2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot \{(\hat{i} + 2\hat{j} - \hat{k}) \times (3\hat{i} - \hat{j} - 2\hat{k})\}$$

$$= (2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$$

$$= (2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot (-5\hat{i} - \hat{j} - 7\hat{k})$$

$$= -10 + 3 - 28$$

$$= -35$$

The volume = 35 sq units

**Q. 30**

Mark (✓) against the correct answer in each of the following:

If the volume of a parallelepiped having  $\vec{a} = (5\hat{i} - 4\hat{j} + \hat{k})$ ,  $\vec{b} = (4\hat{i} + 3\hat{j} + \lambda\hat{k})$  and  $\vec{c} = (\hat{i} - 2\hat{j} + 7\hat{k})$  as coterminal edges, is 216 cubic units then the value of  $\lambda$  is

A.  $\frac{5}{3}$

B.  $\frac{4}{3}$

C.  $\frac{2}{3}$

D.  $\frac{1}{3}$

**Answer :**

Given - The three coterminal edges of a parallelepiped are  $\vec{a} = 5\hat{i} - 4\hat{j} + \hat{k}$ ,

$$\vec{b} = 4\hat{i} + 3\hat{j} + \lambda\hat{k} \text{ \& } \vec{c} = \hat{i} - 2\hat{j} + 7\hat{k}$$

To find - The value of  $\lambda$

Formula to be used -  $[\hat{a} \ \hat{b} \ \hat{c}] = \hat{a} \cdot (\hat{b} \times \hat{c})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \text{ where } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \text{ and } \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

Tip - The volume of the parallelepiped =  $|[\hat{a} \ \hat{b} \ \hat{c}]|$

Hence,

$$[\hat{a} \ \hat{b} \ \hat{c}]$$

$$= \hat{a} \cdot (\hat{b} \times \hat{c})$$

$$= (5\hat{i} - 4\hat{j} + \hat{k}) \cdot \{(4\hat{i} + 3\hat{j} + \lambda\hat{k}) \times (\hat{i} - 2\hat{j} + 7\hat{k})\}$$

$$= (5\hat{i} - 4\hat{j} + \hat{k}) \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & \lambda \\ 1 & -2 & 7 \end{vmatrix}$$

$$= (5\hat{i} - 4\hat{j} + \hat{k}) \cdot ((21 + 2\lambda)\hat{i} + (\lambda - 28)\hat{j} - 11\hat{k})$$

$$= 5(21 + 2\lambda) - 4(\lambda - 28) - 11$$

$$= 206 + 6\lambda$$

$$\text{The volume} = 206 + 6\lambda$$

$$\text{But, the volume} = 216 \text{ sq units}$$

$$\text{So, } 206 + 6\lambda = 216 \Rightarrow \lambda = \frac{10}{6} = \frac{5}{3}$$

### Q. 31

Mark (✓) against the correct answer in each of the following:

It is given that the vectors  $\vec{a} = (2\hat{i} - 2\hat{k})$ ,  $\vec{b} = \hat{i} + (\lambda + 1)\hat{j}$  and  $\vec{c} = (4\hat{i} + 2\hat{k})$  are coplanar. Then, the value of  $\lambda$  is

A.  $\frac{1}{2}$

B.  $\frac{1}{3}$

C. 2

D. 1

Answer :

Given – The vectors  $\vec{a} = 2\hat{i} - 2\hat{k}$ ,  $\vec{b} = \hat{i} + (\lambda + 1)\hat{j}$  &  $\vec{c} = 4\hat{i} + 2\hat{k}$  are coplanar

To find – The value of  $\lambda$

Formula to be used -  $[\hat{a} \ \hat{b} \ \hat{c}] = \hat{a} \cdot (\hat{b} \times \hat{c})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \text{ where } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \text{ and } \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

Tip – For vectors to be coplanar,  $[\hat{a} \ \hat{b} \ \hat{c}] = 0$

Hence,

$$[\hat{a} \ \hat{b} \ \hat{c}] = 0$$

$$\Rightarrow \hat{a} \cdot (\hat{b} \times \hat{c}) = 0$$

$$\Rightarrow (2\hat{i} - 2\hat{k}) \cdot \{(\hat{i} + (\lambda + 1)\hat{j}) \times (4\hat{i} + 2\hat{k})\} = 0$$

$$\Rightarrow (2\hat{i} - 2\hat{k}) \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \lambda + 1 & 0 \\ 4 & 0 & 2 \end{vmatrix} = 0$$

$$\Rightarrow (2\hat{i} - 2\hat{k}) \cdot (2(\lambda + 1)\hat{i} - 2\hat{j} - 4(\lambda + 1)\hat{k}) = 0$$

$$\Rightarrow 4(\lambda - 1) + 8(\lambda - 1) = 0$$

$$\Rightarrow 12(\lambda - 1) = 0 \text{ i.e. } \lambda = 1$$

**Q. 32**

**Mark (✓) against the correct answer in each of the following:**

**Which of the following is meaningless?**

**A.**  $\vec{a} \cdot (\vec{b} \times \vec{c})$

**B.**  $\vec{a} \times (\vec{b} \cdot \vec{c})$

**C.**  $(\vec{a} \times \vec{b}) \cdot \vec{c}$

**D. none of these**

**Answer :**

Tip -  $[\hat{a} \hat{b} \hat{c}] = \hat{a} \cdot (\hat{b} \times \hat{c}) = \hat{b} \cdot (\hat{c} \times \hat{a}) = \hat{c} \cdot (\hat{a} \times \hat{b}) = (\hat{a} \times \hat{b}) \cdot \hat{c}$  since, dot product is commutative

Hence,  $\hat{a} \times (\hat{b} \cdot \hat{c})$  is meaningless.

**Q. 33**

**Mark (✓) against the correct answer in each of the following:**

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = ?$$

**A. 0**

**B. 1**

**C.  $a^2b$**

**D. meaningless**

**Answer :**

Tip - The cross product of two vectors is the vector perpendicular to both the vectors.

$\therefore \vec{a} \times \vec{b}$  gives a vector perpendicular to both  $\vec{a}$  and  $\vec{b}$ .

Now,

$$\vec{a} \cdot (\vec{a} \times \vec{b})$$

$$= |\vec{a}| |\vec{b}| \cos \theta$$

$$= |\vec{a}| |\vec{b}| \cos \frac{\pi}{2}$$

$$= 0$$

**Q. 34**

**Mark (✓) against the correct answer in each of the following:**

**For any three vectors  $\vec{a}, \vec{b}, \vec{c}$  the value of  $[\vec{a} - \vec{b} \ \vec{b} - \vec{c} \ \vec{c} - \vec{a}]$  is**

**A.  $2[\vec{a} \ \vec{b} \ \vec{c}]$**

**B. 1**

**C. 0**

**D. none of these**

**Answer :**

Formula to be used -  $[\hat{a} \ \hat{b} \ \hat{c}] = \hat{a} \cdot (\hat{b} \times \hat{c}) = \hat{b} \cdot (\hat{c} \times \hat{a})$  for any three arbitrary vectors

$$\therefore [\hat{a} - \hat{b} \ \hat{b} - \hat{c} \ \hat{c} - \hat{a}]$$

$$= (\hat{a} - \hat{b}) \cdot \{(\hat{b} - \hat{c}) \times (\hat{c} - \hat{a})\}$$

$$= (\hat{a} - \hat{b}) \cdot (\hat{b} \times \hat{c} - \hat{c} \times \hat{c} - \hat{b} \times \hat{a} + \hat{c} \times \hat{a})$$

$$= (\hat{a} - \hat{b}) \cdot (\hat{b} \times \hat{c} - \hat{b} \times \hat{a} + \hat{c} \times \hat{a})$$

$$= [\hat{a} \cdot (\hat{b} \times \hat{c}) - \hat{b} \cdot (\hat{b} \times \hat{c}) - \hat{a} \cdot (\hat{b} \times \hat{a}) + \hat{b} \cdot (\hat{b} \times \hat{a}) + \hat{a} \cdot (\hat{c} \times \hat{a}) - \hat{b} \cdot (\hat{c} \times \hat{a})]$$

$$= [\hat{a} \ \hat{b} \ \hat{c}] - [\hat{a} \ \hat{b} \ \hat{c}] = 0$$