EXERCISE 24

1. Find $\left(\vec{a} \times \vec{b}\right)$ **and** $\left|\vec{a} \times \vec{b}\right|$**, when**

(i)

$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

(ii)

$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

(iii)

$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

(iv)

$\vec{a} = 4\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{k}$

(v)

$\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

Solution:

(i) It is given that

$\vec{a} = i - j + 2k$

$\vec{b} = 2i + 3j - 4k$

Here we know that

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

$a_1 = 1, a_2 = -1, a_3 = 2$

$b_1 = 2, b_2 = 3, b_3 = -4$

By substituting the values we get

$\vec{a} \times \vec{b} = ((-1 \times -4) - 3 \times 2)i + (2 \times 2 - (-4) \times 1)j + (1 \times 3 - 2 \times (-1))k$

$= \left(-2\hat{i} + 8\hat{j} + 5\hat{k}\right)$

So we get

$|a \times b| = \sqrt{(-2)^2 + 8^2 + 5^2} = \sqrt{93}$

(ii) It is given that

$\vec{a} = 2i - j + 3k$

$\vec{b} = 3i + 5j - 2k$

Here we know that

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

$a_1 = 2$, $a_2 = -1$, $a_3 = 3$

$b_1 = 3$, $b_2 = 5$, $b_3 = -2$

By substituting the values we get

$$\vec{a} \times \vec{b} = \big((-1 \times -2) - 5 \times 3\big)i + (3 \times 3 - (-2) \times 2)j + (2 \times 5 - 3 \times (-1))k$$

$$= (-17)i + (13)j + (7)k$$

So we get

$$|a \times b| = \sqrt{(-17)^2 + 13^2 + 7^2}$$

$$= 13\sqrt{3}$$

(iii) It is given that

$$\vec{a} = i - 7j + 7k$$

$$\vec{b} = 3i - 2j + 2k$$

Here we know that

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

$a_1 = 1$, $a_2 = -7$, $a_3 = 7$

$b_1 = 3$, $b_2 = -2$, $b_3 = 2$

By substituting the values we get

$$\vec{a} \times \vec{b} = \big((-7 \times 2) - (-2) \times 7\big)i + (7 \times 3 - 1 \times 2)j + ((-2) \times 1 - 3 \times (-7))k$$

$$= (0)i + (19)j + (19)k$$

So we get

$$|a \times b| = \sqrt{(0)^2 + 19^2 + 19^2}$$

$$= 19\sqrt{2}$$

(iv) It is given that

$$\vec{a} = 4i + j - 2k$$

$\vec{b} = 3i + 0j + k$

Here we know that

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

$a_1 = 4$, $a_2 = 1$, $a_3 = -2$

$b_1 = 3$, $b_2 = 0$, $b_3 = 1$

By substituting the values we get

$\vec{a} \times \vec{b} = (1 \times 1 - (0) \times -2)i + (-2 \times 3 - 1 \times 4)j + (4 \times 0 - 3 \times 1)k$

$= i - 10j - 3k$

So we get

$|a \times b| = \sqrt{1^2 + (-10)^2 + (-3)^2}$

$= \sqrt{110}$

(v) It is given that

$\vec{a} = 3i + 4j + 0k$

$\vec{b} = i + j + k$

Here we know that

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

$a_1 = 3$, $a_2 = 4$, $a_3 = 0$

$b_1 = 1$, $b_2 = 1$, $b_3 = 1$

By substituting the values we get

$\vec{a} \times \vec{b} = (4 \times 1 - 1 \times 0)i + (0 \times 1 - 1 \times 3)j + (3 \times 1 - 1 \times 4)k$

$= 4i - 3j - k$

So we get

$|a \times b| = \sqrt{4^2 + (-3)^2 + (-1)^2}$

$= \sqrt{26}$

2. Find λ if $\left(2\hat{i} + 6\hat{j} + 14\hat{k}\right) \times \left(\hat{i} - \lambda\hat{j} + 7\hat{k}\right) = \vec{0}$.

Solution:

It is given that

$\vec{a} = 2i + 6j + 14k$

$\vec{b} = i - \lambda j + 7k$

Here we know that

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

$a_1 = 2, a_2 = 6, a_3 = 14$

$b_1 = 1, b_2 = \lambda, b_3 = 7$

By substituting the values we get

$\vec{a} \times \vec{b} = (6 \times 7 - (-\lambda) \times 14)i + (14 \times 1 - 2 \times 7)j + (2 \times (-\lambda) - 1 \times 6)k$

$\vec{a} \times \vec{b} = 0i + 0j + 0k$

So we get

$42 + 14\lambda = 0$

$\lambda = -3$

3. If $\vec{a} = (-3\hat{i} + 4\hat{j} - 7\hat{k})$ and $\vec{b} = (6\hat{i} + 2\hat{j} - 3\hat{k})$, find $(\vec{a} \times \vec{b})$.

Verify that (i) $\vec{a}$ **and** $(\vec{a} \times \vec{b})$ **are perpendicular to each other**

and (ii) $\vec{b}$ **and** $(\vec{a} \times \vec{b})$ **are perpendicular to each other.**

Solution:

It is given that

$\vec{a} = -3i + 4j - 7k$

$\vec{b} = 6i + 2j - 3k$

Here we know that

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

$a_1 = -3, a_2 = 4, a_3 = -7$

$b_1 = 6, b_2 = 2, b_3 = -3$

By substituting the values we get

$\vec{a} \times \vec{b} = (4 \times (-3) - 2 \times (-7))i + ((-7) \times 6 - (-3) \times (-3))j + ((-3) \times 2 - 6 \times 4)k$

$= 2i - 51j - 30k$

Here if $\vec{a}$ and $(\vec{a} \times \vec{b})$ are perpendicular to each other

$\vec{a}.(\vec{a} \times \vec{b}) = 0$

We get

$\vec{a}.(\vec{a} \times \vec{b}) = (-6) - (204) + (210) = 0$

Similarly if $\vec{b}$ and $(\vec{a} \times \vec{b})$ are perpendicular to each other

$\vec{b}.(\vec{a} \times \vec{b}) = (12) - (102) + (90) = 0$

Therefore, it is proved.

4. Find the value of:

(i)

$(i \times j).k + i.j$

(ii)

$(j \times k).i + j.k$

(iii)

$i \times (j + k) + j \times (k + i) + k \times (i + j)$

Solution:

(i) It is given that

$(i \times j).k + i.j$

We know that

$i \times j = k \text{ and } i.j = 0$

So we get

$(k).k + 0 = 1$

(ii) It is given that

$(j \times k).i + j.k$

We know that

$j \times k = i \text{ and } j.k = 0$

So we get

$-(\mathrm{i}).\mathrm{i}+0 = 1$

$(\mathrm{j}\times\mathrm{k}).\mathrm{i}+\mathrm{j}.\mathrm{k}_{=-1+0=-1}$

(iii) It is given that

$\mathrm{i}\times(\mathrm{j}+\mathrm{k})+\mathrm{j}\times(\mathrm{k}+\mathrm{i})+\mathrm{k}\times(\mathrm{i}+\mathrm{j})$

We know that

$\mathrm{i}\times\mathrm{k}=-\mathrm{j}, \mathrm{i}\times\mathrm{j}=\mathrm{k}, \mathrm{j}\times\mathrm{k}=\mathrm{i}, \mathrm{j}\times\mathrm{i}=-\mathrm{k}, \mathrm{k}\times\mathrm{i}=\mathrm{j}, \mathrm{k}\times\mathrm{j}=-\mathrm{i}$

So we get

$\mathrm{k}-\mathrm{j}+\mathrm{i}-\mathrm{k}+\mathrm{j}-\mathrm{i}=0$

5. Find the unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ when

(i)

$\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$

(ii)

$\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$

(iii)

$\vec{a}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{b}=-\hat{i}+3\hat{k}$

(iv)

$\vec{a}=4\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+4\hat{j}-\hat{k}$

Solution:

(i) Consider $\vec{r}$ as the vector perpendicular to $\vec{a}$ and $\vec{b}$

So we get

$\vec{r}=\mathrm{k}.(\vec{a}\times\vec{b})$ where k is scalar

It is given that

$\vec{a}=3\mathrm{i}+\mathrm{j}-2\mathrm{k}$

$\vec{b}=2\mathrm{i}+3\mathrm{j}-\mathrm{k}$

We know that

$\vec{a}\times\vec{b}=(a_2b_3-b_2a_3)\mathrm{i}+(a_3b_1-b_3a_1)\mathrm{j}+(a_1b_2-b_1a_2)\mathrm{k}$

$a_1 = 3, a_2 = 1, a_3 = -2$

$b_1 = 2, b_2 = 3, b_3 = -1$

By substituting the values

$$\vec{a} \times \vec{b} = (1 \times -1 - 3 \times -2)i + (-2 \times 2 - (-1) \times 3)j + (3 \times 3 - 2 \times 1)k$$

We get

$$= \frac{5i - 1j + 7k}{5\sqrt{3}}$$

$$|a \times b| = \sqrt{(5)^2 + (-1)^2 + (7)^2}$$

$$= 5\sqrt{3}$$

So the unit vector perpendicular is

$$\vec{r} = \pm \frac{5i - 1j + 7k}{5\sqrt{3}}$$

(ii) Consider $\vec{r}$ as the vector perpendicular to $\vec{a}$ and $\vec{b}$

So we get

$\vec{r} = k.(\vec{a} \times \vec{b})$ where k is scalar

It is given that

$$\vec{a} = i - 2j + 3k$$

$$\vec{b} = i + 2j - k$$

We know that

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

$a_1 = 1, a_2 = -2, a_3 = 3$

$b_1 = 1, b_2 = 2, b_3 = -1$

By substituting the values

$$\vec{a} \times \vec{b} = (-2 \times -1 - 2 \times 3)i + (3 \times 1 - (-1) \times 1)j + (1 \times 2 - (-2) \times 1)k$$

We get

$$= \frac{-4i + 4j + 4k}{4\sqrt{3}}$$

$$|a \times b| = \sqrt{(-4)^2 + (4)^2 + (4)^2}$$

$$= 4\sqrt{3}$$

So the unit vector perpendicular is

$$\vec{r} = \pm \frac{-i+j+k}{\sqrt{3}}$$

(iii) Consider $\vec{r}$ as the vector perpendicular to $\vec{a}$ and $\vec{b}$

So we get

$\vec{r} = k.(\vec{a} \times \vec{b})$ where k is scalar

It is given that

$$\vec{a} = i + 3j - 2k$$

$$\vec{b} = -i + 0j + 3k$$

We know that

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

$a_1 = 1$, $a_2 = 3$, $a_3 = -2$

$b_1 = -1$, $b_2 = 0$, $b_3 = 3$

By substituting the values

$$\vec{a} \times \vec{b} = (9 - 0)i + (2 - 3)j + (0 - (-3))k$$

We get

$$= \frac{9i-j+3k}{\sqrt{91}}$$

$$|a \times b| = \sqrt{(9)^2 + (-1)^2 + (3)^2}$$

$$= \sqrt{91}$$

So the unit vector perpendicular is

$$\vec{r} = \pm \frac{9i-j+3k}{\sqrt{91}}$$

(iv) Consider $\vec{r}$ as the vector perpendicular to $\vec{a}$ and $\vec{b}$
So we get

$\vec{r} = k.(\vec{a} \times \vec{b})$ where k is scalar

It is given that

$\vec{a} = 4i + 2j - k$

$\vec{b} = i + 4j - k$

We know that

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

$a_1 = 4, a_2 = 2, a_3 = -1$

$b_1 = 1, b_2 = 4, b_3 = -1$

By substituting the values

$\vec{a} \times \vec{b} = (2 \times -1 - (-1) \times 4)i + (-1 \times 1 - (-1) \times 4)j + (4 \times 4 - 1 \times 2)k$

We get

$= \frac{2i+3j+14k}{\sqrt{209}}$

$|a \times b| = \sqrt{(2)^2 + (3)^2 + (14)^2}$

$= \sqrt{209}$

So the unit vector perpendicular is

$\vec{r} = \pm \frac{2i+3j+14k}{\sqrt{209}}$

6. Find the unit vectors perpendicular to the plane of the vectors $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$.

Solution:

Consider $\vec{r}$ as the vector perpendicular to $\vec{a}$ and $\vec{b}$

So we get

$\vec{r} = k.(\vec{a} \times \vec{b})$ where k is scalar

It is given that

$\vec{a} = 2i - 6j - 3k$

$\vec{b} = 4i + 3j - k$

We know that

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

$a_1 = 2$, $a_2 = -6$, $a_3 = -3$

$b_1 = 4$, $b_2 = 3$, $b_3 = -1$

By substituting the values

$$\vec{a} \times \vec{b} = (-6 \times (-1) - 3 \times (-3))i + (-3 \times 4 - (-1) \times 2)j + (2 \times 3 - 4 \times (-6))k$$

We get

$$= \frac{3i - 2j + 6k}{7}$$

$$|a \times b| = \sqrt{(15)^2 + (-10)^2 + (30)^2}$$

$$= \sqrt{1225}$$

So the unit vector perpendicular is

$$\vec{r} = \pm \frac{3i - 2j + 6k}{7}$$

7. Find a vector of magnitude 6 which is perpendicular to both the vectors $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$.

Solution:

Consider $\vec{r}$ as the vector perpendicular to $\vec{a}$ and $\vec{b}$

So we get

$\vec{r} = k.(\vec{a} \times \vec{b})$ where k is scalar

Vector r is of magnitude 6

It is given that

$$\vec{a} = 4i - j + 3k$$

$$\vec{b} = -2i + j - 2k$$

We know that

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

$a_1 = 4$, $a_2 = -1$, $a_3 = 3$

$b_1 = -2$, $b_2 = 1$, $b_3 = -2$

By substituting the values

$$\vec{a} \times \vec{b} = (-1 \times (-2) - 1 \times (3))i + (3 \times (-2) - (-2) \times 4)j + (4 \times 1 - (-2) \times (-1))k$$

We get

$$= \frac{-i+2j+2k}{3}$$

$$|a \times b| = \sqrt{(-1)^2 + (2)^2 + (2)^2}$$

= 3

So the unit vector perpendicular is

$$\vec{r} = \pm k.\frac{-i + 2j + 2k}{3}$$

It is given that r is of magnitude 6

i.e. k = 6

We get

$$\vec{r} = \pm 2(-i + 2j + 2k)$$

8. Find a vector of magnitude 5 units, perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = (\hat{i} + \hat{j} + \hat{k})$ and $\vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k})$

Solution:

We know that

$$\vec{a} + \vec{b} = 2i + 3j + 4k = \vec{l}$$

$$\vec{a} - \vec{b} = 0i - i - 2k = \vec{m}$$

Consider $\vec{r}$ as the vector perpendicular to $\vec{l}$ *and* $\vec{m}$

So we get

$\vec{r} = k.(\hat{l} \times \hat{m})$ where k is scalar

Vector r is of magnitude 5

It is given that

$\vec{l} = 2i + 3j + 4k$

$\vec{m} = 0i - j - 2k$

We know that

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

$a_1 = 2$, $a_2 = 3$, $a_3 = 4$

$b_1 = 0$, $b_2 = -1$, $b_3 = -2$

By substituting the values

$\vec{l} \times \vec{m} = (-2)i + (4)j + (-2)k$

We get

$= \frac{-i+2j-k}{\sqrt{6}}$

$|a \times b| = \sqrt{(-2)^2 + (4)^2 + (-2)^2}$

$= \sqrt{24}$

So the unit vector perpendicular is

$\vec{r} = \pm k.\frac{-i + 2j - k}{\sqrt{6}}$

It is given that r is of magnitude 5

i.e. k = 5

We get

$\vec{r} = \pm 5(\frac{-i + 2j - k}{\sqrt{6}})$

9. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and $|\vec{a} \times \vec{b}| = \sqrt{3}$.
Solution:

It is given that

$\overrightarrow{|a|} = 1$ and $\overrightarrow{|b|} = 2$

$|\vec{a} \times \vec{b}| = \sqrt{3}$

We know that

$$|\vec{a} \times \vec{b}| = \overrightarrow{|a|}.\overrightarrow{|b|}\sin\theta = \sqrt{3}$$

So we get

$$\overrightarrow{|a|}.\overrightarrow{|b|}\sin\theta = 1 \times 2 \times \sin\theta$$

On further simplification

$$2\sin\theta = \sqrt{3}$$

We get

$$\theta = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

10. If $\vec{a} = (\hat{i} - \hat{j})$, $\vec{b} = (3\,\hat{j} - \hat{k})$ and $\vec{c} = (7\,\hat{i} - \hat{k})$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and for which $\vec{c}.\vec{d} = 1$.

Solution:

Consider $\vec{d}$ as the vector perpendicular to $\vec{a}$ and $\vec{b}$

So we get

$\vec{d} = k.(\hat{a} \times \hat{b})$ where k is scalar

It is given that

$$\vec{a} = i - j$$

$$\vec{b} = 0i + 3j - k$$

We know that

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

$a_1 = 1$, $a_2 = -1$, $a_3 = 0$

$b_1 = 0$, $b_2 = 3$, $b_3 = -1$

By substituting the values

$$\vec{a} \times \vec{b} = (1)i + (1)j + (3)k$$

We get

$$= \frac{i+j+3k}{\sqrt{11}}$$

$$|a \times b| = \sqrt{(1)^2 + (1)^2 + (3)^2}$$

$$= \sqrt{11}$$

So the unit vector perpendicular is

$$\vec{d} = \pm k.\frac{i + j + 3k}{\sqrt{11}}$$

It is given that

$$\vec{c}.\vec{d} = 1$$

We get

$$\vec{c} = 7i - k$$

$$\vec{c}.\vec{d} = \frac{7k - 3k}{\sqrt{11}} = 1$$

Here

$$k = \frac{\sqrt{11}}{4}$$

So we get

$$\vec{d} = \frac{i + j + 3k}{4}$$

11. If $\bar{a} = (4\hat{i} + 5\hat{j} - \hat{k}),\ \bar{b} = (\hat{i} - 4\hat{j} + 5\hat{k})$ and $\bar{c} = (3\hat{i} + \hat{j} - \hat{k})$**, find a vector** $\vec{d}$ **which is perpendicular to both** $\vec{a}$ **and** $\vec{b}$ **and for which** $\vec{c} \cdot \vec{d} = 21$
Solution:

Consider $\vec{d}$ as the vector perpendicular to $\vec{a}$ *and* $\vec{b}$

$\vec{d} = k.(\hat{a} \times \hat{b})$ where k is a scalar

It is given that

$$\vec{a} = 4i + 5j - k$$

$$\vec{b} = i - 4j + 5k$$

We know that

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

$a_1 = 4$, $a_2 = 5$, $a_3 = -1$

$b_1 = 1$, $b_2 = -4$, $b_3 = 5$

By substituting these values in the above equation

$$\vec{a} \times \vec{b} = 21i - 21j - 21k$$

$$\vec{d} = k.\ 21i - 21j - 21k$$

We know that

$$\vec{c}.\vec{d} = 21$$

$$\vec{c} = 3i + j - k$$

So we get

$$(3i + j - k)\,k(21i - 21j - 21k) = 21$$

On further calculation

$63k - 21k + 21k = 21$

$63k = 21$

$k = 1/3$

By substituting the value of k in $\vec{d}$ we get

$\vec{d} = 7i - 7j - 7k$

Taking 7 as common

$\vec{d} = 7\,(i - j - k)$

12. Prove that $|\vec{a} \times \vec{b}| = (\vec{a} \cdot \vec{b})\tan\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.
Solution:

Here we know that

$$|\vec{a}.\vec{b}| = ||\vec{a}||\vec{b}|\cos\theta|$$

Similarly

$$|\vec{a} \times \vec{b}| = ||\vec{a}||\vec{b}|\sin\theta|$$

We know that

$$\tan\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}.\vec{b}|}$$

Therefore, it is proved.

13. Write the value of p for which $\vec{a} = \left(3\,\hat{i} + 2\,\hat{j} + 9\,\hat{k}\right)$ and $\vec{b} = \left(\hat{i} + p\,\hat{j} + 3\,\hat{k}\right)$ are parallel vectors.
Solution:

If two vectors are parallel

$\vec{a} \times \vec{b} = 0$

It is given that

$\vec{a} = 3i + 2j + 9k$

$\vec{b} = i + pj + 3k$

We know that

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

$a_1 = 3, a_2 = 2, a_3 = 9$

$b_1 = 1, b_2 = p, b_3 = 3$

By substituting the values

$\vec{a} \times \vec{b} = (6 - 9p)i + (0)j + (3p - 2)k = 0$

We get

$6 - 9p = 0$

Here $p = 2/3$