

13. Method of Integration

Exercise 13A

1. Question

Evaluate the following integrals:

$$\int (2x + 9)^5 dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{(n+1)}}{n+1} + c$$

Therefore ,

$$\text{Put } 2x + 9 = t \Rightarrow 2 dx = dt$$

$$\begin{aligned} \int t^5 \left(\frac{dt}{2}\right) &= \frac{1}{2} \int t^5 dt = \frac{1}{2} \frac{t^6}{6} + c = \frac{t^6}{12} + c \\ &= \frac{(2x + 9)^6}{12} + c \end{aligned}$$

2. Question

Evaluate the following integrals:

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{(n+1)}}{n+1} + c$$

Therefore ,

$$\text{Put } 7 - 3x = t \Rightarrow -3 dx = dt$$

$$\begin{aligned} \int t^4 \left(\frac{dt}{-3}\right) &= \frac{1}{-3} \int t^4 dt = \frac{1}{-3} \frac{t^5}{5} + c = -\frac{t^5}{15} + c \\ &= -\frac{(7 - 3x)^5}{15} + c \end{aligned}$$

3. Question

Evaluate the following integrals:

$$\int \sqrt{3x - 5} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{(n+1)}}{n+1} + c$$

Therefore ,

$$\text{Put } 3x - 5 = t \Rightarrow 3 dx = dt$$

$$\begin{aligned} \int t^{0.5} \left(\frac{dt}{3}\right) &= \frac{1}{3} \int t^{0.5} dt = \frac{1}{3} \times \frac{t^{1.5}}{1.5} + c = \frac{2}{1} \times \frac{t^{1.5}}{9} + c \\ &= \frac{2(3x - 5)^{1.5}}{9} + c \end{aligned}$$

4. Question

Evaluate the following integrals:

$$\int \frac{1}{\sqrt{4x+3}} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{(n+1)}}{n+1} + c$$

Therefore ,

$$\text{Put } 4x + 3 = t \Rightarrow 4 dx = dt$$

$$\begin{aligned} \int t^{-0.5} \left(\frac{dt}{4}\right) &= \frac{1}{4} \int t^{-0.5} dt = \frac{1}{4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{4} \times \frac{t^{0.5}}{1} + c \\ &= \frac{\sqrt{4x+3}}{2} + c \end{aligned}$$

5. Question

Evaluate the following integrals:

$$\int \frac{1}{\sqrt{3-4x}} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{(n+1)}}{n+1} + c$$

Therefore ,

$$\text{Put } 3 - 4x = t \Rightarrow -4 dx = dt$$

$$\begin{aligned} \int t^{-0.5} \left(\frac{dt}{-4}\right) &= \frac{1}{-4} \int t^{-0.5} dt = \frac{1}{-4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{-4} \times \frac{t^{0.5}}{1} + c \\ &= -\frac{\sqrt{3-4x}}{2} + c \end{aligned}$$

6. Question

Evaluate the following integrals:

$$\int \frac{1}{(2x-3)^{3/2}} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{(n+1)}}{n+1} + c$$

Therefore ,

$$\text{Put } 2x - 3 = t \Rightarrow 2 dx = dt$$

$$\begin{aligned} \int t^{-\frac{3}{2}} \left(\frac{dt}{2}\right) &= \frac{1}{2} \int t^{-\frac{3}{2}} dt = \frac{1}{2} \times \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + c = \frac{-2}{2} \times \frac{t^{-0.5}}{1} + c \\ &= -\frac{1}{\sqrt{2x-3}} + c \end{aligned}$$

7. Question

Evaluate the following integrals:

$$\int e^{(2x-1)} dx$$

Answer

$$\text{Formula} = \int e^x dx = e^x + c$$

Therefore ,

$$\text{Put } 2x - 1 = t \Rightarrow 2 dx = dt$$

$$\begin{aligned} \int e^t \left(\frac{dt}{2}\right) &= \frac{1}{2} \int e^t dt = \frac{1}{2} \times e^t + c = \frac{e^{2x-1}}{2} + c \\ &= \frac{e^{(2x-1)}}{2} + c \end{aligned}$$

8. Question

Evaluate the following integrals:

$$\int e^{(1-3x)} dx$$

Answer

$$\text{Formula} = \int e^x dx = e^x + c$$

Therefore ,

$$\text{Put } 1 - 3x = t \Rightarrow -3 dx = dt$$

$$\begin{aligned} \int e^t \left(\frac{dt}{-3}\right) &= \frac{1}{-3} \int e^t dt = \frac{1}{-3} \times e^t + c = \frac{e^{1-3x}}{-3} + c \\ &= -\frac{e^{(1-3x)}}{3} + c \end{aligned}$$

9. Question

Evaluate the following integrals:

$$\int 3^{(2-3x)} dx$$

Answer

$$\text{Formula} = \int a^x dx = \frac{a^x}{\log a} + c$$

Therefore ,

$$\text{Put } 2 - 3x = t \Rightarrow -3 dx = dt$$

$$\begin{aligned} \int 3^t \left(\frac{dt}{-3}\right) &= \frac{1}{-3} \int 3^t dt = \frac{1}{-3} \times \left(\frac{3^t}{\log 3}\right) + c = \frac{3^t}{-3 \log 3} + c \\ &= -\frac{3^{(2-3x)}}{3 \log 3} + c \end{aligned}$$

10. Question

Evaluate the following integrals:

$$\int \sin 3x dx$$

Answer

$$\text{Formula} = \int \sin x dx = -\cos x + c$$

Therefore ,

$$\text{Put } 3x = t \Rightarrow 3 dx = dt$$

$$\begin{aligned}\int \sin t \left(\frac{dt}{3}\right) &= \frac{1}{3} \int \sin t dt = \frac{1}{3} \times (-\cos t) + c = \frac{-\cos 3x}{3} + c \\ &= -\frac{\cos 3x}{3} + c\end{aligned}$$

11. Question

Evaluate the following integrals:

$$\int \cos(5 + 6x) dx$$

Answer

$$\text{Formula } = \int \cos x dx = \sin x + c$$

Therefore ,

$$\text{Put } 5 + 6x = t \Rightarrow 6 dx = dt$$

$$\begin{aligned}\int \cos t \left(\frac{dt}{6}\right) &= \frac{1}{6} \int \cos t dt = \frac{1}{6} \times (\sin t) + c = \frac{\sin 5 + 6x}{6} + c \\ &= \frac{\sin(5 + 6x)}{6} + c\end{aligned}$$

12. Question

Evaluate the following integrals:

$$\int \sin x \sqrt{1 + \cos 2x} dx$$

Answer

$$\text{Formula } \int \cos x dx = \sin x + c$$

$$1 + \cos 2x = 2\cos^2 x$$

Therefore ,

$$\int \sin x \sqrt{1 + \cos 2x} dx = \int \sin x \sqrt{2} \cos x + c$$

$$\int \sqrt{2} \sin x \cos x dx$$

$$\text{Put } \sin x = t \Rightarrow \cos x dx = dt$$

$$\begin{aligned}\int \sqrt{2} \sin x \cos x dx &= \int \sqrt{2} t dt = \sqrt{2} \frac{t^2}{2} + c \\ &= \frac{(\sin x)^2}{\sqrt{2}} + c\end{aligned}$$

13. Question

Evaluate the following integrals:

$$\int \operatorname{cosec}^2(2x + 5) dx$$

Answer

$$\text{Formula } \int \operatorname{cosec}^2 x dx = -\cot x + c$$

Therefore ,

Put $2x + 5 = t \Rightarrow 2 dx = dt$

$$\begin{aligned}\int \operatorname{cosec}^2 t \frac{dt}{2} &= -\frac{1}{2} \cot t + c = -\frac{1}{2} \cot(2x + 5) + c \\ &= -\frac{1}{2} \cot(2x + 5) + c\end{aligned}$$

14. Question

Evaluate the following integrals:

$$\int \sin x \cos x dx$$

Answer

Formula $\int \sin x dx = -\cos x + c$

Therefore ,

Put $\sin x = t \Rightarrow \cos x dx = dt$

$$\begin{aligned}\int t dt &= \frac{t^2}{2} + c \\ &= \frac{(\sin x)^2}{2} + c\end{aligned}$$

15. Question

Evaluate the following integrals:

$$\int \sin^3 x \cos x dx$$

Answer

Formula $\int \sin x dx = -\cos x + c$

Therefore ,

Put $\sin x = t \Rightarrow \cos x dx = dt$

$$\begin{aligned}\int t^3 dt &= \frac{t^4}{4} + c \\ &= \frac{(\sin x)^4}{4} + c\end{aligned}$$

16. Question

Evaluate the following integrals:

$$\int (\sqrt{\cos x}) \sin x dx$$

Answer

Formula $\int \sin x dx = -\cos x + c$

Therefore ,

Put $\cos x = t \Rightarrow -\sin x dx = dt$

$$\int t^{0.5} (-1) dt = -\frac{t^{1.5}}{1.5} + c$$

$$= -\frac{2(\cos x)^3}{3} + c$$

17. Question

Evaluate the following integrals:

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

Answer

$$\text{Formula } \int x^n dx = \frac{x^{(n+1)}}{n+1} + c \quad \frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Therefore ,

$$\text{Put } \sin^{-1} x = t \Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\int t^1 dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin^{-1} x)^2}{2} + c$$

18. Question

Evaluate the following integrals:

$$\int \frac{\sin(2 \tan^{-1} x)}{(1+x^2)} dx.$$

Answer

$$\text{Formula } \int \sin t dx = -\cos t + c \quad \frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$$

Therefore ,

$$\text{Put } \tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$$

$$\int \sin 2t dt = \frac{-\cos 2t}{2} + c$$

$$= -\frac{\cos(2 \tan^{-1} x)}{2} + c$$

19. Question

Evaluate the following integrals:

$$\int \frac{\cos(\log x)}{x} dx$$

Answer

$$\text{Formula } \int \cos t dx = \sin t + c \quad \frac{d(\log x)}{dx} = \frac{1}{x}$$

Therefore ,

$$\text{Put } \log x = t \Rightarrow \frac{1}{x} dx = dt$$

$$\int \cos t \, dt = \sin t + c$$

$$= \sin(\log x) + c$$

20. Question

Evaluate the following integrals:

$$\int \frac{\operatorname{cosec}^2(\log x)}{x} dx$$

Answer

$$\text{Formula } \int \operatorname{cosec}^2 x \, dx = -\cot x + c \quad \frac{d(\log x)}{dx} = \frac{1}{x}$$

Therefore ,

$$\text{Put } \log x = t \Rightarrow \frac{1}{x} dx = dt$$

$$\int \operatorname{cosec}^2 t \frac{dt}{1} = -\cot t + c = -\cot(\log x) + c$$

$$= -\cot(\log x) + c$$

21. Question

Evaluate the following integrals:

$$\int \frac{1}{x \log x} dx$$

Answer

$$\text{Formula } \frac{d(\log x)}{dx} = \frac{1}{x} \int \frac{1}{x} dx = \log x$$

Therefore ,

$$\text{Put } \log x = t \Rightarrow \frac{1}{x} dx = dt$$

$$\int \frac{dt}{t} = \log t + c = \log(\log x) + c$$

$$= \log(\log x) + c$$

22. Question

Evaluate the following integrals:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx$$

Answer

$$\text{Formula } \frac{d(\log x)}{dx} = \frac{1}{x} \int \frac{1}{x} dx = \log x$$

$$\begin{aligned} \int \frac{(x+1)(x+\log x)^2}{x} dx &= \int \frac{x+1}{x} \times \frac{(x+\log x)^2}{1} dx \\ &= \int \left(1 + \frac{1}{x}\right) \times \frac{(x+\log x)^2}{1} dx \end{aligned}$$

Therefore ,

$$\text{Put } x + \log x = t \Rightarrow \left(1 + \frac{1}{x}\right) dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{(x + \log x)^3}{3} + c$$

23. Question

Evaluate the following integrals:

$$\int \frac{(\log x)^2}{x} dx$$

Answer

$$\text{Formula } \frac{d(\log x)}{dx} = \frac{1}{x} \int \frac{1}{x} dx = \log x$$

Therefore ,

$$\text{Put } \log x = t \Rightarrow \frac{1}{x} dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c = \frac{(\log x)^3}{3} + c$$

$$= \frac{(\log x)^3}{3} + c$$

24. Question

Evaluate the following integrals:

$$\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$$

Answer

$$\text{Formula } \int \cos t dx = \sin t + c \quad \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

Therefore ,

$$\text{Put } \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\int \cos t \cdot 2 dt = 2 \sin t + c$$

$$= 2 \sin(\sqrt{x}) + c$$

25. Question

Evaluate the following integrals:

$$\int e^{\tan x} \sec^2 x dx$$

Answer

$$\text{Formula } = \int e^x dx = e^x + c \quad \frac{d(\tan x)}{dx} = \sec^2 x$$

Therefore ,

$$\text{Put } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\int e^t dt = e^t + c$$

$$= e^{\tan x} + c$$

26. Question

Evaluate the following integrals:

$$\int e^{\cos^2 x} \sin 2x \, dx$$

Answer

$$\text{Formula} = \int e^x dx = e^x + c \quad \frac{d(\cos^2 x)}{dx} = 2 \cos x (-\sin x) = -\sin 2x$$

Therefore ,

$$\text{Put } \cos^2 x = t \Rightarrow -\sin 2x \, dx = dt$$

$$\int -e^t dt = -e^t + c$$

$$= -e^{\cos^2 x} + c$$

27. Question

Evaluate the following integrals:

$$\int \sin(ax + b) \cos(ax + b) \, dx$$

Answer

$$\text{Formula} = \int \sin x \, dx = -\cos x + c$$

Therefore ,

$$\text{Put } ax+b = t \Rightarrow a \, dx = dt$$

$$\int \sin t \cos t \frac{dt}{a} = \frac{1}{a} \int \sin t \cos t \, dt$$

$$\text{Put } \sin t = z \Rightarrow \cos t \, dt = dz$$

$$\frac{1}{a} \int z \, dz = \frac{1}{a} \times \frac{z^2}{2} + c$$

$$= \frac{(\sin ax + b)^2}{2a} + c$$

28. Question

Evaluate the following integrals:

$$\int \cos^3 x \, dx$$

Answer

$$\text{Formula} = \int \cos x \, dx = \sin x + c$$

$$\cos 3x = 3 \cos x - 4 \cos^3 x$$

Therefore ,

$$\int \left(\frac{3 \cos x}{4} - \frac{\cos 3x}{4} \right) dx = \frac{3 \sin x}{4} - \frac{\sin 3x}{4 \times 3} + c$$

$$= \frac{3 \sin x}{4} - \frac{\sin 3x}{12} + c$$

29. Question

Evaluate the following integrals:

$$\int \frac{1}{x^2} e^{-1/x} dx$$

Answer

$$\text{Formula} = \int e^x dx = e^x + c$$

Therefore ,

$$\text{Put } -\frac{1}{x} = t \Rightarrow \frac{1}{x^2} dx = dt$$

$$\int e^t (dt) = \int e^t dt = e^t + c = e^{-\frac{1}{x}} + c$$

$$= e^{-\frac{1}{x}} + c$$

30. Question

Evaluate the following integrals:

$$\int \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$$

Answer

$$\text{Formula} = \int \cos x dx = \sin x + c$$

Therefore ,

$$\text{Put } -\frac{1}{x} = t \Rightarrow \frac{1}{x^2} dx = dt$$

$$\int \cos t (dt) = \int \cos t dt = \sin t + c = \sin\left(-\frac{1}{x}\right) + c$$

$$= -\sin\frac{1}{x} + c$$

31. Question

Evaluate the following integrals:

$$\int \frac{dx}{(e^x + e^{-x})}$$

Answer

$$\text{Formula} = \int e^x dx = e^x + c$$

Therefore ,

$$\int \frac{e^x}{1 + e^{2x}} dx$$

$$\text{Put } e^x = t \Rightarrow e^x dx = dt$$

$$\int \frac{1}{1 + t^2} (dt) = \int \frac{1}{1 + t^2} dt = \tan^{-1} t + c$$

$$= \tan^{-1}(e^x) + c$$

32. Question

Evaluate the following integrals:

$$\int \frac{e^{2x}}{(e^{2x} - 2)} dx$$

Answer

$$\text{Formula} = \int e^x dx = e^x + c$$

Therefore ,

$$\text{Put } e^{2x} - 2 = t \Rightarrow 2e^{2x} dx = dt$$

$$\begin{aligned} \int \frac{1}{t} \left(\frac{dt}{2} \right) &= \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \log t + c \\ &= \frac{1}{2} \log(e^{2x} - 2) + c \end{aligned}$$

33. Question

Evaluate the following integrals:

$$\int \cot x \log(\sin x) dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\text{Put } \log(\sin x) = t \Rightarrow \frac{\cos x}{\sin x} dx = dt \quad \diamond \quad \cot x dx = dt$$

$$\begin{aligned} \int t dt &= \frac{t^2}{2} + c \\ &= \frac{(\log \sin x)^2}{2} + c \end{aligned}$$

34. Question

Evaluate the following integrals:

$$\int \frac{\cot x}{\log(\sin x)} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\text{Put } \log(\sin x) = t \Rightarrow \frac{\cos x}{\sin x} dx = dt \quad \diamond \quad \cot x dx = dt$$

$$\begin{aligned} \int \frac{1}{t} dt &= \log t + c \\ &= \log(\log \sin x) + c \end{aligned}$$

35. Question

Evaluate the following integrals:

$$\int 2x \sin(x^2 + 1) dx$$

Answer

$$\text{Formula} = \int \sin x \, dx = -\cos x + c$$

Therefore ,

$$\text{Put } x^2 + 1 = t \Rightarrow 2x \, dx = dt$$

$$\int \sin t \, dt = -\cos t + c$$

$$= -\cos(x^2 + 1) + c$$

36. Question

Evaluate the following integrals:

$$\int \sec x \log(\sec x + \tan x) \, dx$$

Answer

$$\text{Formula} = \int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\text{Put } \log(\sec x + \tan x) = t$$

$$\frac{1}{\sec x + \tan x} \times (\sec x \tan x + \sec^2 x) \, dx = dt$$

$$\frac{1}{\sec x + \tan x} \times \sec x (\sec x + \tan x) \, dx = dt$$

$$\sec x \, dx = dt$$

$$\int t \, dt = \frac{t^2}{2} + c$$

$$= \frac{(\log(\sec x + \tan x))^2}{2} + c$$

37. Question

Evaluate the following integrals:

$$\int \frac{\tan \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}} \, dx$$

Answer

$$\text{Formula} = \int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\tan \sqrt{x} = t$$

$$\sec^2 \sqrt{x} \times \left(\frac{1}{2\sqrt{x}}\right) \, dx = dt$$

$$\int t \, dt = \frac{t^2}{2} + c$$

$$= \frac{(\tan \sqrt{x})^2}{2} + c$$

38. Question

Evaluate the following integrals:

$$\int \frac{x \tan^{-1} x^2}{(1+x^4)} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\text{Put } \tan^{-1} x^2 = t \Rightarrow \frac{1}{1+(x^2)^2} \times 2x \times dx = dt \quad \diamond \quad \frac{2x}{1+x^4} dx = dt$$

$$\int t \left(\frac{dt}{2} \right) = \frac{1}{2} \int t dt = \frac{t^2}{4} + c$$

$$= \frac{(\tan^{-1} x^2)^2}{4} + c$$

39. Question

Evaluate the following integrals:

$$\int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\text{Put } \sin^{-1} x^2 = t \Rightarrow \frac{1}{\sqrt{1-(x^2)^2}} \times 2x \times dx = dt \quad \diamond \quad \frac{2x}{\sqrt{1-x^4}} dx = dt$$

$$\int t \left(\frac{dt}{2} \right) = \frac{1}{2} \int t dt = \frac{t^2}{4} + c$$

$$= \frac{(\sin^{-1} x^2)^2}{4} + c$$

40. Question

Evaluate the following integrals:

$$\int \frac{1}{(\sqrt{1-x^2}) \sin^{-1} x} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\text{Put } \sin^{-1} x = t \Rightarrow \frac{1}{\sqrt{1-(x^2)^2}} \times dx = dt \quad \diamond \quad \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\int \frac{1}{t} \left(\frac{dt}{1} \right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log \sin^{-1} x + c$$

41. Question

Evaluate the following integrals:

$$\int \frac{\sqrt{(2 + \log x)}}{x} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\text{Put } 2 + \log x = t \Rightarrow \frac{1}{x} \times dx = dt$$

$$\int \sqrt{t} \left(\frac{dt}{1} \right) = \int \sqrt{t} dt = \frac{2t^{1.5}}{3} + c$$

$$= \frac{2(2 + \log x)^{\frac{3}{2}}}{3} + c$$

42. Question

Evaluate the following integrals:

$$\int \frac{\sec^2 x}{(1 + \tan x)} dx$$

Answer

$$\text{Formula} = \int \frac{1}{x} dx = \log x + c$$

Therefore ,

$$\text{Put } 1 + \tan x = t \Rightarrow \sec^2 x \times dx = dt$$

$$\int \left(\frac{dt}{t} \right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(1 + \tan x) + c$$

43. Question

Evaluate the following integrals:

$$\int \frac{\sin x}{(1 + \cos x)} dx$$

Answer

$$\text{Formula} = \int \cos x dx = \sin x + c$$

Therefore ,

$$\text{Put } 1 + \cos x = t \Rightarrow -\sin x \times dx = dt$$

$$\int \left(\frac{-dt}{t} \right) = - \int \frac{1}{t} dt = -\log t + c$$

$$= -\log(1 + \cos x) + c$$

44. Question

Evaluate the following integrals:

$$\int \left(\frac{1 + \tan x}{1 - \tan x} \right) dx$$

Answer

$$\text{Formula} = \int \cos x \, dx = \sin x + c$$

Therefore ,

$$\int \left(\frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} \right) dx = \int \left(\frac{\cos x + \sin x}{\cos x - \sin x} \right) dx$$

$$\text{Put } \cos x - \sin x = t \Rightarrow (-\cos x - \sin x) \, dx = dt$$

$$\int \left(\frac{-dt}{t} \right) = - \int \frac{1}{t} dt = -\log t + c$$

$$= -\log(\cos x - \sin x) + c$$

45. Question

Evaluate the following integrals:

$$\text{i. } \int \frac{(1 + \tan x)}{(x + \log \sec x)} dx$$

$$\text{ii. } \int \frac{(1 - \sin 2x)}{(x + \cos^2 x)} dx$$

Answer

(i)

$$\text{Formula} = \int \frac{1}{x} dx = \log x + c$$

Therefore ,

$$\text{Put } x + \log(\sec x) = t \Rightarrow 1 + \frac{1}{\sec x} \times \sec x \tan x \, dx = dt$$

$$(1 + \tan x) dx = dt$$

$$\int \left(\frac{dt}{t} \right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(x + \log(\sec x)) + c$$

(ii)

$$\text{Formula} = \int \frac{1}{x} dx = \log x + c$$

Therefore ,

$$\text{Put } x + \cos^2 x = t \Rightarrow 1 + 2 \cos x \times (-\sin x) dx = dt$$

$$(1 - \sin 2x) dx = dt$$

$$\int \left(\frac{dt}{t} \right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(x + \cos^2 x) + c$$

46. Question

Evaluate the following integrals:

$$\int \frac{\sin 2x}{(a^2 + b^2 \sin^2 x)} dx$$

Answer

$$\text{Formula} = \int \frac{1}{x} dx = \log x + c$$

Therefore ,

$$\text{Put } a^2 + b^2 \sin^2 x = t \Rightarrow b^2 \times 2 \sin x \times \cos x dx = dt$$

$$(b^2 \sin 2x) dx = dt$$

$$\int \frac{1}{t} \left(\frac{dt}{b^2} \right) = \frac{1}{b^2} \int \frac{1}{t} dt = \frac{1}{b^2} \log t + c$$

$$= \frac{1}{b^2} \log |a^2 + b^2 \sin^2 x| + c$$

47. Question

Evaluate the following integrals:

$$\int \frac{\sin 2x}{(a^2 \cos^2 x + b^2 \sin^2 x)} dx$$

Answer

$$\text{Formula} = \int \frac{1}{x} dx = \log x + c$$

Therefore ,

$$\text{Put } a^2 \cos^2 x + b^2 \sin^2 x = t$$

$$(a^2 \times 2 \cos x \times (-\sin x) + b^2 \times 2 \sin x \times \cos x) dx = dt$$

$$(b^2 - a^2) \sin 2x dx = dt$$

$$\int \frac{1}{t} \left(\frac{dt}{b^2 - a^2} \right) = \frac{1}{b^2 - a^2} \int \frac{1}{t} dt = \frac{1}{b^2 - a^2} \log t + c$$

$$= \frac{1}{b^2 - a^2} \log |a^2 \cos^2 x + b^2 \sin^2 x| + c$$

48. Question

Evaluate the following integrals:

$$\int \left(\frac{2 \cos x - 3 \sin x}{3 \cos x + 2 \sin x} \right) dx$$

Answer

$$\text{Formula} = \int \cos x dx = \sin x + c$$

Therefore ,

$$\text{Put } 3 \cos x + 2 \sin x = t \Rightarrow (2 \cos x - 3 \sin x) dx = dt$$

$$\int \left(\frac{dt}{t} \right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(3 \cos x + 2 \sin x) + c$$

49. Question

Evaluate the following integrals:

$$\int \frac{4x}{(2x^2 + 3)} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\text{Put } 2x^2 + 3 = t \Rightarrow (4x) dx = dt$$

$$\int \left(\frac{dt}{t} \right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(2x^2 + 3) + c$$

50. Question

Evaluate the following integrals:

$$\int \frac{(x+1)}{(x^2 + 2x - 3)} dx$$

Answer

$$\text{Formula} = \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Therefore ,

$$\text{Put } x^2 + 2x + 3 = t \Rightarrow (2x+2) dx = dt \quad \diamond \quad 2(x+1)dx = dt$$

$$\int \frac{1}{t} \left(\frac{dt}{2} \right) = \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \log t + c$$

$$= \frac{1}{2} \log(x^2 + 2x + 3) + c$$

51. Question

Evaluate the following integrals:

$$\int \frac{(4x-5)}{(2x^2 - 5x + 1)} dx$$

Answer

$$\text{To find: Value of } \int \frac{4x-5}{(2x^2 - 5x + 1)} dx$$

$$\text{Formula used: } \int \frac{1}{x} dx = \log|x| + c$$

$$\text{We have, } I = \int \frac{4x-5}{(2x^2 - 5x + 1)} dx \dots (i)$$

$$\text{Let } 2x^2 - 5x + 1 = t$$

$$\Rightarrow \frac{d(2x^2 - 5x + 1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow 4x - 5 = \frac{dt}{dx}$$

$$\Rightarrow (4x - 5)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t} [2x^2 - 5x + 1 = t]$$

$$I = \log|t| + c$$

$$I = \log|2x^2 - 5x + 1| + c$$

$$\text{Ans) } \log|2x^2 - 5x + 1| + c$$

52. Question

Evaluate the following integrals:

$$\int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$$

Answer

$$\text{To find: Value of } \int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$$

$$\text{Formula used: } \int \frac{1}{x} dx = \log|x| + c$$

$$\text{We have, } I = \int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx \dots (i)$$

$$\text{Let } 3x^3 - 2x^2 + 5x + 1 = t$$

$$\Rightarrow \frac{d(3x^3 - 2x^2 + 5x + 1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow 9x^2 - 4x + 5 = \frac{dt}{dx}$$

$$\Rightarrow (9x^2 - 4x + 5)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t} [3x^3 - 2x^2 + 5x + 1 = t]$$

$$I = \log|t| + c$$

$$I = \log|3x^3 - 2x^2 + 5x + 1| + c$$

$$\text{Ans) } \log|3x^3 - 2x^2 + 5x + 1| + c$$

53. Question

Evaluate the following integrals:

$$\int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$$

Answer

$$\text{To find: Value of } \int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx \dots (i)$

Let $\log(\tan x) = t$

$$\Rightarrow \frac{d(\log(\tan x))}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(\log(\tan x))}{d \tan x} \cdot \frac{d \tan x}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{\tan x} \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \sec x \operatorname{cosec} x = \frac{dt}{dx}$$

$$\Rightarrow (\sec x \operatorname{cosec} x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t} [\log(\tan x) = t]$$

$$I = \log|t| + c$$

$$I = \log|\log(\tan x)| + c$$

Ans) $\log|\log(\tan x)| + c$

54. Question

Evaluate the following integrals:

$$\int \frac{(1 + \cos x)}{(x + \sin x)^3} dx$$

Answer

To find: Value of $\int \frac{(1 + \cos x)}{(x + \sin x)^3} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(1 + \cos x)}{(x + \sin x)^3} dx \dots (i)$

Let $x + \sin x = t$

$$\Rightarrow \frac{d(x + \sin x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(x)}{dx} + \frac{d(\sin x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (1 + \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (1 + \cos x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^3} [x + \sin x = t]$$

$$\Rightarrow I = -\frac{1}{2t^2} + c$$

$$I = -\frac{1}{2(x + \sin x)^2} + c$$

$$\text{Ans) } -\frac{1}{2(x + \sin x)^2} + c$$

55. Question

Evaluate the following integrals:

$$\int \frac{\sin x}{(1 + \cos x)^2} dx$$

Answer

$$\text{To find: Value of } \int \frac{\sin x}{(1 + \cos x)^2} dx$$

$$\text{Formula used: } \int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

$$\text{We have, } I = \int \frac{\sin x}{(1 + \cos x)^2} dx \dots (i)$$

$$\text{Let } 1 + \cos x = t$$

$$\Rightarrow \frac{d(1 + \cos x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(1)}{dx} + \frac{d(\cos x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (0 - \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (-\sin x) dx = dt$$

Putting this value in equation (i)

$$I = \int -\frac{dt}{t^2} [1 + \cos x = t]$$

$$\Rightarrow I = \frac{1}{t} + c$$

$$I = \frac{1}{1 + \cos x} + c$$

$$\text{Ans) } \frac{1}{1 + \cos x} + c$$

56. Question

Evaluate the following integrals:

$$\int \frac{(2x+3)}{\sqrt{x^2+3x-2}} dx$$

Answer

To find: Value of $\int \frac{(2x+3)}{\sqrt{x^2+3x-2}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2} dx \dots (i)$

Let $x^2 + 3x - 2 = t$

$$\Rightarrow (2x + 3) = \frac{dt}{dx}$$

$$\Rightarrow (2x + 3) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\sqrt{t}} [x^2 + 3x - 2 = t]$$

$$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$I = 2t^{\frac{1}{2}} + c$$

$$I = 2\sqrt{x^2 + 3x - 2} + c$$

$$\text{Ans) } 2\sqrt{x^2 + 3x - 2} + c$$

57. Question

Evaluate the following integrals:

$$\int \frac{(2x-1)}{\sqrt{x^2-x-1}} dx$$

Answer

To find: Value of $\int \frac{(2x-1)}{\sqrt{x^2-x-1}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2} dx \dots (i)$

Let $x^2 - x - 1 = t$

$$\Rightarrow \frac{d(x^2 - x - 1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(x^2)}{dx} - \frac{d(x)}{dx} - \frac{d(1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (2x - 1) = \frac{dt}{dx}$$

$$\Rightarrow (2x - 1) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^2} [x^2 - x - 1 = t]$$

$$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\Rightarrow I = \frac{2\sqrt{t}}{1} + c$$

$$I = \frac{2\sqrt{x^2 - x - 1}}{1} + c$$

$$\text{Ans) } 2\sqrt{x^2 - x - 1} + c$$

58. Question

Evaluate the following integrals:

$$\int \frac{dx}{(\sqrt{x+a} + \sqrt{x+b})}$$

Answer

To find: Value of $\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \dots (i)$

$$I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(\sqrt{x+a})^2 - (\sqrt{x+b})^2} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x+a-x-b} dx$$

$$I = \frac{1}{a-b} \left[\int \sqrt{x+a} dx - \int \sqrt{x+b} dx \right]$$

$$I = \frac{1}{a-b} \left[\int (x+a)^{\frac{1}{2}} dx - \int (x+b)^{\frac{1}{2}} dx \right]$$

$$I = \frac{1}{a-b} \left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}} \right]$$

$$I = \frac{2}{3(a-b)} \left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + c$$

$$\text{Ans) } \frac{2}{3(a-b)} \left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + c$$

59. Question

Evaluate the following integrals:

$$\int \frac{dx}{(\sqrt{1-3x} - \sqrt{5-3x})}$$

Answer

To find: Value of $\int \frac{dx}{\sqrt{1-3x} - \sqrt{5-3x}}$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{1-3x} - \sqrt{5-3x}} \dots (i)$

$$I = \int \frac{dx}{\sqrt{1-3x} - \sqrt{5-3x}} \times \frac{\sqrt{1-3x} + \sqrt{5-3x}}{\sqrt{1-3x} + \sqrt{5-3x}}$$

$$I = \int \frac{\sqrt{1-3x} + \sqrt{5-3x}}{(\sqrt{1-3x})^2 - (\sqrt{5-3x})^2} dx$$

$$I = \int \frac{\sqrt{1-3x} + \sqrt{5-3x}}{(1-3x) - (5-3x)} dx$$

$$I = \int \frac{\sqrt{1-3x} + \sqrt{5-3x}}{1-3x-5+3x} dx$$

$$I = -\frac{1}{4} \left[\int \sqrt{1-3x} dx + \int \sqrt{5-3x} dx \right]$$

$$I = -\frac{1}{4} \left[\int (1-3x)^{\frac{1}{2}} dx + \int (5-3x)^{\frac{1}{2}} dx \right]$$

$$I = -\frac{1}{4} \left[\frac{(1-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)} + \frac{(5-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)} \right]$$

$$I = -\frac{2}{-9 \times 4} \left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}} \right] + c$$

$$I = \frac{1}{18} \left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}} \right] + c$$

$$\text{Ans) } \frac{1}{18} \left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}} \right] + c$$

60. Question

Evaluate the following integrals:

$$\int \frac{x^2}{(1+x^6)} dx$$

Answer

To find: Value of $\int \frac{x^2}{(1+x^6)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x^2}{(1+x^6)} dx \dots (i)$

$$I = \int \frac{x^2}{1+(x^3)^2} dx$$

Let $x^3 = t$

$$\Rightarrow \frac{d(x^3)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (3x^2) = \frac{dt}{dx}$$

$$\Rightarrow (x^2)dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \frac{1}{3} \int \frac{dt}{1+t^2} [1 + \cos x = t]$$

$$\Rightarrow I = \frac{1}{3} \tan^{-1}(t) + c$$

$$I = \frac{1}{3} \tan^{-1}(x^3) + c$$

$$\text{Ans) } \frac{1}{3} \tan^{-1}(x^3) + c$$

61. Question

Evaluate the following integrals:

$$\int \frac{x^3}{(1+x^8)} dx$$

Answer

To find: Value of $\int \frac{x^3}{(1+x^8)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x^3}{(1+x^8)} dx \dots (i)$

$$I = \int \frac{x^3}{1+(x^4)^2} dx$$

Let $x^4 = t$

$$\Rightarrow \frac{d(x^4)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (4x^3) = \frac{dt}{dx}$$

$$\Rightarrow (x^3)dx = \frac{dt}{4}$$

Putting this value in equation (i)

$$I = \frac{1}{4} \int \frac{dt}{1+t^2} [1 + \cos x = t]$$

$$\Rightarrow I = \frac{1}{4} \tan^{-1}(t) + c$$

$$I = \frac{1}{4} \tan^{-1}(x^4) + c$$

$$\text{Ans) } \frac{1}{4} \tan^{-1}(x^4) + c$$

62. Question

Evaluate the following integrals:

$$\int \frac{x}{(1+x^4)} dx$$

Answer

To find: Value of $\int \frac{x}{(1+x^4)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x}{(1+x^4)} dx \dots (i)$

$$I = \int \frac{x}{1+(x^2)^2} dx$$

Let $x^2 = t$

$$\Rightarrow \frac{d(x^2)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (2x) = \frac{dt}{dx}$$

$$\Rightarrow (x)dx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \frac{1}{2} \int \frac{dt}{1+t^2} [1 + \cos x = t]$$

$$\Rightarrow I = \frac{1}{2} \tan^{-1}(t) + c$$

$$I = \frac{1}{2} \tan^{-1}(x^2) + c$$

$$\text{Ans) } \frac{1}{2} \tan^{-1}(x^2) + c$$

63. Question

Evaluate the following integrals:

$$\int \frac{x^5}{\sqrt{1+x^3}} dx$$

Answer

To find: Value of $\int \frac{x^5}{\sqrt{1+x^3}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{x^5}{\sqrt{1+x^3}} dx \dots (i)$

Let $1 + x^3 = t$

$$\Rightarrow x^3 = t - 1$$

$$\Rightarrow \frac{d(x^3)}{dx} = \frac{d(t-1)}{dx}$$

$$\Rightarrow (3x^2) = \frac{dt}{dx}$$

$$\Rightarrow x^2 dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \frac{x^3 x^2}{\sqrt{1+x^3}} dx$$

$$I = \int \frac{(t-1) dt}{\frac{1}{t^2} \cdot 3} [1+x^3=t]$$

$$\Rightarrow I = \frac{1}{3} \int \frac{t}{t^2} dt - \frac{1}{3} \int \frac{1}{t^2} dt$$

$$\Rightarrow I = \frac{1}{3} \left[\int t^{-1} dt - \int t^{-2} dt \right]$$

$$\Rightarrow I = \frac{1}{3} \left[\frac{t^3}{3} - \frac{1}{t} \right]$$

$$\Rightarrow I = \frac{2}{3} \left[\frac{(1+x^3)^{\frac{3}{2}}}{3} - \frac{(1+x^3)^{\frac{1}{2}}}{1} \right]$$

$$\Rightarrow I = \frac{2(1+x^3)^{\frac{3}{2}}}{9} - \frac{2(1+x^3)^{\frac{1}{2}}}{3} + c$$

$$\text{Ans) } \frac{2(1+x^3)^{\frac{3}{2}}}{9} - \frac{2(1+x^3)^{\frac{1}{2}}}{3} + c$$

64. Question

Evaluate the following integrals:

$$\int \frac{x}{\sqrt{1+x}} dx$$

Answer

To find: Value of $\int \frac{x}{\sqrt{1+x}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{x}{\sqrt{1+x}} dx \dots (i)$

Let $1 + x = t$

$\Rightarrow x = t - 1$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$$I = \int \frac{t-1}{\sqrt{t}} dx [1+x=t]$$

$$\Rightarrow I = \int \sqrt{t} dt - \int \frac{1}{\sqrt{t}} dt$$

$$\Rightarrow I = \left[\int t^{\frac{1}{2}} dt - \int t^{-\frac{1}{2}} dt \right]$$

$$\Rightarrow I = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right] + c$$

$$\Rightarrow I = 2 \left[\frac{(1+x)^{\frac{3}{2}}}{3} - \frac{(1+x)^{\frac{1}{2}}}{1} \right] + c$$

$$\Rightarrow I = \frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$$

$$\text{Ans) } \frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$$

65. Question

Evaluate the following integrals:

$$\int \frac{1}{x\sqrt{x^4-1}} dx$$

Answer

To find: Value of $\int \frac{1}{x\sqrt{x^4-1}} dx$

Formula used: $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

We have, $I = \int \frac{1}{x\sqrt{x^4-1}} dx \dots (i)$

Multiplying numerator and denominator with x

$$I = \int \frac{x}{x^2\sqrt{(x^2)^2-1}} dx$$

Let $x^2 = t$

$$\Rightarrow 2x = \frac{dt}{dx}$$

$$\Rightarrow xdx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2-1}} [x^2 = t]$$

$$\Rightarrow I = \frac{1}{2} \sec^{-1} t + c$$

$$\Rightarrow I = \frac{1}{2} \sec^{-1}(x^2) + c$$

$$\text{Ans) } \frac{1}{2} \sec^{-1}(x^2) + c$$

66. Question

Evaluate the following integrals:

$$\int x\sqrt{x-1} dx$$

Answer

To find: Value of $\int x\sqrt{x-1} dx$

$$\text{Formula used: } \int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

We have, $I = \int x\sqrt{x-1} dx \dots (i)$

$$\text{Let } x - 1 = t$$

$$x = t + 1$$

$$\Rightarrow dx = dt$$

Putting this value in equation (i)

$$I = \int (t+1)\sqrt{t} dt [x = t+1]$$

$$\Rightarrow I = \int t\sqrt{t} dx + \int \sqrt{t} dx$$

$$\Rightarrow I = \int t^{\frac{3}{2}} dx + \int t^{\frac{1}{2}} dx$$

$$\Rightarrow I = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{2}{5} (x-1)^{\frac{5}{2}} + \frac{2}{3} (x-1)^{\frac{3}{2}} + c$$

$$\text{Ans) } \frac{2}{5} (x-1)^{\frac{5}{2}} + \frac{2}{3} (x-1)^{\frac{3}{2}} + c$$

67. Question

Evaluate the following integrals:

$$\int (1-x)\sqrt{1+x} \, dx$$

Answer

To find: Value of $\int (1-x)\sqrt{1+x} \, dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int (1-x)\sqrt{1+x} \, dx \dots (i)$

Let $1+x = t$

$$x = t - 1$$

$$\Rightarrow dx = dt$$

Putting this value in equation (i)

$$I = \int \{1 - (t-1)\}\sqrt{t} \, dt \quad [x = t-1]$$

$$\Rightarrow I = \int \{1 - t + 1\}\sqrt{t} \, dt$$

$$\Rightarrow I = \int \{2 - t\}\sqrt{t} \, dt$$

$$\Rightarrow I = \int 2\sqrt{t} \, dt - \int t\sqrt{t} \, dt$$

$$\Rightarrow I = 2 \int t^{\frac{1}{2}} dx - \int t^{\frac{3}{2}} dx$$

$$\Rightarrow I = 2 \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$\Rightarrow I = \frac{4}{3} (1+x)^{\frac{3}{2}} - \frac{2}{5} (1+x)^{\frac{5}{2}} + c$$

$$\text{Ans) } \frac{4}{3} (1+x)^{\frac{3}{2}} - \frac{2}{5} (1+x)^{\frac{5}{2}} + c$$

68. Question

Evaluate the following integrals:

$$\int x\sqrt{x^2-1} \, dx$$

Answer

To find: Value of $\int x\sqrt{x^2-1} \, dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int x\sqrt{x^2-1} \, dx \dots (i)$

Let $x^2 - 1 = t$

$$\Rightarrow 2x = \frac{dt}{dx}$$

$$\Rightarrow x dx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \int \frac{1}{2} \sqrt{t} dt [x = x^2 - 1]$$

$$\Rightarrow I = \frac{1}{2} \int t^{\frac{1}{2}} dx$$

$$\Rightarrow I = \frac{1}{2} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{1}{3} t^{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{1}{3} (x^2 - 1)^{\frac{3}{2}} + c$$

$$\text{Ans) } \frac{1}{3} (x^2 - 1)^{\frac{3}{2}} + c$$

69. Question

Evaluate the following integrals:

$$\int x\sqrt{3x-2} dx$$

Answer

To find: Value of $\int x\sqrt{3x-2} dx$

$$\text{Formula used: } \int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

$$\text{We have, } I = \int x\sqrt{3x-2} dx \dots (i)$$

$$\text{Let } 3x - 2 = t$$

$$\Rightarrow 3x = t + 2$$

$$\Rightarrow x = \frac{t+2}{3}$$

$$\Rightarrow 3 = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \left(\frac{t+2}{3} \right) \sqrt{t} \frac{dt}{3} [t = 3x - 2]$$

$$\Rightarrow I = \frac{1}{9} \left[\int t^{\frac{3}{2}} dx + 2 \int t^{\frac{1}{2}} dx \right]$$

$$\Rightarrow I = \frac{1}{9} \left[\frac{t^{\frac{5}{2}}}{\frac{5}{2}} + 2 \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right] + c$$

$$\Rightarrow I = \frac{1}{9} \left[\frac{2}{5} (3x-2)^{\frac{5}{2}} + \frac{4}{3} (3x-2)^{\frac{3}{2}} \right] + c$$

$$\Rightarrow I = \frac{2}{45} (3x-2)^{\frac{5}{2}} + \frac{4}{27} (3x-2)^{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{2}{45} (3x-2)^{\frac{5}{2}} + \frac{4}{27} (3x-2)^{\frac{3}{2}} + c$$

$$\text{Ans) } \frac{2}{45} (3x-2)^{\frac{5}{2}} + \frac{4}{27} (3x-2)^{\frac{3}{2}} + c$$

70. Question

Evaluate the following integrals:

$$\int \frac{dx}{x \cos^2(1 + \log x)}$$

Answer

To find: Value of $\int \frac{dx}{x \cos^2(1 + \log x)}$

Formula used: $\int \sec^2 x \, dx = \tan x + c$

We have, $I = \int \frac{dx}{x \cos^2(1 + \log x)} \dots (i)$

Let $1 + \log x = t$

$$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{x} dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\cos^2(t)} [t = 1 + \log x]$$

$$\Rightarrow I = \int \sec^2 t \, dt$$

$$\Rightarrow I = \tan(t) + c$$

$$\Rightarrow I = \tan(1 + \log x) + c$$

$$\text{Ans) } \tan(1 + \log x) + c$$

71. Question

Evaluate the following integrals:

$$\int x^2 \sin x^3 \, dx$$

Answer

To find: Value of $\int x^2 \sin x^3 \, dx$

Formula used: $\int \sin x \, dx = -\cos x + c$

We have, $I = \int x^2 \sin x^3 \, dx \dots (i)$

Let $x^3 = t$

$$\Rightarrow 3x^2 = \frac{dt}{dx}$$

$$\Rightarrow x^2 dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \sin t \frac{dt}{3} [t = x^3]$$

$$\Rightarrow I = \frac{1}{3} \left[\int \sin t dt \right]$$

$$\Rightarrow I = \frac{1}{3} (-\cos t) + c$$

$$\Rightarrow I = \frac{1}{3} (-\cos x^3) + c$$

Ans) $\frac{-\cos x^3}{3} + c$

72. Question

Evaluate the following integrals:

$$\int (2x + 4) \sqrt{x^2 + 4x + 3} dx$$

Answer

To find: Value of $\int (2x + 4) \sqrt{x^2 + 4x + 3} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int (2x + 4) \sqrt{x^2 + 4x + 3} dx \dots (i)$

Let $x^2 + 4x + 3 = t$

$$\Rightarrow (2x + 4) = \frac{dt}{dx}$$

$$\Rightarrow (2x + 4) dx = dt$$

Putting this value in equation (i)

$$I = \int \sqrt{t} dt [t = (x^2 + 4x + 3)]$$

$$\Rightarrow I = \int t^{\frac{1}{2}} dx$$

$$\Rightarrow I = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{2}{3} \left[(t)^{\frac{3}{2}} \right] + c$$

$$\Rightarrow I = \frac{2}{3} \left[(x^2 + 4x + 3)^{\frac{3}{2}} \right] + c$$

Ans) $\frac{2}{3} \left[(x^2 + 4x + 3)^{\frac{3}{2}} \right] + c$

73. Question

Evaluate the following integrals:

$$\int \frac{\sin x}{(\sin x - \cos x)} dx$$

Answer

To find: Value of $\int \frac{\sin x}{(\sin x - \cos x)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\sin x}{(\sin x - \cos x)} dx \dots (i)$

$$\Rightarrow I = \frac{1}{2} \int \frac{2\sin x}{(\sin x - \cos x)} dx$$

$$\Rightarrow I = \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x - \cos x)} dx$$

$$\Rightarrow I = \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2} \int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx$$

Let $\sin x - \cos x = t$

$$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x + \sin x) dx = dt$$

Putting this value in equation (i)

$$I = \frac{1}{2} \int \frac{dt}{t} + \frac{1}{2} \int dx$$

$$\Rightarrow I = \frac{1}{2} \log|\sin x - \cos x| + \frac{1}{2} x + c$$

$$\Rightarrow I = \frac{x}{2} + \frac{1}{2} \log|\sin x - \cos x| + c$$

Ans) $\frac{x}{2} + \frac{1}{2} \log|\sin x - \cos x| + c$

74. Question

Evaluate the following integrals:

$$\int \frac{dx}{(1 - \tan x)}$$

Answer

To find: Value of $\int \frac{dx}{(1 - \tan x)}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \tan x)} \dots (i)$

$$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\sin x}{\cos x}\right)}$$

$$\Rightarrow I = \int \frac{dx}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$$

$$\Rightarrow I = \frac{1}{2} \int \frac{2 \cos x dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2} \int \frac{(\cos x + \sin x) + (\cos x - \sin x) dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2} \int \frac{(\cos x + \sin x)}{(\cos x - \sin x)} dx + \frac{1}{2} \int \frac{(\cos x - \sin x)}{(\cos x - \sin x)} dx$$

Let $(\cos x - \sin x) = t$

$$\Rightarrow (-\sin x - \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\sin x + \cos x) dx = -dt$$

Putting this value in equation (i)

$$I = -\frac{1}{2} \int \frac{dt}{(t)} dx + \frac{1}{2} \int dx$$

$$\Rightarrow I = -\frac{1}{2} \log |\cos x - \sin x| + \frac{1}{2} x + c$$

$$\Rightarrow I = \frac{1}{2} x - \frac{1}{2} \log |\sin x - \cos x| + c$$

$$\text{Ans) } \frac{1}{2} x - \frac{1}{2} \log |\sin x - \cos x| + c$$

75. Question

Evaluate the following integrals:

$$\int \frac{dx}{(1 - \cot x)}$$

Answer

To find: Value of $\int \frac{dx}{(1 - \cot x)}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \cot x)} \dots (i)$

$$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\cos x}{\sin x}\right)}$$

$$\Rightarrow I = \int \frac{dx}{\left(\frac{\sin x - \cos x}{\sin x}\right)}$$

$$\Rightarrow I = \frac{1}{2} \int \frac{2 \sin x dx}{(\sin x - \cos x)}$$

$$I = \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x) dx}{(\sin x - \cos x)}$$

$$I = \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2} \int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx$$

$$\text{Let } (\sin x - \cos x) = t$$

$$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x + \sin x) dx = dt$$

Putting this value in equation (i)

$$I = \frac{1}{2} \int \frac{dt}{(t)} dx + \frac{1}{2} \int dx$$

$$\Rightarrow I = \frac{1}{2} \log |\sin x - \cos x| + \frac{1}{2} x + c$$

$$\text{Ans) } \frac{1}{2} x + \frac{1}{2} \log |\sin x - \cos x| + c$$

76. Question

Evaluate the following integrals:

$$\int \frac{\cos 2x}{(\sin x + \cos x)} dx$$

Answer

$$\text{To find: Value of } \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$$

$$\text{Formula used: } \int \frac{1}{x} dx = \log |x| + c$$

$$\text{We have, } I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx \dots (i)$$

$$\Rightarrow I = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx$$

$$\text{Let } (\cos x + \sin x) = t$$

$$\Rightarrow (-\sin x + \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$$\Rightarrow I = \log|t| + c$$

$$\Rightarrow I = \log|\cos x + \sin x| + c$$

$$\text{Ans) } \log|\cos x + \sin x| + c$$

77. Question

Evaluate the following integrals:

$$\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$$

Answer

$$\text{To find: Value of } \int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$$

$$\text{Formula used: } \int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

$$\text{We have, } I = \int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx \dots (i)$$

$$\Rightarrow I = \int \frac{\cos x - \sin x}{\cos^2 x + \sin^2 x + 2 \sin x \cos x} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\cos x + \sin x)^2} dx$$

$$\text{Let } (\sin x + \cos x) = t$$

$$\Rightarrow (\cos x - \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^2}$$

$$\Rightarrow I = -\frac{1}{t} + c$$

$$\Rightarrow I = -\frac{1}{\sin x + \cos x} + c$$

$$\text{Ans) } \frac{-1}{\sin x + \cos x} + c$$

78. Question

Evaluate the following integrals:

$$\int \frac{(x+1)(x + \log x)^2}{x} dx$$

Answer

$$\text{To find: Value of } \int \frac{(x+1)(x + \log x)^2}{x} dx$$

$$\text{Formula used: } \int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

We have, $I = \int \frac{(x+1)(x+\log x)^2}{x} dx \dots (i)$

Let $(x + \log x) = t$

$$\Rightarrow \left(1 + \frac{1}{x}\right) = \frac{dt}{dx}$$

$$\Rightarrow \left(\frac{x+1}{x}\right) = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int t^2 dt$$

$$\Rightarrow I = \frac{t^3}{3} + c$$

$$\Rightarrow I = \frac{(x + \log x)^3}{3} + c$$

Ans) $\frac{(x + \log x)^3}{3} + c$

79. Question

Evaluate the following integrals:

$$\int x \sin^3 x^2 \cos x^2 dx$$

Answer

To find: Value of $\int x \sin^3 x^2 \cos x^2 dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int x \sin^3 x^2 \cos x^2 dx \dots (i)$

Let $(\sin x^2) = t$

$$\Rightarrow (\sin x^2 \cdot 2x) = \frac{dt}{dx}$$

$$\Rightarrow (\sin x^2 \cdot x) dx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \int t^3 \frac{dt}{2}$$

$$I = \frac{1}{2} \int t^3 dt$$

$$\Rightarrow I = \frac{1}{2} \frac{t^4}{4} + c$$

$$\Rightarrow I = \frac{t^4}{8} + c$$

$$\Rightarrow I = \frac{\sin^4 x^2}{8} + c$$

Ans) $\frac{\sin^4 x^2}{8} + c$

80. Question

Evaluate the following integrals:

$$\int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$$

Answer

To find: Value of $\int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$

Formula used: $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

We have, $I = \int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx \dots (i)$

Let $(\tan x) = t$

$$\Rightarrow (\sec^2 x) = \frac{dt}{dx}$$

$$\Rightarrow (\sec^2 x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\sqrt{1 - t^2}}$$

$$\Rightarrow I = \sin^{-1}(t) + c$$

$$\Rightarrow I = \sin^{-1}(\tan x) + c$$

Ans) $\sin^{-1}(\tan x) + c$

81. Question

Evaluate the following integrals:

$$\int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5) dx$$

Answer

To find: Value of $\int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5) dx$

Formula used: $\int \operatorname{cosec}^2 x dx = -\cot x + c$

We have, $I = \int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5) dx \dots (i)$

Let $(2e^{-x} + 5) = t$

$$\Rightarrow (2e^{-x}(-1)) = \frac{dt}{dx}$$

$$\Rightarrow (e^{-x}) dx = \frac{dt}{-2}$$

Putting this value in equation (i)

$$I = \int \operatorname{cosec}^2(t) \frac{dt}{-2}$$

$$I = \frac{1}{-2} \int \operatorname{cosec}^2(t) dt$$

$$\Rightarrow I = \frac{1}{-2} (-\cot t) + c$$

$$\Rightarrow I = \frac{1}{2} \cot(2e^{-x} + 5) + c$$

$$\text{Ans) } \frac{1}{2} \cot(2e^{-x} + 5) + c$$

82. Question

Evaluate the following integrals:

$$\int 2x \sec^3(x^2 + 3) \tan(x^2 + 3) dx$$

Answer

To find: Value of $\int 2x \sec^3(x^2 + 3) \tan(x^2 + 3) dx$

$$\text{Formula used: } \int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

$$\text{We have, } I = \int 2x \sec^2(x^2 + 3) \sec(x^2 + 3) \tan(x^2 + 3) dx \dots (i)$$

$$\text{Let } \sec(x^2 + 3) = t$$

$$\Rightarrow \sec(x^2 + 3) = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2 + 3) \tan(x^2 + 3) \cdot 2x = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2 + 3) \tan(x^2 + 3) \cdot 2x = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int t^2 dt$$

$$\Rightarrow I = \frac{t^3}{3} + c$$

$$\Rightarrow I = \frac{\sec^3(x^2 + 3)}{3} + c$$

$$\text{Ans) } \frac{\sec^3(x^2 + 3)}{3} + c$$

83. Question

Evaluate the following integrals:

$$\int \frac{\sin 2x}{(a + b \cos x)^2} dx$$

Answer

To find: Value of $\int \frac{\sin 2x}{(a + b \cos x)^2} dx$

Formula used: (i) $\int \frac{1}{x} dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin 2x}{(a + b \cos x)^2} dx \dots (i)$

$$I = \int \frac{2 \sin x \cos x}{(a + b \cos x)^2} dx$$

Let $(a + b \cos x) = t$

$$\Rightarrow (\cos x) = \frac{t - a}{b}$$

$$\Rightarrow (\sin x) dx = \frac{dt}{-b}$$

Putting this value in equation (i)

$$I = \frac{2}{-b^2} \int \frac{t - a}{t^2} dt$$

$$I = \frac{2}{-b^2} \left[\int \frac{t}{t^2} dt - \int \frac{a}{t^2} dt \right]$$

$$I = \frac{2}{-b^2} \left[\int \frac{1}{t} dt - a \int \frac{1}{t^2} dt \right]$$

$$I = \frac{2}{-b^2} \left[\log |t| - a \left(-\frac{1}{t} \right) + c \right]$$

$$I = -\frac{2}{b^2} \left[\log |a + b \cos x| + \left(\frac{a}{a + b \cos x} \right) \right] + c$$

$$\text{Ans) } -\frac{2}{b^2} \left[\log |a + b \cos x| + \left(\frac{a}{a + b \cos x} \right) \right] + c$$

84. Question

Evaluate the following integrals:

$$\int \frac{dx}{(3 - 5x)}$$

Answer

To find: Value of $\int \frac{dx}{(3 - 5x)}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(3 - 5x)} \dots (i)$

Let $(3 - 5x) = t$

$$\Rightarrow (-5) = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{dt}{-5}$$

Putting this value in equation (i)

$$I = \int \frac{1}{t} \frac{dt}{-5}$$

$$I = \frac{1}{-5} \int \frac{dt}{t}$$

$$\Rightarrow I = \frac{1}{-5} \log |t| + c$$

$$\Rightarrow I = -\frac{1}{5} \log |3 - 5x| + c$$

$$\text{Ans) } -\frac{1}{5} \log |3 - 5x| + c$$

85. Question

Evaluate the following integrals:

$$\int \sqrt{1+x} \, dx$$

Answer

To find: Value of $\int \sqrt{1+x} \, dx$

$$\text{Formula used: } \int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

$$\text{We have, } I = \int \sqrt{1+x} \, dx \quad \dots (i)$$

$$\text{Let } (1+x) = t$$

$$\Rightarrow dx = dt$$

Putting this value in equation (i)

$$I = \int \sqrt{t} \, dt$$

$$I = \int t^{\frac{1}{2}} \, dt$$

$$\Rightarrow I = \frac{2}{3} (1+x)^{\frac{3}{2}} + c$$

$$\text{Ans) } \frac{2}{3} (1+x)^{\frac{3}{2}} + c$$

86. Question

Evaluate the following integrals:

$$\int x^2 e^{x^3} \cos(e^{x^3}) \, dx$$

Answer

To find: Value of $\int x^2 e^{x^3} \cos(e^{x^3}) \, dx$

$$\text{Formula used: } \int \cos x \, dx = \sin x + c$$

We have, $I = \int x^2 e^{x^3} \cos(e^{x^3}) dx \dots (i)$

Let $e^{x^3} = t$

$$\Rightarrow e^{x^3} \cdot 3x^2 = \frac{dt}{dx}$$

$$\Rightarrow e^{x^3} \cdot x^2 \cdot dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \cos(t) \frac{dt}{3}$$

$$I = \frac{\sin(t)}{3} + c$$

$$I = \frac{\sin(e^{x^3})}{3} + c$$

$$\text{Ans) } \frac{\sin(e^{x^3})}{3} + c$$

87. Question

Evaluate the following integrals:

$$\int \frac{e^{m \tan^{-1} x}}{(1+x^2)} dx$$

Answer

To find: Value of $\int \frac{e^{m \tan^{-1} x}}{(1+x^2)} dx$

Formula used: $\int e^t dx = e^t + c$

We have, $I = \int \frac{e^{m \tan^{-1} x}}{(1+x^2)} dx \dots (i)$

Let $(m \tan^{-1} x) = t$

$$\Rightarrow m \left(\frac{1}{1+x^2} \right) = \frac{dt}{dx}$$

$$\Rightarrow \left(\frac{1}{1+x^2} \right) dx = \frac{dt}{m}$$

Putting this value in equation (i)

$$I = \int e^t \frac{dt}{m}$$

$$\Rightarrow I = \frac{e^t}{m} + c$$

$$\Rightarrow I = \frac{e^{m \tan^{-1} x}}{m} + c$$

Ans) $\frac{e^{m \tan^{-1} x}}{m} + c$

88. Question

Evaluate the following integrals:

$$\int \frac{(x+1)e^x}{\cos^2(xe^x)} dx$$

Answer

To find: Value of $\int \frac{(x+1)e^x dx}{\cos^2(xe^x)}$

Formula used: $\int \sec^2 x dx = \tan x + c$

We have, $I = \int \frac{(x+1)e^x dx}{\cos^2(xe^x)} \dots (i)$

Let $(xe^x) = t$

$$\Rightarrow xe^x + e^x \cdot 1 = \frac{dt}{dx}$$

$$\Rightarrow e^x(x+1) = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\cos^2(t)}$$

$$\Rightarrow I = \int \sec^2(t) dt$$

$$\Rightarrow I = \tan(t) + c$$

$$\Rightarrow I = \tan(xe^x) + c$$

Ans) $\tan(xe^x) + c$

89. Question

Evaluate the following integrals:

$$\int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}} dx$$

Answer

To find: Value of $\int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}}) dx}{\sqrt{x}}$

Formula used: $\int \cos x dx = \sin x + c$

We have, $I = \int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}}) dx}{\sqrt{x}} \dots (i)$

Let $(e^{\sqrt{x}}) = t$

$$\Rightarrow e^{\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\Rightarrow \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$$

Putting this value in equation (i)

$$I = \int \cos(t) 2dt$$

$$I = 2 \sin(e^{\sqrt{x}}) + c$$

$$\text{Ans) } 2 \sin(e^{\sqrt{x}}) + c$$

90. Question

Evaluate the following integrals:

$$\int \sqrt{e^x - 1} dx$$

Answer

To find: Value of $\int \sqrt{e^x - 1} dx$

Formula used: $\int \frac{1}{x^2 + 1} dx = \tan^{-1} x + c$

We have, $I = \int \sqrt{e^x - 1} dx \dots (i)$

$$\text{Let } (e^x - 1) = t^2$$

$$\Rightarrow e^x - 1 = t^2$$

$$\Rightarrow e^x = t^2 + 1$$

$$\Rightarrow e^x = \frac{2tdt}{dx}$$

$$\Rightarrow dx = \frac{2tdt}{e^x}$$

$$\Rightarrow dx = \frac{2t}{t^2 + 1} dt$$

Putting this value in equation (i)

$$I = \int \sqrt{t^2} \frac{2t}{t^2 + 1} dt$$

$$\Rightarrow I = \int \frac{2t^2}{t^2 + 1} dt$$

$$\Rightarrow I = 2 \int \frac{t^2 + 1 - 1}{t^2 + 1} dt$$

$$\Rightarrow I = 2 \int \left(1 - \frac{1}{t^2 + 1} \right) dt$$

$$\Rightarrow I = 2 [t - \tan^{-1} t] + c$$

$$\Rightarrow I = 2 [\sqrt{e^x - 1} - \tan^{-1} \sqrt{e^x - 1}] + c$$

$$\text{Ans) } 2 [\sqrt{e^x - 1} - \tan^{-1} \sqrt{e^x - 1}] + c$$

91. Question

Evaluate the following integrals:

Answer

To find: Value of $\int \frac{dx}{(x-\sqrt{x})}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(x-\sqrt{x})} \dots (i)$

$$\Rightarrow I = \int \frac{dx}{\sqrt{x}(\sqrt{x}-1)}$$

Let $(\sqrt{x}-1) = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{x}} dx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \int \frac{1}{t} \frac{dt}{2}$$

$$I = \frac{1}{2} \log|t| + c$$

$$I = \frac{1}{2} \log|\sqrt{x}-1| + c$$

$$\text{Ans) } \frac{1}{2} \log|\sqrt{x}-1| + c$$

92. Question

Evaluate the following integrals:

$$\int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)} dx$$

Answer

To find: Value of $\int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)} dx$

Formula used: $\int \sec^2 x dx = \tan x + c$

We have, $I = \int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)} dx \dots (i)$

Let $2\tan^{-1}x = t$

$$\Rightarrow \frac{2}{1+x^2} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{1+x^2} dx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \int \sec^2(t) \frac{dt}{2}$$

$$I = \frac{1}{2} \tan(t) + c$$

$$I = \frac{1}{2} \tan(2 \tan^{-1} x) + c$$

$$\text{Ans) } \frac{1}{2} \tan(2 \tan^{-1} x) + c$$

93. Question

Evaluate the following integrals:

$$\int \left(\frac{1 + \sin 2x}{x + \sin^2 x} \right) dx$$

Answer

To find: Value of $\int \left(\frac{1 + \sin 2x}{x + \sin^2 x} \right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \left(\frac{1 + \sin 2x}{x + \sin^2 x} \right) dx \dots (i)$

Let $x + \sin^2 x = t$

$$\Rightarrow 1 + 2\sin x \cdot \cos x = \frac{dt}{dx}$$

$$\Rightarrow (1 + \sin 2x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$$I = \log|t| + c$$

$$I = \log|x + \sin^2 x| + c$$

$$\text{Ans) } \log|x + \sin^2 x| + c$$

94. Question

Evaluate the following integrals:

$$\int \left(\frac{1 - \tan x}{x + \log \cos x} \right) dx$$

Answer

To find: Value of $\int \left(\frac{1 - \tan x}{x + \log(\cos x)} \right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \left(\frac{1 - \tan x}{x + \log(\cos x)} \right) dx \dots (i)$

$$\text{Let } x + \log(\cos x) = t$$

$$\Rightarrow 1 + \frac{1 \cdot (-\sin x)}{\cos x} = \frac{dt}{dx}$$

$$\Rightarrow 1 - \tan x = \frac{dt}{dx}$$

$$\Rightarrow (1 - \tan x)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$$I = \log |t| + c$$

$$I = \log |x + \log(\cos x)| + c$$

$$\text{Ans) } \log |x + \log(\cos x)| + c$$

95. Question

Evaluate the following integrals:

$$\int \frac{(1 + \cot x)}{(x + \log \sin x)} dx$$

Answer

$$\text{To find: Value of } \int \left(\frac{1 + \cot x}{x + \log(\sin x)} \right) dx$$

$$\text{Formula used: } \int \frac{1}{x} dx = \log |x| + c$$

$$\text{We have, } I = \int \left(\frac{1 + \cot x}{x + \log(\sin x)} \right) dx \dots (i)$$

$$\text{Let } x + \log(\sin x) = t$$

$$\Rightarrow 1 + \frac{1 \cdot (\cos x)}{\sin x} = \frac{dt}{dx}$$

$$\Rightarrow 1 + \cot x = \frac{dt}{dx}$$

$$\Rightarrow (1 + \cot x)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$$I = \log |x + \log(\sin x)| + c$$

$$I = \log |x + \log(\sin x)| + c$$

$$\text{Ans) } \log |x + \log(\sin x)| + c$$

96. Question

Evaluate the following integrals:

$$\int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$$

Answer

To find: Value of $\int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx \dots (i)$

Let $1 - \tan^2 x = t$

$$\Rightarrow 0 - 2 \tan x \cdot \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow (\tan x \cdot \sec^2 x) dx = \frac{dt}{-2}$$

$$\Rightarrow (1 + \cot x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{1}{t} \frac{dt}{(-2)}$$

$$I = \frac{1}{2} \log |t| + c$$

$$I = \frac{1}{2} \log |1 - \tan^2 x| + c$$

$$\text{Ans) } \frac{1}{2} \log |1 - \tan^2 x| + c$$

97. Question

Evaluate the following integrals:

$$\int \frac{\sin(2 \tan^{-1} x)}{(1 + x^2)} dx$$

Answer

To find: Value of $\int \frac{\sin(2 \tan^{-1} x)}{(1 + x^2)} dx$

Formula used: $\int \sin x dx = -\cos x + c$

We have, $I = \int \frac{\sin(2 \tan^{-1} x)}{(1 + x^2)} dx \dots (i)$

Let $2 \tan^{-1} x = t$

$$\Rightarrow 2 \frac{1}{1 + x^2} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dx}{1 + x^2} = \frac{dt}{2}$$

$$\Rightarrow (1 + \cot x) dx = dt$$

Putting this value in equation (i)

$$I = \int \sin(t) \frac{dt}{(2)}$$

$$I = -\frac{1}{2} \cos(t) + c$$

$$I = -\frac{1}{2} \cos(2 \tan^{-1} x) + c$$

$$\text{Ans) } -\frac{1}{2} \cos(2 \tan^{-1} x) + c$$

98. Question

Evaluate the following integrals:

$$\int \frac{dx}{(x^{1/2} + x^{1/3})}$$

Answer

$$\text{To find: Value of } \int \frac{dx}{(x^{1/2} + x^{1/3})}$$

$$\text{Formula used: (i) } \int \frac{1}{x} dx = \log|x| + c$$

$$\text{(ii) } \int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

$$\text{We have, } I = \int \frac{dx}{(x^{1/2} + x^{1/3})} \dots (i)$$

$$\text{Let } x = t^6$$

$$\Rightarrow x^{1/6} = t$$

$$\Rightarrow 6t^5 dt = dx$$

Putting this value in equation (i)

$$I = \int \frac{6t^5 dt}{(t^3 + t^2)}$$

$$I = \int \frac{6t^5 dt}{t^2(t+1)}$$

$$I = 6 \int \frac{t^3 dt}{(t+1)}$$

$$I = 6 \int \frac{t^3 + 1 - 1}{(t+1)} dt$$

$$I = 6 \int \frac{(t+1)(t^2 - t + 1)}{(t+1)} dt - \int \frac{1}{(t+1)} dt$$

$$I = 6 \left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1| \right] + c$$

$$I = [2t^3 - 3t^2 + 6t - 6\log|t+1|] + c$$

$$I = \left[2 \left(x^{\frac{1}{6}} \right)^3 - 3 \left(x^{\frac{1}{6}} \right)^2 + 6 \left(x^{\frac{1}{6}} \right) - 6 \log \left| \left(x^{\frac{1}{6}} \right) + 1 \right| \right] + c$$

$$I = \left[2\sqrt{x} - 3 \left(x^{\frac{1}{3}} \right) + 6 \left(x^{\frac{1}{6}} \right) - 6 \log \left| \left(x^{\frac{1}{6}} \right) + 1 \right| \right] + c$$

$$\text{Ans) } \left[2\sqrt{x} - 3 \left(x^{\frac{1}{3}} \right) + 6 \left(x^{\frac{1}{6}} \right) - 6 \log \left| \left(x^{\frac{1}{6}} \right) + 1 \right| \right] + c$$

99. Question

Evaluate the following integrals:

$$\int (\sin^{-1} x)^2 dx$$

Answer

To find: Value of $\int (\sin^{-1} x)^2 dx$

Formula used: $\int \sin x dx = \cos x + c$

We have, $I = \int (\sin^{-1} x)^2 dx \dots (i)$

Let $\sin^{-1} x = t$, $x = \sin t$,

$$\Rightarrow \cos t = \sqrt{1 - x^2}$$

$$\Rightarrow \frac{1}{\sqrt{1 - x^2}} = \frac{dt}{dx}$$

$$\Rightarrow \sqrt{1 - x^2} dt = dx$$

$$\Rightarrow \sqrt{1 - (\sin t)^2} dt = dx$$

$$\Rightarrow \sqrt{1 - \sin^2 t} dt = dx$$

$$\Rightarrow \cos t dt = dx$$

Putting this value in equation (i)

$$I = \int t^2 \cos t dt$$

$$I = \int t^2 \cos t dt - \int \left[\frac{d(t^2)}{dt} \int \cos t dt \right] dt$$

$$I = t^2 \sin t - \int [2t \cdot \sin t] dt$$

$$I = t^2 \sin t - 2 \left\{ \int t [\sin t] dt - \int \left[\frac{dt}{dt} \int \sin t dt \right] dt \right\}$$

$$I = t^2 \sin t - 2 \left[-t \cos t + \int 1 \cdot \cos t dt \right]$$

$$I = t^2 \sin t + 2t \cos t - 2 \sin t + c$$

$$I = (\sin^{-1} x)^2 x + 2(\sin^{-1} x) \sqrt{1 - x^2} - 2x + c$$

Ans) $(\sin^{-1} x)^2 x + 2(\sin^{-1} x)\sqrt{1-x^2} - 2x + c$

100. Question

Evaluate the following integrals:

$$\int \frac{2x \tan^{-1} x^2}{(1+x^4)} dx$$

Answer

To find: Value of $\int \frac{2x \tan^{-1}(x^2)}{(1+x^4)} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{2x \tan^{-1}(x^2)}{(1+x^4)} dx \dots (i)$

Let $\tan^{-1}(x^2) = t$

$$\Rightarrow \frac{1}{1+(x^2)^2} \cdot 2x = \frac{dt}{dx}$$

$$\Rightarrow \frac{2x}{1+x^4} dx = dt$$

Putting this value in equation (i)

$$I = \int t \cdot dt$$

$$I = \frac{t^2}{2} + c$$

$$I = \frac{\{\tan^{-1}(x^2)\}^2}{2} + c$$

Ans) $\frac{\{\tan^{-1}(x^2)\}^2}{2} + c$

101. Question

Evaluate the following integrals:

$$\int \frac{(x^2+1)}{(x^4+1)} dx$$

Answer

To find: Value of $\int \frac{(x^2+1)}{(x^4+1)} dx$

Formula used: $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

We have, $I = \int \frac{(x^2+1)}{(x^4+1)} dx \dots (i)$

Dividing Numerator and Denominator by x^2 ,

$$I = \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x^2 + \frac{1}{x^2} + 2 - 2\right)} dx$$

$$I = \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x^2 - 2 \cdot x \cdot \frac{1}{x} + \left(\frac{1}{x}\right)^2 + 2\right)} dx$$

$$I = \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(\left(x - \frac{1}{x}\right)^2 + (\sqrt{2})^2\right)} dx$$

$$\text{Let } x - \frac{1}{x} = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{1}{(t)^2 + (\sqrt{2})^2} dt$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t}{\sqrt{2}} \right) + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x - \frac{1}{x}}{\sqrt{2}} \right) + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + c$$

$$\text{Ans) } \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + c$$

102. Question

Evaluate the following integrals:

$$\int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$$

Answer

$$\text{To find: Value of } \int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$$

$$\text{Formula used: } \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$$

$$\text{We have, } I = \int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx \dots (i)$$

$$\text{Let } (\sin x - \cos x) = t$$

$$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x + \sin x) dx = dt$$

$$\Rightarrow t^2 = \sin^2 x - 2\sin x \cdot \cos x + \cos^2 x$$

$$\Rightarrow t^2 = 1 - 2\sin x \cdot \cos x$$

$$\Rightarrow 2\sin x \cdot \cos x = 1 - t^2$$

$$\Rightarrow \sin 2x = 1 - t^2$$

Putting this value in equation (i)

$$\Rightarrow I = \int \frac{dt}{\sqrt{1-t^2}}$$

$$I = \sin^{-1} t$$

$$I = \sin^{-1} (\sin x - \cos x)$$

$$\text{Let } \sin^{-1} (\sin x - \cos x) = \theta$$

$$\Rightarrow I = \sin^{-1} (\sin x - \cos x) = \theta \dots (ii)$$

$$\Rightarrow \sin \theta = \sin x - \cos x$$

$$\text{Now if } \sin \theta = \sin x - \cos x$$

$$\text{Then } \cos \theta = \sqrt{1 - (\sin x - \cos x)^2}$$

$$\Rightarrow \cos \theta = \sqrt{1 - (\sin^2 x - 2\sin x \cdot \cos x + \cos^2 x)}$$

$$\Rightarrow \cos \theta = \sqrt{1 - (1 - 2\sin x \cdot \cos x)}$$

$$\Rightarrow \cos \theta = \sqrt{2\sin x \cdot \cos x}$$

$$\text{Now } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\text{Now } \tan \theta = \frac{\sin x - \cos x}{\sqrt{2\sin x \cdot \cos x}}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\sin x - \cos x}{\sqrt{2\sin x \cdot \cos x}} \right)$$

Comparing the value θ from eqn. (ii)

$$I = \theta = \tan^{-1} \left(\frac{\sin x - \cos x}{\sqrt{2\sin x \cdot \cos x}} \right)$$

Dividing Numerator and denominator from $\cos x$

$$I = \theta = \tan^{-1} \left(\frac{\tan x - 1}{\sqrt{2\tan x}} \right)$$

$$\text{Ans.) } \tan^{-1} \left(\frac{\tan x - 1}{\sqrt{2\tan x}} \right)$$

Objective Questions I

1. Question

Mark (✓) against the correct answer in each of the following:

$$\int (2x + 3)^5 dx = ?$$

A. $\frac{(2x+3)^6}{6} + C$

B. $\frac{(2x+3)^4}{8} + C$

C. $\frac{(2x+3)^6}{12} + C$

D. none of these

Answer

Given = $\int (2x+3)^5$

Let, $2x + 3 = z$

$\Rightarrow 2dx = dz$

So,

$\int (2x+3)^5 dx$

$= \int \frac{z^5}{2} dz$

$= \frac{1}{2} \frac{z^6}{6} + c$ where c is the integrating constant.

$= \frac{z^6}{12} + c$

$= \frac{(2x+3)^6}{12} + c$

2. Question

Mark (✓) against the correct answer in each of the following:

$\int (3-5x)^7 dx = ?$

A. $-5(3-5x)^6 + C$

B. $\frac{(3-5x)^8}{-40} + C$

C. $\frac{-5(3-5x)^8}{8} + C$

D. none of these

Answer

Given = $\int (3-5x)^7$

Let, $3 - 5x = z$

$\Rightarrow -5dx = dz$

So,

$$\int (3-5x)^7 dx$$

$$= -\int \frac{z^7}{5} dz$$

$$= -\frac{1}{5} \frac{z^8}{8} + c \quad \text{where } c \text{ is the integrating constant.}$$

$$= -\frac{z^8}{40} + c$$

$$= -\frac{(3-5x)^8}{40} + c$$

3. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{(2-3x)^4} dx = ?$$

A. $\frac{1}{15(2-3x)^5} + C$

B. $\frac{1}{-12(2-3x)^3} + C$

C. $\frac{1}{9(2-3x)^3} + C$

D. none of these

Answer

$$\text{Given} = \int \frac{1}{(2-3x)^4}$$

$$\text{Let, } 2-3x = z$$

$$\Rightarrow -3dx = dz$$

So,

$$\int \frac{1}{(2-3x)^4} dx$$

$$= \int \frac{1}{z^4} \left(\frac{dz}{-3} \right)$$

$$= -\frac{1}{3} \int \frac{dz}{z^4}$$

where c is the integrating constant.

$$= -\frac{1}{3} \int z^{-4} dz$$

$$= -\frac{1}{3} \frac{z^{-3}}{-3} + c$$

$$= \frac{1}{9(2-3x)^3} + c$$

4. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{ax+b} \, dx = ?$$

A. $\frac{2(ax+b)^{3/2}}{3a} + C$ B. $\frac{3(ax+b)^{3/2}}{2a} + C$

C. $\frac{1}{2\sqrt{ax+b}} + C$

D. none of these

Answer

$$\text{Given} = \int \sqrt{ax+b}$$

$$\text{Let, } ax + b = z^2$$

$$\Rightarrow adx = 2zdz$$

So,

$$\int \sqrt{ax+b} \, dx$$

$$= \int z \frac{2zdz}{a}$$

$$= \frac{2}{a} \int z^2 dz$$

where c is the integrating constant.

$$= \frac{2}{a} \frac{z^3}{3} + c$$

$$= \frac{2}{3a} z^3 + c$$

$$= \frac{2(ax+b)^{3/2}}{3a} + c$$

5. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sec^2(7 - 4x) dx = ?$$

- A. $\frac{1}{4} \tan(7 - 4x) + C$
- B. $\frac{-1}{4} \tan(7 - 4x) + C$
- C. $4 \tan(7 - 4x) + C$
- D. $-4 \tan(7 - 4x) + C$

Answer

$$\text{Given} = \int \sec^2(7 - 4x)$$

$$\text{Let, } 7 - 4x = z$$

$$\Rightarrow -4dx = dz$$

So,

$$\int \sec^2(7 - 4x) dx$$

$$= \int \sec^2 z \frac{dz}{-4}$$

$$= -\frac{1}{4} \int \sec^2 z dz \quad \text{where } c \text{ is the integrating constant.}$$

$$= -\frac{1}{4} \tan z + c$$

$$= -\frac{1}{4} \tan(7 - 4x) + c$$

6. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cos 3x \, dx = ?$$

- A. $-\frac{1}{3} \sin 3x + C$
- B. $\frac{1}{3} \sin 3x + C$
- C. $3 \sin 3x + C$
- D. $-3 \sin 3x + C$

Answer

$$\text{Given} = \int \cos 3x$$

$$\text{So, } \int \cos 3x dx = \frac{\sin 3x}{3} + c \quad \text{where } c \text{ is the integrating constant.}$$

7. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^{(5-3x)} dx = ?$$

A. $-3e^{(5-3x)} + C$

B. $\frac{1}{3}e^{(5-3x)} + C$

C. $\frac{e^{(5-3x)}}{-3} + C$

D. none of these

Answer

$$\text{Given} = \int e^{(5-3x)}$$

$$\text{Let, } 5 - 3x = z$$

$$\Rightarrow -3dx = dz$$

So,

$$\int e^{(5-3x)} dx$$

$$= \int e^z \frac{dz}{-3}$$

$$= -\frac{1}{3} \int e^z dz \quad \text{where } c \text{ is the integrating constant.}$$

$$= -\frac{1}{3} e^z + c$$

$$= -\frac{1}{3} e^{(5-3x)} + c$$

8. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^{(3x+4)} dx = ?$$

A. $\frac{3}{(\log 2)} \cdot 2^{(3x+4)} + C$

B. $\frac{2^{(3x+4)}}{3(\log 2)} + C$

C. $\frac{2^{(3x+4)}}{2(\log 3)} + C$

D. none of these

Answer

$$\text{Given} = \int e^{(3x+4)}$$

$$\text{Let, } 3x + 4 = z$$

$$\Rightarrow 3dx = dz$$

So,

$$\int e^{(3x+4)} dx$$

$$= \int e^z \frac{dz}{3}$$

$$= \frac{1}{3} \int e^z dz$$

$$= \frac{1}{3} e^z + c$$

$$= \frac{1}{3} e^{(3x+4)} + c$$

where c is the integrating constant.

9. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^2 \frac{x}{2} dx = ?$$

A. $\tan \frac{x}{2} - x + C$

B. $\tan \frac{x}{2} + x + C$

C. $2 \tan \frac{x}{2} + x + C$

D. $2 \tan \frac{x}{2} - x + C$

Answer

$$\text{Given} = \int \tan^2 \frac{x}{2}$$

$$\text{Let, } \frac{x}{2} = z$$

$$\Rightarrow dx = 2dz$$

So,

$$\begin{aligned}
& \int \tan^2 \frac{x}{2} dx \\
&= 2 \int \tan^2 z dz \\
&= 2 \int \frac{\sin^2 z}{\cos^2 z} dz \\
&= 2 \int \frac{1 - \cos^2 z}{\cos^2 z} dz \\
&= 2 \int (\sec^2 z - 1) dz \\
&= 2 [\tan z - z] + c \\
&= 2 \left[\tan \frac{x}{2} - \frac{x}{2} \right] + c
\end{aligned}$$

where c is the integrating constant.

10. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{1 - \cos x} dx = ?$$

A. $-\sqrt{2} \cos \frac{x}{2} + C$

B. $-2\sqrt{2} \cos \frac{x}{2} + C$

C. $\frac{-1}{2} \cos \frac{x}{2} + C$

D. $\frac{-1}{\sqrt{2}} \cos \frac{x}{2} + C$

Answer

$$\text{Given} = \int \sqrt{1 - \cos x}$$

So,

$$\begin{aligned}
& \int \sqrt{1 - \cos x} dx \\
&= \int \sqrt{1 - \cos x} \frac{\sqrt{1 + \cos x}}{\sqrt{1 + \cos x}} dx \\
&= \int \frac{\sqrt{1 - \cos^2 x}}{\sqrt{1 + \cos x}} dx \\
&= \int \frac{\sin x}{\sqrt{1 + \cos x}} dx
\end{aligned}$$

$$\text{Let } 1 + \cos x = u^2$$

$$\text{So, } -\sin x dx = 2u du$$

$$-\int \frac{2u}{u} du = -2 \int du = -2u + c = -2\sqrt{1 + \cos x} + c$$

where c is the integrating constant.

11. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{1 + \sin x} \, dx = ?$$

A. $-\sqrt{2} \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) + C$

B. $\sqrt{2} \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) + C$

C. $-2\sqrt{2} \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) + C$

D. none of these

Answer

$$\text{Given} = \int \sqrt{1 + \sin x}$$

So,

$$\begin{aligned} & \int \sqrt{1 + \sin x} \, dx \\ &= \int \sqrt{1 + \sin x} \cdot \frac{\sqrt{1 - \sin x}}{\sqrt{1 - \sin x}} \, dx \\ &= \int \frac{\sqrt{1 - \sin^2 x}}{\sqrt{1 - \sin x}} \, dx \\ &= \int \frac{\cos x}{\sqrt{1 - \sin x}} \, dx \end{aligned}$$

$$\text{Let } 1 - \sin x = u^2$$

$$\text{So, } -\cos x \, dx = 2u \, du$$

$$-\int \frac{2u}{u} \, du = -2 \int du = -2u + c = -2\sqrt{1 - \sin x} + c$$

where c is the integrating constant.

12. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin^3 x \, dx = ?$$

A. $-\frac{3}{4} \cos x + \frac{\cos 3x}{12} + C$

B. $\frac{3}{4} \cos x + \frac{\cos 3x}{12} + C$

C. $-\frac{3}{4} \cos x - \frac{\cos 3x}{12} + C$

D. none of these

Answer

$$\text{Given} = \int \sin^3 x dx$$

So,

$$\begin{aligned} \int \sin^3 x dx \\ &= \int \sin^2 x \sin x dx \\ &= \int (1 - \cos^2 x) \sin x dx \end{aligned}$$

$$\text{Let } \cos x = u$$

$$\text{So, } -\sin x dx = du$$

$$\begin{aligned} &-\int (1 - u^2) du \\ &= -\int du + \int u^2 du \\ &= -u + \frac{u^3}{3} + c \\ &= -\cos x + \frac{\cos^3 x}{3} + c \end{aligned}$$

where c is the integrating constant.

13. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\log x}{x} dx = ?$$

A. $\frac{1}{2}(\log x)^2 + C$

B. $-\frac{1}{2}(\log x)^2 + C$

C. $\frac{2}{x^2} + C$

D. $\frac{-2}{x^2} + C$

Answer

$$\text{Given} = \int \frac{\log x}{x}$$

$$\text{Let, } \log x = u$$

$$\text{So, } \frac{1}{x} dx = du$$

So,

$$\begin{aligned}
 & \int \frac{\log x}{x} dx \\
 &= \int u du \\
 &= \frac{u^2}{2} + c \\
 &= \frac{(\log x)^2}{2} + c
 \end{aligned}$$

where c is the integrating constant.

14. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sec^2(\log x)}{x} dx = ?$$

- A. $\log(\tan x) + C$
- B. $-\log(\tan x) + C$
- C. $\tan(\tan x) + C$
- D. $-\tan(\log x) + C$

Answer

$$\text{Given} = \int \frac{\sec^2(\log x)}{x} dx$$

Let, $\log x = z$

$$\Rightarrow \frac{dx}{x} = dz$$

So,

$$\begin{aligned}
 & \int \frac{\sec^2(\log x)}{x} dx \\
 &= \int \sec^2 z dz \\
 &= \tan z + c \\
 &= \tan(\log x) + c
 \end{aligned}$$

where c is the integrating constant.

15. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{x(\log x)} dx = ?$$

- A. $\log|x| + C$
- B. $\frac{-2}{x^2} + C$

C. $(\log x)^2 + C$

D. $\log |\log x| + C$

Answer

$$\text{Given} = \int \frac{1}{x(\log x)}$$

Let, $\log x = z$

$$\Rightarrow \frac{dx}{x} = dz$$

So,

$$\int \frac{1}{x(\log x)} dx$$

$$= \int \frac{1}{z} dz$$

$$= \log z + c$$

$$= \log(\log x) + c$$

where c is the integrating constant.

16. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^{x^3} x^2 dx = ?$$

A. $e^{x^3} + C$

B. $\frac{1}{3}e^{x^3} + C$

C. $\frac{1}{6}e^{x^3} + C$

D. none of these

Answer

$$\text{Given} = \int e^{x^3} x^2$$

Let, $x^3 = z$

$$\Rightarrow 3x^2 dx = dz$$

$$\Rightarrow x^2 dx = \frac{dz}{3}$$

So,

$$\begin{aligned} & \int e^{x^3} x^2 dx \\ &= \frac{1}{3} \int e^z dz \\ &= \frac{1}{3} e^z + c \\ &= \frac{1}{3} e^{x^3} + c \end{aligned}$$

where c is the integrating constant.

17. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = ?$$

- A. $e^{\sqrt{x}} + C$
- B. $\frac{1}{2} e^{\sqrt{x}} + C$
- C. $2e^{\sqrt{x}} + C$
- D. none of these

Answer

$$\text{Given} = \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$\text{Let, } x = z^2$$

$$\Rightarrow dx = 2z dz$$

So,

$$\begin{aligned} & \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx \\ &= \int \frac{e^z}{z} 2z dz \\ &= 2 \int e^z dz \\ &= 2e^z + c \\ &= 2e^{\sqrt{x}} + c \end{aligned}$$

where c is the integrating constant.

18. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{e^{\tan^{-1} x}}{(1+x^2)} dx = ?$$

A. $\frac{e^{\tan^{-1}x}}{x} + C$

B. $e^{\tan^{-1}x} + C$

C. $e^x \tan^{-1}x + C$

D. none of these

Answer

Given = $\int \frac{e^{\tan^{-1}x}}{(1+x^2)}$

Let, $\tan^{-1}x = z$

$\Rightarrow \frac{1}{1+x^2} dx = dz$

So,

$\int \frac{e^{\tan^{-1}x}}{(1+x^2)} dx$

$= \int e^z dz$

$= e^z + c$

$= e^{\tan^{-1}x} + c$

where c is the integrating constant.

19. Question

Mark (✓) against the correct answer in each of the following:

$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = ?$

A. $2 \cos \sqrt{x} + C$

B. $-2 \cos \sqrt{x} + C$

C. $-\frac{\cos \sqrt{x}}{2} + C$

D. $\frac{\cos \sqrt{x}}{2} + C$

Answer

Given = $\int \frac{\sin \sqrt{x}}{\sqrt{x}}$

Let, $x = z^2$

$\Rightarrow dx = 2z dz$

So,

$$\begin{aligned} & \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx \\ &= \int \frac{\sin z}{z} 2z dz \\ &= 2 \int \sin z dz \\ &= -2 \cos z + c \\ &= -2 \cos \sqrt{x} + c \end{aligned}$$

where c is the integrating constant.

20. Question

Mark (✓) against the correct answer in each of the following:

$$\int (\sqrt{\sin x}) \cos x \, dx = ?$$

- A. $\frac{2}{3}(\cos x)^{3/2} + C$
- B. $\frac{3}{2}(\cos x)^{3/2} + C$
- C. $\frac{2}{3}(\sin x)^{3/2} + C$
- D. $\frac{3}{2}(\sin x)^{3/2} + C$

Answer

$$\text{Given} = \int (\sqrt{\sin x}) \cos x \, dx$$

$$\text{Let, } \sin x = z^2$$

$$\Rightarrow \cos x \, dx = 2z \, dz$$

So,

$$\begin{aligned} & \int (\sqrt{\sin x}) \cos x \, dx \\ &= 2 \int z^2 \, dz \\ &= 2 \frac{z^3}{3} + c \\ &= \frac{2}{3} \sin^{3/2} x + c \end{aligned}$$

where c is the integrating constant.

21. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{(1+x^2)\sqrt{\tan^{-1}x}}$$

A. $\frac{1}{2} \log |\tan^{-1}x| + C$

B. $2\sqrt{\tan^{-1}x} + C$

C. $\frac{1}{2\sqrt{\tan^{-1}x}} + C$

D. none of these

Answer

Given = $\int \frac{1}{(1+x^2)\sqrt{\tan^{-1}x}}$

Let, $\tan^{-1}x = z^2$

$$\Rightarrow \frac{1}{1+x^2} dx = 2z dz$$

So,

$$\int \frac{1}{(1+x^2)\sqrt{\tan^{-1}x}} dx$$

$$= \int \frac{2z}{z} dz$$

$$= 2 \int dz$$

$$= 2z + c$$

$$= 2\sqrt{\tan^{-1}x} + c$$

where c is the integrating constant.

22. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cot x}{\log(\sin x)} dx = ?$$

A. $\log |\cot x| + C$

B. $\log |\cot x \operatorname{cosec} x| + C$

C. $\log |\log \sin x| + C$

D. none of these

Answer

Given = $\int \frac{\cot x}{\log(\sin x)}$

Let, $\sin x = z$

$$\Rightarrow \cos x dx = dz$$

So,

$$\begin{aligned} & \int \frac{\cot x}{\log(\sin x)} dx \\ &= \int \frac{\cos x}{\sin x \log(\sin x)} dx \\ &= \int \frac{dz}{z \log z} \end{aligned}$$

Let, $\log z = u$

$$\Rightarrow \frac{1}{z} dz = du$$

So,

$$\begin{aligned} & \int \frac{dz}{z \log z} \\ &= \int \frac{du}{u} \\ &= \log u + c \\ &= \log |\log z| + c \end{aligned}$$

where c is the integrating constant.

23. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{x \cos^2(1 + \log x)} dx = ?$$

- A. $\tan(1 + \log x) + C$
- B. $\cot(1 + \log x) + C$
- C. $\sec(1 + \log x) + C$
- D. none of these

Answer

$$\text{Given} = \int \frac{1}{x \cos^2(1 + \log x)} dx$$

Let, $1 + \log x = z$

$$\Rightarrow \frac{1}{x} dx = dz$$

So,

$$\begin{aligned}
 & \int \frac{1}{x \cos^2(1 + \log x)} dx \\
 &= \int \frac{dz}{\cos^2 z} \\
 &= \int \sec^2 z dz \\
 &= \tan z + c \\
 &= \tan(1 + \log x) + c
 \end{aligned}$$

where c is the integrating constant.

24. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x^2 \tan^{-1} x^3}{(1 + x^6)} dx = ?$$

A. $\frac{1}{3}(\tan^{-1} x^3) + C$

B. $\log |\tan^{-1} x^3| + C$

C. $\frac{1}{6}(\tan^{-1} x^3)^2 + C$

D. none of these

Answer

$$\text{Given} = \int \frac{x^2 \tan^{-1} x^3}{(1 + x^6)} dx$$

$$\text{Let, } \tan^{-1} x^3 = z$$

$$\Rightarrow \frac{1}{1 + x^6} \times 3x^2 dx = dz$$

$$\Rightarrow \frac{x^2}{1 + x^6} dx = \frac{dz}{3}$$

So,

$$\frac{1}{3} \int z dz$$

$$= \frac{1}{3} \frac{z^2}{2} + c$$

$$= \frac{z^2}{6} + c$$

$$= \frac{(\tan^{-1} x^3)^2}{6} + c$$

where c is the integrating constant.

25. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sec^5 x \tan x \, dx = ?$$

A. $5 \tan^5 x + C$

B. $\frac{1}{5} \tan^5 x + C$

C. $5 \log |\cos x| + C$

D. none of these

Answer

Given = $\int \sec^5 x \tan x \, dx$

So, $\int \sec^5 x \tan x \, dx = \int \sec^4 x (\sec x \tan x) \, dx$

Let, $\sec x = z$

$\Rightarrow \sec x \tan x \, dx = dz$

$$\int \sec^4 x (\sec x \tan x) \, dx$$

$$= \int z^4 \, dz$$

$$= \frac{z^5}{5} + c$$

$$= \frac{\sec^5 x}{5} + c$$

where c is the integrating constant.

26. Question

Mark (✓) against the correct answer in each of the following:

$$\int \operatorname{cosec}^3(2x+1) \cot(2x+1) \, dx = ?$$

A. $\frac{1}{4} \operatorname{cosec}^4(2x+1) + C$

B. $-\frac{1}{3} \operatorname{cosec}^3(2x+1) + C$

C. $-\frac{1}{6} \operatorname{cosec}^3(2x+1) + C$

D. $\frac{1}{2} \operatorname{cosec}(2x+1) \cot(2x+1) + C$

Answer

Given = $\int \operatorname{cosec}^3(2x+1) \cot(2x+1) \, dx$

So,

$$\int \operatorname{cosec}^3(2x+1) \cot(2x+1) dx$$

$$= \int \operatorname{cosec}^2(2x+1) \operatorname{cosec}(2x+1) \cot(2x+1) dx$$

Let, $\operatorname{cosec}(2x+1) = z$

$$\Rightarrow -2 \operatorname{cosec}(2x+1) \cot(2x+1) dx = dz$$

$$\int \operatorname{cosec}^2(2x+1) \operatorname{cosec}(2x+1) \cot(2x+1) dx$$

$$= \int z^2 \frac{dz}{-2} =$$

$$= -\frac{1}{2} \frac{z^3}{3} + c$$

$$= -\frac{\operatorname{cosec}^6(2x+1)}{6} + c$$

where c is the integrating constant.

27. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\tan(\sin^{-1} x)}{\sqrt{1-x^2}} dx = ?$$

A. $\log |\sec(\sin^{-1} x)| + C$

B. $\log |\cos(\sin^{-1} x)| + C$

C. $\tan(\sin^{-1} x) + C$

D. none of these

Answer

$$\text{Given} = \int \frac{\tan(\sin^{-1} x)}{\sqrt{1-x^2}} dx$$

Let, $\sin^{-1} x = z$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dz$$

So,

$$\int \frac{\tan(\sin^{-1} x)}{\sqrt{1-x^2}} dx$$

$$= \int \tan z dz$$

$$= \log |\sec z| + c$$

$$= \log |\sec(\sin^{-1} x)| + c$$

where c is the integrating constant.

28. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\tan(\log x)}{x} dx = ?$$

- A. $x \tan(\log x) + C$
- B. $\log |\tan x| + C$
- C. $\log |\cos(\log x)| + C$
- D. $-\log |\cos(\log x)| + C$

Answer

$$\text{Given} = \int \frac{\tan(\log x)}{x}$$

Let, $\log x = z$

$$\Rightarrow \frac{1}{x} dx = dz$$

So,

$$\begin{aligned} \int \frac{\tan(\log x)}{x} dx &= \int \tan z dz \\ &= \log |\sec z| + c \\ &= \log |\sec(\log x)| + c \\ &= -\log |\cos(\log x)| + c \end{aligned}$$

where c is the integrating constant.

29. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \cot(e^x) dx = ?$$

- A. $\cot(e^x) + C$
- B. $\log |\sin e^x| + C$
- C. $\log |\operatorname{cosec} e^x| + C$
- D. none of these

Answer

$$\text{Given} = \int e^x \cot(e^x) dx$$

Let, $e^x = z$

$$\Rightarrow e^x dx = dz$$

So,

$$\begin{aligned}
 & \int e^x \cot(e^x) dx \\
 &= \int \cot z dz \\
 &= \log |\sin z| + c \\
 &= \log |\sin(e^x)| + c
 \end{aligned}$$

where c is the integrating constant.

30. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{e^x}{\sqrt{1+e^x}} dx = ?$$

A. $2\sqrt{1+e^x} + C$

B. $\frac{1}{2}\sqrt{1+e^x} + C$

C. $\frac{1}{\sqrt{1+e^x}} + C$

D. none of these

Answer

$$\text{Given} = \int \frac{e^x}{\sqrt{1+e^x}}$$

$$\text{Let, } 1 + e^x = z^2$$

$$\Rightarrow e^x dx = 2z dz$$

So,

$$\int \frac{e^x}{\sqrt{1+e^x}} dx$$

$$= \int \frac{2z dz}{z}$$

$$= 2 \int dz$$

$$= 2z + c$$

$$= 2\sqrt{1+e^x} + c$$

where c is the integrating constant.

31. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x}{\sqrt{1-x^2}} dx = ?$$

A. $\sin^{-1} x + C$

B. $\sin^{-1} \sqrt{x} + C$

C. $\sqrt{1-x^2} + C$

D. $-\sqrt{1-x^2} + C$

Answer

Given = $\int \frac{x}{\sqrt{1-x^2}} dx$

Let, $1 - x^2 = z^2$

$\Rightarrow -2x dx = 2z dz$

So,

$\int \frac{x}{\sqrt{1-x^2}} dx$

$= -\int \frac{z dz}{z}$

$= -\int dz$

$= -z + c$

$= -\sqrt{1-x^2} + c$

where c is the integrating constant.

32. Question

Mark (✓) against the correct answer in each of the following:

$\int \frac{e^x (1+x)}{\cos^2(xe^x)} dx = ?$

A. $\tan(xe^x) + C$

B. $\cot(xe^x) + C$

C. $ex^x \tan x + C$

D. none of these

Answer

Given = $\int \frac{e^x (1+x)}{\cos^2(xe^x)} dx$

Let, $xe^x = z$

$\Rightarrow e^x(1+x)dx = dz$

So,

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(xe^x) + c$$

where c is the integrating constant.

33. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(e^x + e^{-x})} = ?$$

A. $\cot^{-1}(e^x) + C$

B. $\tan^{-1}(e^x) + C$

C. $\log |e^x + 1| + C$

D. none of these

Answer

Given =

$$\int \frac{dx}{(e^x + e^{-x})}$$

$$= \int \frac{e^x}{(e^x + 1)} dx$$

Let, $e^x + 1 = z$

$$\Rightarrow e^x dx = dz$$

So,

$$\int \frac{e^x dx}{(e^x + 1)}$$

$$= \int \frac{dz}{z}$$

$$= \log |z| + c$$

$$= \log |e^x + 1| + c$$

where c is the integrating constant.

34. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{2^x}{1-4^x} dx = ?$$

- A. $\sin^{-1}(2^x) + C$
- B. $(\log e^2) \sin^{-1}(2^x) + C$
- C. $(\log e^2) \cos^{-1}(2^x) + C$
- D. $\log_2 e \sin^{-1}(2^x) + C$

Answer

Given =

$$\begin{aligned} & \int \frac{2^x dx}{1-4^x} \\ &= \int \frac{2^x}{1-(2^x)^2} dx \end{aligned}$$

Let, $2^x = z$

$$\Rightarrow 2^x (\log 2) dx = dz$$

So,

$$\begin{aligned} & \int \frac{2^x dx}{1-(2^x)^2} \\ &= \frac{1}{\log 2} \int \frac{dz}{1-z^2} \\ &= \frac{1}{\log 2} \sin^{-1} z + c \\ &= \frac{\sin^{-1} 2^x}{\log 2} + c \end{aligned}$$

where c is the integrating constant.

35. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(e^x - 1)} = ?$$

- A. $\log |e^x - 1| + C$
- B. $\log |1 - e^x| + C$
- C. $\log |e^x - 1| + C$
- D. none of these

Answer

Given =

$$\begin{aligned}
 & \int \frac{dx}{e^x - 1} \\
 &= -\int \frac{-1 + e^x - e^x}{e^x - 1} dx \\
 &= -\int \frac{e^x - 1}{e^x - 1} dx + \int \frac{e^x}{e^x - 1} dx \\
 &= -\int dx + \int \frac{e^x}{e^x - 1} dx
 \end{aligned}$$

Let, $e^x - 1 = z$

$$\Rightarrow e^x dx = dz$$

So,

$$\begin{aligned}
 & -\int dx + \int \frac{e^x}{e^x - 1} dx \\
 &= -x + \int \frac{dz}{z} \\
 &= -x + \log z + c \\
 &= -x + \log |e^x - 1| + c
 \end{aligned}$$

where c is the integrating constant.

36. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{(\sqrt{x} + x)} dx = ?$$

A. $\log |1 + \sqrt{x}| + C$

B. $2 \log |1 + \sqrt{x}| + C$

C. $\frac{1}{\sqrt{x}} \tan^{-1} \sqrt{x} + C$

D. none of these

Answer

Given =

$$\begin{aligned}
 & \int \frac{dx}{(\sqrt{x} + x)} \\
 &= \int \frac{1}{\sqrt{x}} \frac{1}{(1 + \sqrt{x})} dx
 \end{aligned}$$

Let, $1 + \sqrt{x} = z$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dz$$

So,

$$\begin{aligned} & \int \frac{1}{\sqrt{x}} \frac{1}{(1+\sqrt{x})} dx \\ &= 2 \int \frac{dz}{z} \\ &= 2 \log|z| + c \\ &= 2 \log|1+\sqrt{x}| + c \end{aligned}$$

where c is the integrating constant.

37. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1+\sin x)} = ?$$

A. $\tan x + \sec x + C$

B. $\tan x - \sec x + C$

C. $\frac{1}{2} \tan \frac{x}{2} + C$

D. none of these

Answer

Given

$$\begin{aligned} & \int \frac{dx}{1+\sin x} \\ &= \int \frac{dx}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}} \\ &= \int \frac{dx}{\left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2} \\ &= \int \frac{\sec^2 \frac{x}{2} dx}{\left(\tan \frac{x}{2} + 1 \right)^2} \end{aligned}$$

Let, $\tan \frac{x}{2} + 1 = z$

$$\Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dz$$

So,

$$\int \frac{2dz}{z^2}$$

$$= -\frac{2}{z} + c$$

$$= -\frac{2}{\tan \frac{x}{2} + 1} + c$$

where c is the integrating constant.

38. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1 + \sin x)} dx = ?$$

- A. $x + \tan x - \sec x + C$
- B. $x - \tan x - \sec x + C$
- C. $x - \tan x + \sec x + C$
- D. none of these

Answer

Given

$$\int \frac{\sin x}{1 + \sin x} dx$$

$$= \int dx - \int \frac{dx}{1 + \sin x}$$

$$= x - \int \frac{dx}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}$$

$$= x - \int \frac{dx}{\left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2}$$

$$= x - \int \frac{\sec^2 \frac{x}{2} dx}{\left(\tan \frac{x}{2} + 1 \right)^2}$$

$$\text{Let, } \tan \frac{x}{2} + 1 = z$$

$$\Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dz$$

So,

$$\begin{aligned}
 & x - \int \frac{2dz}{z^2} \\
 &= x + \frac{2}{z} + c \\
 &= x + \frac{2}{\tan \frac{x}{2} + 1} + c
 \end{aligned}$$

where c is the integrating constant.

39. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1 - \sin x)} dx = ?$$

- A. $-x + \sec x - \tan x + C$
- B. $x + \cos x - \sin x + C$
- C. $-\log |1 - \sin x| + C$
- D. none of these

Answer

Given

$$\begin{aligned}
 & \int \frac{\sin x}{1 - \sin x} dx \\
 &= -\int dx + \int \frac{dx}{1 - \sin x} \\
 &= -x + \int \frac{dx}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2}} \\
 &= -x + \int \frac{dx}{\left(\sin \frac{x}{2} - \cos \frac{x}{2} \right)^2} \\
 &= -x + \int \frac{\sec^2 \frac{x}{2} dx}{\left(\tan \frac{x}{2} - 1 \right)^2}
 \end{aligned}$$

$$\text{Let, } \tan \frac{x}{2} - 1 = z$$

$$\Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dz$$

So,

$$\begin{aligned}
 & -x + \int \frac{2dz}{z^2} \\
 & = -x - \frac{2}{z} + c \\
 & = -x - \frac{2}{\tan \frac{x}{2} + 1} + c
 \end{aligned}$$

where c is the integrating constant.

40. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1 + \cos x)} = ?$$

A. $\frac{1}{2} \tan \frac{x}{2} + C$

B. $-\cot \frac{x}{2} + C$

C. $\tan \frac{x}{2} + C$

D. none of these

Answer

Given

$$\begin{aligned}
 & \int \frac{dx}{1 + \cos x} \\
 & = \int \frac{dx}{1 + 2\cos^2 \frac{x}{2} - 1} \\
 & = \frac{1}{2} \int \frac{dx}{\cos^2 \frac{x}{2}} \\
 & = \frac{1}{2} \int \sec^2 \frac{x}{2} dx \\
 & = \frac{1}{2} 2 \tan \frac{x}{2} + c \\
 & = \tan \frac{x}{2} + c
 \end{aligned}$$

where c is the integrating constant.

41. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1 - \cos x)} = ?$$

A. $\frac{1}{(x - \sin x)} + C$

B. $\log |x - \sin x| + C$

C. $\log \left| \tan \frac{x}{2} \right| + C$

D. $-\cot \frac{x}{2} + C$

Answer

Given

$$\begin{aligned} & \int \frac{dx}{1 - \cos x} \\ &= \int \frac{dx}{1 - 1 + 2 \sin^2 \frac{x}{2}} \\ &= \frac{1}{2} \int \frac{dx}{\sin^2 \frac{x}{2}} \\ &= \frac{1}{2} \int \operatorname{cosec}^2 \frac{x}{2} dx \\ &= -\frac{1}{2} 2 \cot \frac{x}{2} + c \\ &= -\cot \frac{x}{2} + c \end{aligned}$$

where c is the integrating constant.

42. Question

Mark (✓) against the correct answer in each of the following:

$$\int \left\{ \frac{1 - \tan\left(\frac{x}{2}\right)}{1 + \tan\left(\frac{x}{2}\right)} \right\} dx = ?$$

A. $2 \log \left| \sec \frac{x}{2} \right| + C$

B. $2 \log \left| \cos \frac{x}{2} \right| + C$

C. $2 \log \left| \sec \left(\frac{\pi}{4} - \frac{x}{2} \right) \right| + C$

D. $2 \log \left| \cos \left(\frac{\pi}{4} - \frac{x}{2} \right) \right| + C$

Answer

Given

$$\int \frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}} dx$$

$$= \int \frac{1 - \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}}{1 + \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}} dx$$

$$= \int \frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}} dx$$

$$\text{Let, } \cos \frac{x}{2} + \sin \frac{x}{2} = z$$

$$\Rightarrow \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right) dx = dz$$

So,

$$\int \frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}} dx$$

$$= \int \frac{dz}{z}$$

$$= \log z + c$$

$$= \log \left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) + c$$

where c is the integrating constant.

43. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{e^x} dx = ?$$

A. $\sqrt{e^x} + C$

B. $2\sqrt{e^x} + C$

C. $\frac{1}{2}\sqrt{e^x} + C$

D. none of these

Answer

Given

$$\begin{aligned}\int \sqrt{e^x} dx \\&= \int (e^x)^{\frac{1}{2}} dx \\&= \int e^{\frac{1}{2}x} dx \\&= 2e^{\frac{1}{2}x} + c \\&= 2\sqrt{e^x} + c\end{aligned}$$

where c is the integrating constant.

44. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cos x}{(1 + \cos x)} dx = ?$$

- A. $x + \tan \frac{x}{2} + C$
- B. $-x + \tan \frac{x}{2} + C$
- C. $x - \tan \frac{x}{2} + C$
- D. none of these

Answer

Given

$$\begin{aligned}\int \frac{\cos x dx}{1 + \cos x} \\&= \int \frac{1 + \cos x - 1}{1 + \cos x} dx \\&= \int dx - \int \frac{dx}{1 + \cos x} \\&= x - \tan \frac{x}{2} + c\end{aligned}$$

[From Question no. 40] where c is the integrating constant.

45. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sec^2 x \operatorname{cosec}^2 x dx = ?$$

- A. $\tan x - \cot x + C$
- B. $\tan x + \cot x + C$
- C. $-\tan x + \cot x + C$

D. none of these

Answer

Given

$$\begin{aligned} & \int \sec^2 x \operatorname{cosec}^2 x dx \\ &= \int \frac{1}{\sin^2 x \cos^2 x} dx \\ &= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx \\ &= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx \\ &= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx \\ &= \tan x - \cot x + c \end{aligned}$$

where c is the integrating constant.

46. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = ?$$

- A. $\tan x + x + C$
- B. $\tan x - x + C$
- C. $-\tan x + x + C$
- D. none of these

Answer

Given

$$\begin{aligned} & \int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx \\ &= \int \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} dx \\ &= \int \tan^2 \frac{x}{2} dx \\ &= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx \\ &= 2 \tan \frac{x}{2} - x + c \end{aligned}$$

where c is the integrating constant.

47. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 + \cos x)}{(1 - \cos x)} dx = ?$$

A. $-2 \cot \frac{x}{2} - x + C$

B. $-2 \cot \frac{x}{2} + x + C$

C. $2 \cot \frac{x}{2} + x + C$

D. none of these

Answer

Given

$$\begin{aligned} & \int \frac{(1 + \cos 2x)}{(1 - \cos 2x)} dx \\ &= \int \frac{2 \cos^2 \frac{x}{2}}{2 \sin^2 \frac{x}{2}} dx \\ &= \int \cot^2 \frac{x}{2} dx \\ &= \int \left(\operatorname{cosec}^2 \frac{x}{2} - 1 \right) dx \\ &= -2 \cot \frac{x}{2} - x + c \end{aligned}$$

where c is the integrating constant.

48. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = ?$$

A. $\tan x + \cot x + C$

B. $\tan x - \cot x + C$

C. $-\tan x + \cot x + C$

D. none of these

Answer

Given

$$\begin{aligned}
& \int \frac{1}{\sin^2 x \cos^2 x} dx \\
&= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx \\
&= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx \\
&= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx \\
&= \tan x - \cot x + c
\end{aligned}$$

where c is the integrating constant.

49. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = ?$$

- A. $\cot x + \tan x + C$
- B. $-\cot x + \tan x + C$
- C. $\cot x - \tan x + C$
- D. $-\cot x - \tan x + C$

Answer

Given

$$\begin{aligned}
& \int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx \\
&= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx \\
&= \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx \\
&= \int \operatorname{cosec}^2 x dx - \int \sec^2 x dx \\
&= -\tan x - \cot x + c
\end{aligned}$$

where c is the integrating constant.

50. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = ?$$

- A. $\sin x + x \cos \alpha + C$
- B. $2\sin x + x \cos \alpha + C$
- C. $2 \sin x + 2x \cos \alpha + C$
- D. none of these

Answer

Given

$$\begin{aligned}
 & \int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx \\
 &= \int \frac{-2 \sin\left(\frac{2x+2\alpha}{2}\right) \sin\left(\frac{2x-2\alpha}{2}\right)}{-2 \sin\left(\frac{x+\alpha}{2}\right) \sin\left(\frac{x-\alpha}{2}\right)} \\
 &= \int \frac{\sin(x+\alpha) \sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right) \sin\left(\frac{x-\alpha}{2}\right)} \\
 &= \int \frac{2 \sin\left(\frac{x+\alpha}{2}\right) \cos\left(\frac{x+\alpha}{2}\right) \times 2 \sin\left(\frac{x-\alpha}{2}\right) \cos\left(\frac{x-\alpha}{2}\right)}{\sin\left(\frac{x+\alpha}{2}\right) \sin\left(\frac{x-\alpha}{2}\right)} \\
 &= 2 \int 2 \cos\left(\frac{x+\alpha}{2}\right) \cos\left(\frac{x-\alpha}{2}\right) \\
 &= 2 \int \cos\left(\frac{x+\alpha}{2} + \frac{x-\alpha}{2}\right) + \cos\left(\frac{x+\alpha}{2} - \frac{x-\alpha}{2}\right) \\
 &= 2 \int (\cos x + \cos \alpha) dx \\
 &= 2 [\sin x + x \cos \alpha] + c
 \end{aligned}$$

where c is the integrating constant.

51. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1} \left\{ \sqrt{\frac{1-\cos 2x}{1+\cos 2x}} \right\} dx = ?$$

A. $2x^2 + C$

B. $\frac{x^2}{2} + C$

C. $\frac{2}{(1+x^2)} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \cos 2x = 2\cos^2 x$; $1 - \cos 2x = 2\sin^2 x$

Therefore ,

$$\Rightarrow \int \tan^{-1} \sqrt{\frac{1-\cos 2x}{1+\cos 2x}} dx = \int \tan^{-1} \sqrt{\frac{2\sin^2 x}{2\cos^2 x}} dx = \int \tan^{-1} \tan x dx$$

$$\Rightarrow \int x dx = \frac{x^2}{2} + c$$

52. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1}(\sec x + \tan x) dx = ?$$

A. $\frac{\pi x}{4} + \frac{x^2}{4} + C$

B. $\frac{\pi x}{4} - \frac{x^2}{4} + C$

C. $\frac{1}{(1+x^2)} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \sin x = (\cos \frac{x}{2} + \sin \frac{x}{2})^2$

$$\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} ; \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

Therefore ,

$$\Rightarrow \int \tan^{-1}(\sec x + \tan x) dx = \int \tan^{-1} \left(\frac{1+\sin x}{\cos x} \right) dx$$

$$\Rightarrow \int \tan^{-1} \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})^2}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} dx = \int \tan^{-1} \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})^2}{(\cos \frac{x}{2} + \sin \frac{x}{2})(\cos \frac{x}{2} - \sin \frac{x}{2})} dx$$

$$\Rightarrow \int \tan^{-1} \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})^1}{(\cos \frac{x}{2} - \sin \frac{x}{2})} dx = \int \tan^{-1} \frac{1+\tan \frac{x}{2}}{1-\tan \frac{x}{2}} dx$$

(Multiply by $\sec \frac{x}{2}$ in numerator and denominator)

$$\Rightarrow \int \tan^{-1} \frac{1+\tan \frac{x}{2}}{1-\tan \frac{x}{2}} dx = \int \tan^{-1} \frac{\tan \frac{\pi}{4} + \tan \frac{x}{2}}{\tan \frac{\pi}{4} - \tan \frac{x}{2}} dx = \int \tan^{-1} \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) dx$$

$$\Rightarrow \int \left(\frac{\pi}{4} + \frac{x}{2} \right) dx = \frac{\pi x}{4} + \frac{x^2}{4} + c$$

53. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 + \sin x)}{(1 - \sin x)} dx = ?$$

A. $2 \tan x + x - 2 \sec x + C$

B. $2 \tan x - x + 2 \sec x + C$

C. $2 \tan x - x - 2 \sec x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$$\Rightarrow \int \frac{1+\sin x(1+\sin x)}{1-\sin x(1+\sin x)} dx$$

$$\Rightarrow \int \frac{(1+\sin x)^2}{1-\sin^2 x} dx = \int \frac{1+\sin^2 x+2\sin x}{\cos^2 x} dx$$

$$\Rightarrow \int \frac{1}{\cos^2 x} dx + 2 \int \frac{\sin x}{\cos^2 x} dx + \int \frac{\sin^2 x}{\cos^2 x} dx$$

$$\Rightarrow \int \sec^2 x dx + 2 \int \frac{\sin x}{\cos^2 x} dx + \int \tan^2 x dx$$

$$\Rightarrow \int \sec^2 x dx + 2 \int \frac{\sin x}{\cos^2 x} dx + \int (-1 + \sec^2 x) dx$$

$$\Rightarrow 2 \int \sec^2 x dx + 2 \int \frac{\sin x}{\cos^2 x} dx - \int 1 dx$$

Put $\cos x = t$

Therefore $\rightarrow \sin x dx = -dt$

$$\Rightarrow 2 \tan x - 2 \int \frac{dt}{t^2} - x + c$$

$$\Rightarrow 2 \tan x + 2 \frac{1}{t} - x + c$$

$$\Rightarrow 2 \tan x + 2 \sec x - x + c$$

54. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x^4}{(1+x^2)} dx = ?$$

A. $\frac{x^3}{3} + x + \tan^{-1} x + C$

B. $\frac{-x^3}{3} + x - \tan^{-1} x + C$

C. $\frac{x^3}{3} - x + \tan^{-1} x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \frac{x^4+1-1}{1+x^2} dx$$

$$\Rightarrow \int \frac{x^4-1}{1+x^2} dx + \int \frac{1}{1+x^2} dx = \int \frac{(1+x^2)(x^2-1)}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$\Rightarrow \int x^2 - 1 dx + \int \frac{1}{1+x^2} dx$$

$$\Rightarrow \frac{x^3}{3} - x + \tan^{-1} x + c$$

55. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin(x - \alpha)}{\sin(x + \alpha)} dx = ?$$

- A. $x \cos 2\alpha - \sin 2\alpha \cdot \log |\sin(x + \alpha)| + C$
- B. $x \cos 2\alpha + \sin 2\alpha \cdot \log |\sin(x + \alpha)| + C$
- C. $x \cos 2\alpha + \sin \alpha \cdot \log |\sin(x + \alpha)| + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\int \cot x = \log(\sin x) + c$$

Therefore ,

$$\Rightarrow \int \frac{\sin(x+\alpha-2\alpha)}{\sin(x+\alpha)} dx$$

$$\Rightarrow \int \frac{\sin(x+\alpha) \cos(-2\alpha) + \cos(x+\alpha) \sin(-2\alpha)}{\sin(x+\alpha)} dx$$

$$\Rightarrow \int \cos(2\alpha) dx - \sin 2\alpha \int \cot(x+\alpha) dx$$

$$\Rightarrow \cos(2\alpha) x - \sin 2\alpha \log |\sin(x+\alpha)| + c$$

56. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{(\sqrt{x+3} - \sqrt{x+2})} dx = ?$$

- A. $\frac{2}{3}(x+3)^{3/2} - \frac{2}{3}(x+3)^{3/2} + C$
- B. $\frac{2}{3}(x+3)^{3/2} + \frac{2}{3}(x+3)^{3/2} + C$
- C. $\frac{3}{2}(x+3)^{3/2} - \frac{3}{2}(x+3)^{3/2} + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\int \cot x = \log(\sin x) + c$$

Therefore ,

$$\Rightarrow \int \frac{(\sqrt{x+3} + \sqrt{x+2})}{(\sqrt{x+3} - \sqrt{x+2})(\sqrt{x+3} + \sqrt{x+2})} dx \text{ (Rationalizing the denominator)}$$

$$\Rightarrow \int (\sqrt{x+3} + \sqrt{x+2}) dx$$

$$\Rightarrow \int \sqrt{x+3} dx + \int \sqrt{x+2} dx$$

$$\Rightarrow \frac{2(x+3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{2(x+2)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

57. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 + \tan x)}{(1 - \tan x)} dx = ?$$

A. $-\log |\cos x - \sin x| + C$

B. $\log |\cos x - \sin x| + C$

C. $\log |\cos x + \sin x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log (\sin x) + c$

Therefore ,

$$\Rightarrow \int \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} dx \text{ (Rationalizing the denominator)}$$

$$\Rightarrow \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Put $\cos x - \sin x = t$

$(-\sin x - \cos x) dx = dt$

$(\sin x + \cos x) dx = -dt$

$$\Rightarrow \int \frac{-dt}{t} = -\log t + c$$

$$\Rightarrow -\log |\cos x - \sin x| + c$$

59. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{3x^2}{(1+x^6)} dx = ?$$

A. $\sin^{-1} x^3 + C$

B. $\cos^{-1} x^3 + C$

C. $\tan^{-1} x^3 + C$

D. $\cot^{-1} x^3 + C$

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^3 = t$ $3x^2 dx = dt$

$$\Rightarrow \int \frac{dt}{1+t^2}$$

$$\Rightarrow \tan^{-1} t + c$$

$$\Rightarrow \tan^{-1} x^3 + c$$

59. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{x\sqrt{x^6-1}} = ?$$

A. $\frac{1}{3} \sec^{-1} x^3 + C$

B. $\frac{1}{3} \operatorname{cosec}^{-1} x^3 + C$

C. $\frac{1}{3} \cot^{-1} x^3 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Therefore ,

Put $x^3 = t$, $3x^2 dx = dt$

$$\Rightarrow \int \frac{dt}{x \times 3x^2 \sqrt{t^2-1}} = \int \frac{dt}{3t\sqrt{t^2-1}}$$

$$\Rightarrow \frac{1}{3} \int \frac{dt}{t\sqrt{t^2-1}}$$

$$\Rightarrow \frac{1}{3} \sec^{-1} t + c$$

$$\Rightarrow \frac{1}{3} \sec^{-1} x^3 + c$$

60. Question

Mark (✓) against the correct answer in each of the following:

$$\int \left\{ (2x+1)\sqrt{x^2+x+1} \right\} dx = ?$$

A. $\frac{3}{2} (x^2+x+1)^{3/2} + C$

B. $\frac{2}{3} (x^2+x+1)^{3/2} + C$

C. $\frac{3}{2} (2x+1)^{3/2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Therefore ,

Put $x^2 + x + 1 = t$, $(2x + 1)dx = dt$

$$\Rightarrow \int \sqrt{t} dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\Rightarrow \frac{2}{3} t^{\frac{3}{2}} + c$$

$$\Rightarrow \frac{2}{3} (x^2 + x + 1)^{\frac{3}{2}} + c$$

61. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2x+3} + \sqrt{2x-3}} = ?$$

A. $\frac{1}{18} (2x+3)^{3/2} + \frac{1}{18} (2x-3)^{3/2} + C$

B. $\frac{1}{18} (2x+3)^{3/2} - \frac{1}{18} (2x-3)^{3/2} + C$

C. $\frac{1}{12} (2x+3)^{3/2} - \frac{1}{12} (2x-3)^{3/2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log (\sin x) + c$

Therefore ,

$$\Rightarrow \int \frac{(\sqrt{2x+3} - \sqrt{2x-3})}{(\sqrt{2x+3} + \sqrt{2x-3})(\sqrt{2x+3} - \sqrt{2x-3})} dx \text{ (Rationalizing the denominator)}$$

$$\Rightarrow \int \frac{\sqrt{2x+3} - \sqrt{2x-3}}{6} dx$$

$$\Rightarrow \frac{1}{6} \int \sqrt{2x+3} dx - \frac{1}{6} \int \sqrt{2x-3} dx$$

$$\Rightarrow \frac{2(2x+3)^{\frac{3}{2}}}{3 \times 6 \times 2} - \frac{2(2x-3)^{\frac{3}{2}}}{3 \times 6 \times 2} + c$$

$$\Rightarrow \frac{(2x+3)^{\frac{3}{2}}}{18} - \frac{(2x-3)^{\frac{3}{2}}}{18} + c$$

62. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan x dx = ?$$

A. $\log |\cos x| + C$

B. $-\log |\cos x| + C$

C. $\log |\sin x| + C$

D. $-\log |\sin x| + C$

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log (\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin x}{\cos x} dx$

Put $\cos x = t$ $-\sin x dx = dt$

$\Rightarrow \int \frac{-dt}{t}$

$\Rightarrow -\log t + c$

$\Rightarrow -\log |\cos x| + c$

63. Question

Mark (✓) against the correct answer in each of the following:

$\int \sec x dx = ?$

A. $\log |\sec x - \tan x| + C$

B. $-\log |\sec x + \tan x| + C$

C. $\log |\sec x + \tan x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log (\sin x) + c$

Therefore ,

$\Rightarrow \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$

$\Rightarrow \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$

Put $\sec x + \tan x = t$, $(\sec^2 x + \sec x \tan x) dx = dt$

$\Rightarrow \int \frac{dt}{t}$

$\Rightarrow \log t + c$

$\Rightarrow \log |\sec x + \tan x| + c$

64. Question

Mark (✓) against the correct answer in each of the following:

$$\int \operatorname{cosec} x \, dx = ?$$

- A. $\log |\operatorname{cosec} x - \cot x| + C$
- B. $-\log |\operatorname{cosec} x - \cot x| + C$
- C. $\log |\operatorname{cosec} x + \cot x| + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\int \cot x = \log (\sin x) + c$$

Therefore ,

$$\Rightarrow \int \operatorname{cosec} x \frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x - \cot x} dx$$

$$\Rightarrow \int \frac{\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x}{\operatorname{cosec} x - \cot x} dx$$

Put $\operatorname{cosec} x - \cot x = t$, $(\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x) dx = dt$

$$\Rightarrow \int \frac{dt}{t}$$

$$\Rightarrow \log t + c$$

$$\Rightarrow \log |\operatorname{cosec} x - \cot x| + c$$

65. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 + \sin x)}{(1 + \cos x)} dx = ?$$

A. $\tan \frac{x}{2} + 2 \log \left| \cos \frac{x}{2} \right| + C$

B. $-\tan \frac{x}{2} + 2 \log \left| \cos \frac{x}{2} \right| + C$

C. $\tan \frac{x}{2} - 2 \log \left| \cos \frac{x}{2} \right| + C$

- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$$\Rightarrow \int \frac{1 + \sin x}{2 \cos^2 \frac{x}{2}} dx$$

$$\Rightarrow \int \frac{1}{2\cos^2 \frac{x}{2}} + \frac{2\sin \frac{x}{2} \cos \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx = \frac{1}{2} \int \sec^2 \frac{x}{2} dx + \int \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} dx$$

$$\Rightarrow \frac{1}{2} \tan \frac{x}{2} \times 2 + \int \tan \frac{x}{2} dx$$

$$\Rightarrow \tan \frac{x}{2} + 2 \left(-\log \cos \frac{x}{2} \right) + c$$

$$\Rightarrow \tan \frac{x}{2} - 2 \log \left| \cos \frac{x}{2} \right| + c$$

66. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\tan x}{(\sec x + \cos x)} dx = ?$$

- A. $\tan^{-1}(\cos x) + C$
- B. $-\tan^{-1}(\cos x) + C$
- C. $\cot^{-1}(\cos x) + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$$\Rightarrow \int \frac{\sec x \tan x}{\sec^2 x + 1} dx$$

Put $\sec x = t$ ($\sec x \tan x$) $dx = dt$

$$\Rightarrow \int \frac{dt}{1+t^2} = \tan^{-1} t + c$$

$$\Rightarrow \tan^{-1} \sec x + c$$

$$\Rightarrow -\tan^{-1}(\cos x) + c$$

67. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}} dx = ?$$

- A. $\sin^{-1} x + \sqrt{1-x^2} + C$
- B. $\sin^{-1} x + (1+x^2) + C$
- C. $\sin^{-1} x - \sqrt{1-x^2} + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$$\Rightarrow \int \sqrt{\frac{(1+x)^2}{(1+x)(1-x)}} dx$$

$$\Rightarrow \int \frac{1+x}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx$$

$$\text{Put } 1-x^2 = t \quad -2x \, dx = dt$$

$$\Rightarrow \sin^{-1} x - \frac{1}{2} \int \frac{1}{\sqrt{t}} dt + c$$

$$\Rightarrow \sin^{-1} x - \frac{1}{2} \cdot \frac{\sqrt{t}}{\frac{1}{2}} + c$$

$$\Rightarrow \sin^{-1} x - \sqrt{t} + c = \sin^{-1} x - \sqrt{1-x^2} + c$$

68. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{x^2} e^{-1/x} dx = ?$$

A. $e^{-1/x} + C$

B. $-e^{-1/x} + C$

C. $\frac{e^{-1/x}}{x} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$$\text{Put } -\frac{1}{x} = t \quad \frac{1}{x^2} dx = dt$$

$$\Rightarrow \int e^t dt$$

$$\Rightarrow e^t + c$$

$$\Rightarrow e^{-\frac{1}{x}} + c$$

69. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x^3}{(1+x^8)} dx = ?$$

A. $\tan^{-1} x^4 + C$

B. $4 \tan^{-1} x^4 + C$

C. $\frac{1}{4} \tan^{-1} x^4 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^4 = t$ $4x^3 dx = dt$

$$\Rightarrow \frac{1}{4} \int \frac{1}{1+t^2} dt$$

$$\Rightarrow \frac{1}{4} \tan^{-1} t + c$$

$$\Rightarrow \frac{1}{4} \tan^{-1} x^4 + c$$

70. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx = ?$$

A. $\frac{1}{3}(x+\log x)^3 + C$

B. $\frac{x^2}{2} + x + C$

C. $\frac{x^3}{3} + \frac{x^2}{2} + x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^1 + \log x = t$ $(1 + \frac{1}{x})dx = dt \Rightarrow (\frac{x+1}{x})dx = dt$

$$\Rightarrow \int t^2 dt$$

$$\Rightarrow \frac{t^3}{3} + c$$

$$\Rightarrow \frac{(x+\log x)^3}{3} + c$$

71. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{2x \tan^{-1} x^2}{(1+x^4)} dx = ?$$

A. $(\tan^{-1} x^2)^2 + C$

B. $2 \tan^{-1} x^2 + C$

C. $\frac{1}{2} (\tan^{-1} x^2)^2 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $\tan^{-1} x^2 = t$ $\left(\frac{1}{1+(x^2)^2} \times 2x \right) dx = dt \Rightarrow \left(\frac{2x}{1+x^4} \right) dx = dt$

$$\Rightarrow \int t^1 dt$$

$$\Rightarrow \frac{t^2}{2} + c$$

$$\Rightarrow \frac{(\tan^{-1} x^2)^2}{2} + c$$

72. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(2-3x)} = ?$$

A. $-3 \log |2-3x| + C$

B. $-\frac{1}{3} \log |2-3x| + C$

C. $-\log |2-3x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $2-3x = t$ $-3dx = dt$

$$\Rightarrow -\frac{1}{3} \int \frac{1}{t} dt$$

$$\Rightarrow -\frac{1}{3} \log t + c$$

$$\Rightarrow -\frac{1}{3} \log |2-3x| + c$$

73. Question

Mark (✓) against the correct answer in each of the following:

$$\int x\sqrt{x^2-1} dx = ?$$

A. $\frac{2}{3}(x^2-1)^{3/2} + C$

B. $\frac{1}{3}(x^2-1)^{3/2} + C$

C. $\frac{1}{\sqrt{x^2-1}} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x} dx = \log x + c$

Therefore ,

$$\text{Put } x^2 - 1 = t \quad 2x dx = dt$$

$$\Rightarrow \int \sqrt{t} dt$$

$$\Rightarrow \frac{1}{2} t^{\frac{2}{2}} + c \Rightarrow \frac{t^{\frac{2}{2}}}{\frac{2}{2}} + c$$

$$\Rightarrow \frac{(x^2-1)^{\frac{2}{2}}}{\frac{2}{2}} + c$$

74. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^{(5-3x)} dx = ?$$

A. $\frac{3^{(5-3x)}}{3(\log 3)} + C$

B. $\frac{3^{(4-3x)}}{(\log 3)} + C$

C. $-3^{(5-3x)} \log 3 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

$$\text{Put } 5 - 3x = t \quad -3 dx = dt$$

$$\Rightarrow -\frac{1}{3} \int 3^t dt$$

$$\Rightarrow -\frac{1}{3} \times \frac{3^t}{\log 3} + c \Rightarrow -\frac{1}{3} \times \frac{3^{(5-3x)}}{\log 3} + c$$

$$\Rightarrow -\frac{3^{(5-3x)}}{3 \log 3} + c$$

75. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^{\tan x} \sec^2 x dx = ?$$

A. $e^{\tan x} + \tan x + C$

B. $e^{\tan x} \cdot \tan x + C$

C. $e^{\tan x} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c \Rightarrow e^{\tan x} + c$

76. Question

Mark (✓) against the correct answer in each of the following:

$\int e^{\cos^2 x} \sin 2x dx = ?$

A. $e^{\cos^2 x} + C$

B. $-e^{\cos^2 x} + C$

C. $e^{\sin^2 x} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\cos^2 x = t \Rightarrow 2 \cos x (-\sin x) dx = dt \Rightarrow -\sin 2x dx = dt$

$\Rightarrow -\int e^t dt$

$\Rightarrow -e^t + c \Rightarrow -e^{\cos^2 x} + c$

77. Question

Mark (✓) against the correct answer in each of the following:

$\int x \sin^3 x^2 \cos x^2 dx = ?$

A. $\frac{1}{4} \sin^4 x^2 + C$

B. $\frac{1}{8} \sin^4 x^2 + C$

C. $\frac{1}{2} \sin^4 x^2 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\sin x^2 = t \Rightarrow 2x \cos x^2 dx = dt$

$\Rightarrow \frac{1}{2} \int t^3 dt$

$$\Rightarrow \frac{1}{2} \frac{t^4}{4} + c \Rightarrow \frac{t^4}{8} + c$$

$$\Rightarrow \frac{(\sin x^2)^4}{8} + c$$

78. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}} dx = ?$$

A. $\sin(e^{\sqrt{x}}) + C$

B. $\frac{1}{2} \sin(e^{\sqrt{x}}) + C$

C. $2 \sin(e^{\sqrt{x}}) + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\sin e^{\sqrt{x}} = t \Rightarrow (\cos e^{\sqrt{x}}) \times (e^{\sqrt{x}}) \times (\frac{1}{2\sqrt{x}}) dx = dt$

$$\Rightarrow \int 2 dt$$

$$\Rightarrow 2t + c \Rightarrow 2 \sin e^{\sqrt{x}} + c$$

79. Question

Mark (✓) against the correct answer in each of the following:

$$\int x^2 \sin x^3 dx = ?$$

A. $\cos x^3 + C$

B. $-\cos x^3 + C$

C. $-\frac{1}{3} \cos x^3 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$$\Rightarrow \frac{1}{3} \int \sin t dt$$

$$\Rightarrow -\frac{1}{3} \cos t + c \Rightarrow -\frac{1}{3} \cos x^3 + c$$

80. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(x+1)e^x}{\cos^2(xe^x)} dx = ?$$

- A. $\tan(xe^x) + C$
- B. $-\tan(xe^x) + C$
- C. $\cot(xe^x) + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $xe^x = t \Rightarrow (e^x + xe^x)dx = dt \Rightarrow e^x(1+x)dx = dt$

$$\Rightarrow \int \frac{dt}{\cos^2 t} \Rightarrow \int \sec^2 t dt = \tan t + c$$

$$\Rightarrow \tan(xe^x) + c$$

81. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{x\sqrt{x^4-1}} dx = ?$$

- A. $\sec^{-1} x^2 + C$
- B. $\frac{1}{2} \sec^{-1} x^2 + C$
- C. $\operatorname{cosec}^{-1} x^2 + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x^2 = t \Rightarrow 2x dx = dt$

$$\Rightarrow \int \frac{1}{x\sqrt{t^2-1}} \times \frac{dt}{2x} \Rightarrow \frac{1}{2} \int \frac{1}{t\sqrt{t^2-1}} dt$$

$$\Rightarrow \frac{1}{2} \sec^{-1} t + c \Rightarrow \frac{1}{2} \sec^{-1} x^2 + c$$

82. Question

Mark (✓) against the correct answer in each of the following:

$$\int x\sqrt{x-1} dx = ?$$

- A. $\frac{2}{3}(x-1)^{3/2} + C$
- B. $\frac{2}{5}(x-1)^{5/2} + C$
- C. $\frac{2}{5}(x-1)^{5/2} + \frac{3}{2}(x-1)^{3/2} + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x-1 = t \Rightarrow x = t+1 \Rightarrow dx = dt$

$\Rightarrow \int (t+1) \times \sqrt{t} dt \Rightarrow \int t^{3/2} dt + \int t^{1/2} dt$

$\Rightarrow \frac{t^{5/2}}{5/2} + \frac{t^{3/2}}{3/2} + c \Rightarrow \frac{2t^{5/2}}{5} + \frac{2t^{3/2}}{3} + c$

$\Rightarrow \frac{2(x-1)^{5/2}}{5} + \frac{2(x-1)^{3/2}}{3} + c$

83. Question

Mark (✓) against the correct answer in each of the following:

$\int x\sqrt{x^2-x} dx = ?$

- A. $\frac{1}{3}(x^2-1)^{3/2} + C$
- B. $\frac{2}{3}(x^2-1)^{3/2} + C$
- C. $\frac{1}{\sqrt{x^2-1}} + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

$\Rightarrow \int x\sqrt{x^2-1} dx$

Put $x^2-1 = t \Rightarrow 2x dx = dt$

$\Rightarrow \int \sqrt{t} \frac{dt}{2} \Rightarrow \frac{1}{2} \int t^{1/2} dt$

$\Rightarrow \frac{t^{3/2}}{3/2} + c \Rightarrow \frac{(x^2-1)^{3/2}}{3} + c$

$$\Rightarrow \frac{1}{3}(x^2 - 1)^{\frac{3}{2}} + c$$

84. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1 + \sqrt{x})} = ?$$

- A. $\sqrt{x} - \log|1 + \sqrt{x}| + C$
- B. $\sqrt{x} + \log|1 + \sqrt{x}| + C$
- C. $2\sqrt{x} - 2\log|1 + \sqrt{x}| + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

$$\Rightarrow \int \frac{1}{1+\sqrt{x}} dx$$

Put $x = t^2 \Rightarrow dx = 2t dt$

$$\Rightarrow \int \frac{2t}{1+t} dt \Rightarrow 2 \int \frac{t}{1+t} dt \Rightarrow 2 \int \frac{t+1-1}{1+t} dt \Rightarrow 2 \int dt - 2 \int \frac{1}{1+t} dt$$

$$\Rightarrow 2t - 2 \log(1+t) + c \Rightarrow 2\sqrt{x} - 2 \log(1 + \sqrt{x}) + c$$

85. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{e^x - 1} dx$$

- A. $\frac{3}{2}(e^x - 1)^{\frac{3}{2}} + C$
- B. $\frac{1}{2}(e^x - 1)^{\frac{1}{2}} + C$
- C. $\frac{2}{3}(e^x - 1)^{\frac{3}{2}} + C$
- D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$$\Rightarrow \int \sqrt{e^x - 1} dx$$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$$\Rightarrow \int \sqrt{t} \frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t} dt$$

Put $t = z^2$ $dt = 2z dz$

$$\Rightarrow \int \frac{2z^2}{1+z^2} dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2} dz \Rightarrow 2 \int \frac{1+z^2}{1+z^2} dz - 2 \int \frac{1}{1+z^2} dz$$

$$\Rightarrow 2 \int dz - 2 \int \frac{1}{1+z^2} dz \Rightarrow 2z - 2 \tan^{-1} z + c$$

$$\Rightarrow 2\sqrt{t} - 2 \tan^{-1} \sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2 \tan^{-1} \sqrt{e^x - 1} + c$$

86. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{(\sin x - \cos x)} dx = ?$$

A. $\frac{1}{2}x - \frac{1}{2} \log |\sin x - \cos x| + C$

B. $\frac{1}{2}x + \frac{1}{2} \log |\sin x - \cos x| + C$

C. $\log |\sin x - \cos x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

We can write $\sin x = \frac{1}{2} [(\sin x - \cos x) + (\sin x + \cos x)]$

$$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]}{(\sin x - \cos x)} dx$$

$$\Rightarrow \frac{1}{2} \int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$$

$$\Rightarrow \frac{1}{2} \int dx + \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx \Rightarrow \frac{x}{2} + \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$$

Put $(\sin x - \cos x) = t$ $(\sin x + \cos x) dx = dt$

$$\Rightarrow \frac{x}{2} + \frac{1}{2} \int \frac{1}{t} dt \Rightarrow \frac{x}{2} + \frac{1}{2} \log t + c \Rightarrow \frac{1}{2}x + \frac{1}{2} \log |\sin x - \cos x| + c$$

87. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1 - \tan x)} = ?$$

A. $\frac{1}{2} \log |\sin x - \cos x| + C$

B. $\frac{1}{2}x + \frac{1}{2} \log |\sin x - \cos x| + C$

C. $\frac{1}{2}x - \frac{1}{2}\log |\sin x - \cos x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

$$\Rightarrow \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx \Rightarrow \int \frac{\cos x}{\cos x - \sin x} dx$$

We can write $\cos x = \frac{1}{2}[(\cos x - \sin x) + (\sin x + \cos x)]$

$$\Rightarrow \int \frac{\frac{1}{2}[(\cos x - \sin x) + (\sin x + \cos x)]}{(\cos x - \sin x)} dx$$

$$\Rightarrow \frac{1}{2} \int \frac{(\cos x - \sin x)}{\cos x - \sin x} dx + \frac{1}{2} \int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$$

$$\Rightarrow \frac{1}{2} \int dx + \frac{1}{2} \int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx \Rightarrow \frac{x}{2} + \frac{1}{2} \int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$$

Put $(\cos x - \sin x) = t$ $(\sin x + \cos x) dx = -dt$

$$\Rightarrow \frac{x}{2} - \frac{1}{2} \int \frac{1}{t} dt \Rightarrow \frac{x}{2} - \frac{1}{2} \log t + c \Rightarrow \frac{x}{2} - \frac{1}{2} \log |\cos x - \sin x| + c$$

88. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1 - \cot x)} = ?$$

A. $\log |\sin x - \cos x| + C$

B. $\frac{1}{2} \log |\sin x - \cos x| + C$

C. $\frac{1}{2}x - \frac{1}{2} \log |\sin x - \cos x| + C$

D. $\frac{1}{2}x + \frac{1}{2} \log |\sin x - \cos x| + C$

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

$$\Rightarrow \int \frac{1}{1 - \frac{\cos x}{\sin x}} dx \Rightarrow \int \frac{\sin x}{\sin x - \cos x} dx$$

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]}{(\sin x - \cos x)} dx$$

$$\Rightarrow \frac{1}{2} \int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$$

$$\Rightarrow \frac{1}{2} \int dx + \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx \Rightarrow \frac{x}{2} + \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$$

$$\text{Put } (\sin x - \cos x) = t \quad (\sin x + \cos x) dx = dt$$

$$\Rightarrow \frac{x}{2} + \frac{1}{2} \int \frac{1}{t} dt \Rightarrow \frac{x}{2} + \frac{1}{2} \log t + c \Rightarrow \frac{x}{2} + \frac{1}{2} \log |\sin x - \cos x| + c$$

89. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx = ?$$

- A. $\sin^{-1}(\tan x) + C$
- B. $\cos^{-1}(\sin x) + C$
- C. $\tan^{-1}(\cos x) + C$
- D. $\tan^{-1}(\sin x) + C$

Answer

$$\text{Formula :- } \int x^n dx = \frac{x^{n+1}}{n+1} + c ; \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$$

Therefore ,

$$\text{Put } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{1-t^2}} dt \Rightarrow \sin^{-1} t + c$$

$$\Rightarrow \sin^{-1}(\tan x) + c$$

90. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(x^2 + 1)}{(x^4 + 1)} dx = ?$$

- A. $\frac{1}{\sqrt{2}} \tan^{-1} \left(x - \frac{1}{x} \right) + C$
- B. $\frac{1}{\sqrt{2}} \cot^{-1} \left\{ \left(x - \frac{1}{x} \right) \right\} + C$
- C. $\frac{1}{\sqrt{2}} \tan^{-1} \left\{ \frac{1}{\sqrt{2}} \left(x - \frac{1}{x} \right) \right\} + C$
- D. none of these

Answer

$$\text{Formula :- } \int x^n dx = \frac{x^{n+1}}{n+1} + c ; \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

Therefore ,

$$\Rightarrow \int \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} dx \Rightarrow \int \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2} - 2 + 2} dx \Rightarrow \int \frac{1 + \frac{1}{x^2}}{(x - \frac{1}{x})^2 + 2} dx$$

$$\text{Put } x - \frac{1}{x} = t \Rightarrow (1 + \frac{1}{x^2}) dx = dt$$

$$\Rightarrow \int \frac{1}{t^2+2} dt \Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + c$$

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} [\frac{1}{\sqrt{2}} (x - \frac{1}{x})] + c$$

91. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin^6 x}{\cos^8 x} dx = ?$$

A. $\frac{1}{7} \tan^7 x + C$

B. $\frac{1}{7} \sec^7 x + C$

C. $\log |\cos^6 x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \frac{\sin^6 x}{\cos^6 x \cos^2 x} dx \Rightarrow \int \frac{\tan^6 x}{\cos^2 x} dx \Rightarrow \int \tan^6 x \sec^2 x dx$$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\Rightarrow \int t^6 dt \Rightarrow \frac{t^7}{7} + c$$

$$\Rightarrow \frac{(\tan x)^7}{7} + c$$

92. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sec^5 x \tan x dx = ?$$

A. $\frac{1}{5} \tan^5 x + C$

B. $\frac{1}{5} \sec^5 x + C$

C. $5 \log |\cos x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \sec^4 x \sec x \tan x dx$$

Put $\sec x = t \Rightarrow \sec x \tan x \, dx = dt$

$$\Rightarrow \int t^4 dt \Rightarrow \frac{t^5}{5} + c$$

$$\Rightarrow \frac{(\sec x)^5}{5} + c$$

93. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^5 x \, dx = ?$$

A. $\frac{1}{6} \tan^6 x + C$

B. $\frac{1}{4} \tan^4 x + \frac{1}{2} \tan^2 x + \log |\sec x| + C$

C. $\frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \log |\sec x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \tan^3 x \tan^2 x \, dx \Rightarrow \int \tan^3 x (\sec^2 x - 1) \, dx$$

$$\Rightarrow \int \tan^3 x \sec^2 x \, dx - \int \tan^3 x \, dx \Rightarrow \int \tan^3 x \sec^2 x \, dx - \int \tan^1 x \tan^2 x \, dx$$

$$\Rightarrow \int \tan^3 x \sec^2 x \, dx - \int \tan x (\sec^2 x - 1) \, dx$$

$$\Rightarrow \int \tan^3 x \sec^2 x \, dx - \int \tan x \sec^2 x \, dx + \int \tan x \, dx$$

Put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\Rightarrow \int t^3 dt - \int t^1 dt + \log |\sec x| \Rightarrow \frac{t^4}{4} - \frac{t^2}{2} + \log |\sec x| + c$$

$$\Rightarrow \frac{(\tan x)^4}{4} - \frac{(\tan x)^2}{2} + \log |\sec x| + c$$

94. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin^3 x \cos^3 x \, dx = ?$$

A. $-\frac{1}{4} \cos^4 x + \frac{1}{6} \cos^6 x + C$

B. $\frac{1}{4} \cos^4 x - \frac{1}{6} \cos^6 x + C$

C. $\frac{1}{4} \cos^4 x + \frac{1}{6} \cos^6 x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \cos x (\cos^2 x \sin^3 x) dx \Rightarrow \int \cos x ((1 - \sin^2 x) \sin^3 x) dx$$

$$\Rightarrow \int \cos x (\sin^3 x - \sin^5 x) dx \Rightarrow \int \sin^3 x \cos x dx - \int \sin^5 x \cos x dx$$

$$\text{Put } \sin x = t \Rightarrow \cos x dx = dt$$

$$\Rightarrow \int t^3 dt - \int t^5 dt \Rightarrow \frac{t^4}{4} - \frac{t^6}{6} + c$$

$$\Rightarrow \frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} + c$$

95. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sec^4 x \tan x dx = ?$$

A. $\frac{1}{2} \sec^2 x + \frac{1}{4} \sec^4 x + C$

B. $\frac{1}{2} \tan^2 x + \frac{1}{4} \tan^4 x + C$

C. $\frac{1}{2} \sec x + \log |\sec x + \tan x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \sec^2 x \sec^2 x \tan x dx \Rightarrow \int (1 + \tan^2 x) \sec^2 x \tan x dx$$

$$\Rightarrow \int \sec^2 x \tan x dx + \int \tan^3 x \sec^2 x dx$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow \int t^1 dt + \int t^3 dt \Rightarrow \frac{t^2}{2} + \frac{t^4}{4} + c$$

$$\Rightarrow \frac{(\tan x)^2}{2} + \frac{(\tan x)^4}{4} + c$$

96. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\log \tan x}{\sin x \cos x} dx = ?$$

A. $\log \{ \log (\tan x) \} + C$

B. $\frac{1}{2} (\log \tan x)^2 + C$

C. $\log (\sin x \cos x) + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \sec^2 x \sec^2 x \tan x dx \Rightarrow \int (1 + \tan^2 x) \sec^2 x \tan x dx$$

$$\Rightarrow \int \sec^2 x \tan x dx + \int \tan^3 x \sec^2 x dx$$

Put $\log(\tan x) = t \Rightarrow \frac{1}{\tan x} \sec^2 x dx = dt \Rightarrow \frac{1}{\sin x \cos x} dx = dt$

$$\Rightarrow \int t^1 dt \Rightarrow \frac{t^2}{2} + c$$

$$\Rightarrow \frac{(\log |\tan x|)^2}{2} + c$$

97. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin^3(2x+1) dx = ?$$

A. $\frac{1}{8} \sin^4(2x+1) + C$

B. $\frac{1}{2} \cos(2x+1) + \frac{1}{3} \cos^3(2x+1) + C$

C. $-\frac{1}{2} \cos(2x+1) + \frac{1}{6} \cos^3(2x+1) + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \sin^2(2x+1) \sin(2x+1) dx \Rightarrow \int (1 - \cos^2(2x+1)) \sin(2x+1) dx$$

$$\Rightarrow \int \sin(2x+1) dx - \int \cos^2(2x+1) \sin(2x+1) dx$$

Put $\cos(2x+1) = t \Rightarrow -2 \sin(2x+1) dx = dt$

$$\Rightarrow -\int \frac{dt}{2} - \left(-\frac{1}{2}\right) \int t^2 dt \Rightarrow -\frac{1}{2} \int dt + \frac{1}{2} \int t^2 dt$$

$$\Rightarrow -\frac{1}{2} t + \frac{1}{2} \frac{t^3}{3} + c \Rightarrow -\frac{1}{2} t + \frac{t^3}{6} + c$$

$$\Rightarrow -\frac{1}{2} \cos(2x+1) + \frac{[\cos(2x+1)]^3}{6} + c$$

98. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sqrt{\tan x}}{\sin x + \cos x} dx = ?$$

- A. $2\sqrt{\tan x} + C$
 B. $2\sqrt{\cot x} + C$
 C. $2\sqrt{\sec x} + C$
 D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \frac{\sqrt{\tan x}}{\sin x \times \cos x} dx \Rightarrow \int \frac{\sqrt{\tan x}}{\frac{\tan x}{\sec x} \times \sec x} dx \Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan x}} dx$$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\Rightarrow \int \frac{dt}{\sqrt{t}} \Rightarrow \frac{\sqrt{t}}{\frac{1}{2}} + c \Rightarrow 2\sqrt{t} + c$$

$$\Rightarrow 2\sqrt{\tan x} + c$$

99. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(\cos x + \sin x)}{(1 - \sin 2x)} dx = ?$$

- A. $\log |\sin x - \cos x| + C$
 B. $\frac{1}{(\cos x - \sin x)} + C$
 C. $\log |\cos x + \sin x| + C$
 D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \frac{\cos x + \sin x}{\cos^2 x + \sin^2 x - \sin 2x} dx \Rightarrow \int \frac{\cos x + \sin x}{(\cos x - \sin x)^2} dx$$

Put $\cos x - \sin x = t \Rightarrow (\cos x + \sin x) dx = -dt$

$$\Rightarrow \int \frac{-dt}{t^2} \Rightarrow \frac{1}{t} + c \Rightarrow \frac{1}{\cos x - \sin x} + c$$

100. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{e^x - 1} dx = ?$$

- A. $\frac{2}{3}(e^x - 1)^{3/2} + C$

B. $\frac{1}{2} \cdot \frac{e^x}{\sqrt{e^x - 1}} + C$

C. $2\sqrt{e^x - 1} - 2 \tan^{-1} \sqrt{e^x - 1} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$$\Rightarrow \int \sqrt{e^x - 1} dx$$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$$\Rightarrow \int \sqrt{t} \frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t} dt$$

Put $t = z^2$ $dt = 2z dz$

$$\Rightarrow \int \frac{2z^2}{1+z^2} dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2} dz \Rightarrow 2 \int \frac{1+z^2}{1+z^2} dz - 2 \int \frac{1}{1+z^2} dz$$

$$\Rightarrow 2 \int dz - 2 \int \frac{1}{1+z^2} dz \Rightarrow 2z - 2 \tan^{-1} z + c$$

$$\Rightarrow 2\sqrt{t} - 2 \tan^{-1} \sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2 \tan^{-1} \sqrt{e^x - 1} + c$$

101. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{\sin^3 x \cos x}} = ?$$

A. $2\sqrt{\tan x} + C$

B. $2\sqrt{\cot x} + C$

C. $-2\sqrt{\tan x} + C$

D. $\frac{-2}{\sqrt{\tan x}} + C$

Answer

Let $I = \int \frac{dx}{\sqrt{\sin^3 x \cos x}}$

Now multiplying and dividing by $\cos^2 x$, we get,

$$I = \int \frac{dx}{\sqrt{\sin^3 x \cos x}} \times \frac{1}{\cos^2 x} \times \cos^2 x$$

$$I = \int \frac{(\sec^2 x)}{\sqrt{\frac{\sin^3 x}{\cos^3 x}}} dx$$

$$I = \int \frac{\sec^2 x}{\sqrt{\tan^3 x}} dx$$

Let $\tan x = t$

Differentiating both sides, we get,

$$\sec^2 x \, dx = dt$$

Therefore,

$$I = \int \frac{dt}{t^{3/2}}$$

Integrating, we get,

$$I = \frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$$

$$I = \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$I = -\frac{2}{\sqrt{t}} + C$$

$$I = -\frac{2}{\sqrt{\tan x}} + C$$

Exercise 13B

1. Question

Evaluate the following integrals:

(i) $\int \sin^2 x \, dx$

(ii) $\int \cos^2 x \, dx$

Answer

i) $\int \sin^2 x \, dx$

$$\Rightarrow \int \sin^2 x \, dx$$

Now, we know that $1 - \cos 2x = 2\sin^2 x$

So, applying this identity in the given integral, we get,

$$\int \sin^2 x \, dx = \int \frac{(1 - \cos 2x) \, dx}{2}$$

$$\Rightarrow \frac{1}{2} (\int dx - \int \cos 2x \, dx)$$

$$\Rightarrow \frac{x}{2} - \frac{\sin 2x}{2 \times 2} + c$$

$$\Rightarrow \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$\text{Ans: } \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

ii) $\int \cos^2 x \, dx$

$$\Rightarrow \int \cos^2 x \, dx$$

Now, we know that $1 + \cos 2x = 2\cos^2 x$

So, applying this identity in the given integral, we get,

$$\int \cos^2 x \, dx = \int \frac{(1 + \cos 2x) dx}{2}$$

$$\Rightarrow \frac{1}{2} (\int dx + \int \cos 2x dx)$$

$$\Rightarrow \frac{x}{2} + \frac{\sin 2x}{2 \times 2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$\text{Ans: } \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

2. Question

Evaluate the following integrals:

$$(i) \int \cos^2 (x/2) dx$$

$$(ii) \int \cot^2 (x/2) dx$$

Answer

$$(i) \int \cos^2 (x/2) dx$$

$$\Rightarrow \int \cos^2 \left(\frac{x}{2} \right) dx$$

Now, we know that $1 + \cos x = 2\cos^2 (x/2)$

So, applying this identity in the given integral, we get,

$$\int \cos^2 \left(\frac{x}{2} \right) dx = \int \frac{(1 + \cos x) dx}{2}$$

$$\Rightarrow \frac{1}{2} (\int dx + \int \cos x dx)$$

$$\Rightarrow \frac{x}{2} + \frac{\sin 2x}{2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{\sin 2x}{2} + c$$

$$\text{Ans: } \frac{x}{2} + \frac{\sin 2x}{2} + c$$

$$(ii) \int \cot^2 (x/2) dx$$

$$\Rightarrow \int \cot^2 \left(\frac{x}{2} \right) dx$$

Now, we know that $\operatorname{cosec}^2 x - \cot^2 x = 1$

So, applying this identity in the given integral we get,

$$\Rightarrow \int \cot^2 \left(\frac{x}{2} \right) dx = \int (\operatorname{cosec}^2 \left(\frac{x}{2} \right) - 1) dx$$

$$\Rightarrow \int (\operatorname{cosec}^2 \left(\frac{x}{2} \right) - 1) dx = \int \operatorname{cosec}^2 \left(\frac{x}{2} \right) dx - \int 1 dx$$

$$\Rightarrow \int \operatorname{cosec}^2 \left(\frac{x}{2} \right) dx - \int 1 dx = \frac{-\cot x}{\frac{1}{2}} - x + c$$

$$\Rightarrow -2\cot x - x + c$$

$$\Rightarrow \int \cot^2\left(\frac{x}{2}\right) dx = 2\cot x - x + c$$

Ans: $-2\cot x - x + c$

3. Question

Evaluate the following integrals:

(i) $\int \sin^2 nx \, dx$

(ii) $\int \sin^5 x \, dx$

Answer

i) $\int \sin^2 nx \, dx$

$$\Rightarrow \int \sin^2 nx \, dx$$

Now, we know that $1 - \cos 2nx = 2\sin^2 nx$

So, applying this identity in the given integral, we get,

$$\int \sin^2 nx \, dx = \int \frac{(1 - \cos 2nx) \, dx}{2}$$

$$\Rightarrow \frac{1}{2} (\int dx - \int \cos 2nx \, dx)$$

$$\Rightarrow \frac{x}{2} - \frac{\sin 2nx}{2n \times 2} + c$$

$$\Rightarrow \frac{x}{2} - \frac{\sin 2x}{4n} + c$$

Ans: $\int \sin^2 nx \, dx = \frac{x}{2} - \frac{\sin 2nx}{4n} + c$

(ii) $\int \sin^5 x \, dx$

We know that $1 - \cos^2 x = \sin^2 x$

$$\Rightarrow \int \sin^5 x \, dx = \int (1 - \cos^2 x)^2 \sin x \, dx$$

$\Rightarrow$ Put $\cos x = t$

$\Rightarrow \sin x \, dx = dt$

$$\Rightarrow \int (1 - \cos^2 x)^2 \sin x \, dx = - \int (1 - t^2)^2 dt$$

$$\Rightarrow - \int (1 - t^2)^2 dt = - \int (1 + t^4 - 2t^2) dt$$

$$\Rightarrow - \int dt + \int 2t^2 dt - \int t^4 dt$$

$$\Rightarrow -t + \frac{2t^3}{3} - \frac{t^5}{5} + c$$

Resubstituting the value of $t = \cos x$ we get,

$$\Rightarrow -\cos x + \frac{2\cos^3 x}{3} - \frac{\cos^5 x}{5} + c$$

Ans: $-\cos x + \frac{2\cos^3 x}{3} - \frac{\cos^5 x}{5} + c$

4. Question

Evaluate the following integrals:

$$\int \cos^3(3x+5) dx$$

Answer

Substitute $3x+5=u$

$$\Rightarrow 3dx=du$$

$$\Rightarrow dx=du/3$$

$$\Rightarrow \int \cos^3(3x+5) dx = \frac{1}{3} \int \cos^3(u) du$$

Now We know that $1-\cos^2x=\sin^2x$,

$$\Rightarrow \frac{1}{3} \int \cos^3(u) du = \frac{1}{3} \int (1 - \sin^2(u)) \cos u du$$

$\Rightarrow$ Substitute $\sin u=t$

$$\Rightarrow \cos u du=dt$$

$$\Rightarrow \frac{1}{3} \int (1 - \sin^2(u)) \cos u du = \frac{1}{3} \int (1 - t^2) dt$$

$$\Rightarrow \frac{1}{3} \int dt - \frac{1}{3} \int t^2 dt$$

$$\Rightarrow \frac{t}{3} - \frac{t^3}{3 \times 3} + C$$

$$\Rightarrow \frac{t}{3} - \frac{t^3}{9} + C$$

Resubstituting the value of $t=\sin u$ and $u=3x+5$ we get,

$$\Rightarrow \frac{\sin(3x+5)}{3} - \frac{\sin^3(3x+5)}{9} + C$$

$$\text{Ans: } \frac{\sin(3x+5)}{3} - \frac{\sin^3(3x+5)}{9} + C$$

5. Question

Evaluate the following integrals:

$$\int \sin^7(3-2x) dx$$

Answer

$$\Rightarrow - \int \sin^7(2x-3) dx$$

Substitute $2x-3=u$

$$\Rightarrow 2dx=du$$

$$\Rightarrow dx=du/2$$

$$\Rightarrow - \left(\frac{1}{2} \right) \int \sin^7(u) du$$

$\Rightarrow$ We know that $1-\cos^2x=\sin^2x$

$$\Rightarrow - \left(\frac{1}{2} \right) \int (1 - \cos^2(u))^3 \sin u du$$

$\Rightarrow$ Put $\cos u=t$

$$\Rightarrow -\sin u du=dt$$

$$\Rightarrow \left(\frac{1}{2} \right) \int (1 - t^2)^3 dt$$

$$\Rightarrow \left(\frac{1}{2} \right) \int (1 - t^6 - 3t^2 + 3t^4) dt$$

$$\Rightarrow \left(\frac{1}{2}\right) \left[\int dt - \int t^6 dt - \int 3t^2 dt + \int 3t^4 dt \right]$$

$$\Rightarrow \left(\frac{1}{2}\right) \left[t - \frac{t^7}{7} - \frac{3t^3}{3} + \frac{3t^5}{5} \right] + c$$

$$\Rightarrow \left(\frac{1}{2}\right) \left[t - \frac{t^7}{7} - t^3 + \frac{3t^5}{5} \right] + c$$

Resubstituting the value of $t = \cos u$ and $u = 2x - 3$ we get

$$\Rightarrow \left(\frac{1}{2}\right) \left[\cos(2x - 3) - \frac{\cos^7(2x - 3)}{7} - \cos^3(2x - 3) + \frac{3\cos^5(2x - 3)}{5} \right] + c$$

$$\Rightarrow \frac{\cos(2x - 3)}{2} - \frac{\cos^7(2x - 3)}{14} - \frac{\cos^3(2x - 3)}{2} + \frac{3\cos^5(2x - 3)}{10} + c$$

Now as we know $\cos(-x) = \cos x$

$$\Rightarrow \frac{\cos(2x - 3)}{2} - \frac{\cos^7(2x - 3)}{14} - \frac{\cos^3(2x - 3)}{2} + \frac{3\cos^5(2x - 3)}{10} + c$$

$$= \frac{\cos(3 - 2x)}{2} - \frac{\cos^7(3 - 2x)}{14} - \frac{\cos^3(3 - 2x)}{2} + \frac{3\cos^5(3 - 2x)}{10} + c$$

$$\text{Ans: } \frac{\cos(3 - 2x)}{2} - \frac{\cos^7(3 - 2x)}{14} - \frac{\cos^3(3 - 2x)}{2} + \frac{3\cos^5(3 - 2x)}{10} + c$$

6. Question

Evaluate the following integrals:

$$(i) \int \left(\frac{1 - \cos 2x}{1 + \cos 2x} \right) dx$$

$$(ii) \int \left(\frac{1 + \cos 2x}{1 - \cos 2x} \right) dx$$

Answer

$$(i) \int \left(\frac{1 - \cos 2x}{1 + \cos 2x} \right) dx$$

$$\Rightarrow \int \frac{1 - \cos 2x}{1 + \cos 2x} dx$$

$$1 - \cos 2x = 2\sin^2 x \text{ and } 1 + \cos 2x = 2\cos^2 x$$

$$\Rightarrow \int \frac{1 - \cos 2x}{1 + \cos 2x} dx = \int \frac{2\sin^2 x}{2\cos^2 x} dx$$

$$\Rightarrow \int \tan^2 x dx$$

$$\text{Now } \sec^2 x - 1 = \tan^2 x$$

$$\Rightarrow \int (\sec^2 x - 1) dx$$

$$\Rightarrow \int \sec^2 x dx - \int dx$$

$$\Rightarrow \tan x - x + c$$

$$\text{Ans: } \tan x - x + c$$

$$(ii) \int \left(\frac{1 + \cos 2x}{1 - \cos 2x} \right) dx$$

$$\Rightarrow \int \frac{1 + \cos 2x}{1 - \cos 2x} dx$$

$$1 - \cos 2x = 2\sin^2 x \text{ and } 1 + \cos 2x = 2\cos^2 x$$

$$\Rightarrow \int \frac{1 + \cos 2x}{1 - \cos 2x} dx = \int \frac{2\cos^2 x}{2\sin^2 x} dx$$

$$\Rightarrow \int \cot^2 x dx$$

$$\text{Now } \operatorname{cosec}^2 x - 1 = \cot^2 x$$

$$\Rightarrow \int (\operatorname{cosec}^2 x - 1) dx$$

$$\Rightarrow \int \operatorname{cosec}^2 x dx - \int dx$$

$$\Rightarrow -\cot x - x + c$$

$$\text{Ans: } -\cot x - x + c$$

7. Question

Evaluate the following integrals:

$$(i) \int \frac{1 - \cos x}{1 + \cos x} dx$$

$$(ii) \int \frac{1 + \cos x}{1 - \cos x} dx$$

Answer

$$i) \int \frac{1 - \cos x}{1 + \cos x} dx$$

$$\Rightarrow \int \frac{1 - \cos x}{1 + \cos x} dx$$

$$1 - \cos x = 2\sin^2 x/2 \text{ and } 1 + \cos x = 2\cos^2 x/2$$

$$\Rightarrow \int \frac{1 - \cos x}{1 + \cos x} dx = \int \frac{2\sin^2(\frac{x}{2})}{2\cos^2(\frac{x}{2})} dx$$

$$\Rightarrow \int \tan^2(\frac{x}{2}) dx$$

$$\text{Now } \sec^2(x/2) - 1 = \tan^2(x/2)$$

$$\Rightarrow \int (\sec^2(\frac{x}{2}) - 1) dx$$

$$\Rightarrow \int \sec^2(\frac{x}{2}) dx - \int dx$$

$$\Rightarrow 2\tan(x/2) - x + c$$

$$\text{Ans: } 2\tan(x/2) - x + c$$

$$(ii) \int \frac{1 + \cos x}{1 - \cos x} dx$$

$$\Rightarrow \int \frac{1 + \cos x}{1 - \cos x} dx$$

$$1 - \cos x = 2\sin^2 x/2 \text{ and } 1 + \cos x = 2\cos^2 x/2$$

$$\Rightarrow \int \frac{1 + \cos x}{1 - \cos x} dx = \int \frac{2\cos^2(\frac{x}{2})}{2\sin^2(\frac{x}{2})} dx$$

$$\Rightarrow \int \cot^2(\frac{x}{2}) dx$$

$$\text{Now } \operatorname{cosec}^2(x/2) - 1 = \cot^2(x/2)$$

$$\Rightarrow \int (\operatorname{cosec}^2(\frac{x}{2}) - 1) dx$$

$$\Rightarrow \int \operatorname{cosec}^2(\frac{x}{2}) dx - \int dx$$

$$\Rightarrow -2\cot(x/2) - x + c$$

$$\text{Ans: } \Rightarrow -2\cot(x/2) - x + c$$

8. Question

Evaluate the following integrals:

$$\int \sin 3x \cos 4x \, dx$$

Answer

$$\Rightarrow \int \sin 3x \cos 4x \, dx$$

Applying the formula: $\sin x \times \cos y = \frac{1}{2}(\sin(x+y) - \sin(y-x))$

$$\Rightarrow \frac{1}{2} \int (\sin 7x - \sin x) \, dx$$

$$\Rightarrow \frac{1}{2} \int \sin 7x \, dx - \frac{1}{2} \int \sin x \, dx$$

$$\Rightarrow \frac{-\cos 7x}{14} + \frac{\cos x}{2} + c$$

$$\text{Ans: } \frac{-\cos 7x}{14} + \frac{\cos x}{2} + c$$

9. Question

Evaluate the following integrals:

$$\int \cos 4x \cos 3x \, dx$$

Answer

$$\Rightarrow \int \cos 4x \cos 3x \, dx$$

Applying the formula: $\cos x \times \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$

$$\Rightarrow \frac{1}{2} \int (\cos 7x + \cos x) \, dx$$

$$\Rightarrow \frac{1}{2} \int \cos 7x \, dx + \frac{1}{2} \int \cos x \, dx$$

$$\Rightarrow \frac{\sin 7x}{14} + \frac{\sin x}{2} + c$$

$$\text{Ans: } \frac{\sin 7x}{14} + \frac{\sin x}{2} + c$$

10. Question

Evaluate the following integrals:

$$\int \sin 4x \sin 8x \, dx$$

Answer

$$\Rightarrow \int \sin 4x \sin 8x \, dx$$

Applying the formula: $\sin x \times \sin y = \frac{1}{2}(\cos(y-x) - \cos(y+x))$

$$\Rightarrow \frac{1}{2} \int (\cos 4x - \cos 12x) \, dx$$

$$\Rightarrow \frac{1}{2} \int \cos 4x \, dx - \frac{1}{2} \int \cos 12x \, dx$$

$$\Rightarrow \frac{\sin 4x}{8} - \frac{\sin 12x}{24} + c$$

$$\text{Ans: } \frac{\sin 4x}{8} - \frac{\sin 12x}{24} + c$$

11. Question

Evaluate the following integrals:

$$\int \sin 6x \cos x \, dx$$

Answer

$$\Rightarrow \int \sin 6x \cos x \, dx$$

Applying the formula: $\sin x \times \cos y = \frac{1}{2}(\sin(y+x) - \sin(y-x))$

$$\Rightarrow \frac{1}{2} \int (\sin 7x - \sin(-5x)) \, dx$$

$$\Rightarrow \frac{1}{2} \int \sin 7x \, dx + \frac{1}{2} \int \sin 5x \, dx$$

$$\Rightarrow \frac{-\cos 7x}{14} - \frac{\cos x}{10} + c$$

$$\text{Ans: } \frac{-\cos 7x}{14} - \frac{\cos x}{10} + c$$

12. Question

Evaluate the following integrals:

$$\int \sin x \sqrt{1 + \cos 2x} \, dx$$

Answer

we know that $1 + \cos 2x = 2\cos^2 x$

So, applying this identity in the given integral we get,

$$\Rightarrow \int \sin x \sqrt{1 + \cos 2x} \, dx$$

$$\Rightarrow \int \sin x \sqrt{2\cos^2 x} \, dx$$

$$\Rightarrow \sqrt{2} \int \sin x \cos x \, dx$$

Let $\sin x = t$

$$\Rightarrow \cos x \, dx = dt$$

$$\Rightarrow \sqrt{2} \int t \, dt$$

$$\Rightarrow \sqrt{2} \frac{t^2}{2} + c = \frac{t^2}{\sqrt{2}} + c$$

Resubstituting the value of $t = \sin x$ we get

$$\Rightarrow \frac{\sin^2 x}{\sqrt{2}} + c$$

$$\text{Ans: } \frac{\sin^2 x}{\sqrt{2}} + c$$

13. Question

Evaluate the following integrals:

$$\int \cos^4 x \, dx$$

Answer

$$\Rightarrow \int \cos^2 x \cos^2 x \, dx$$

$$\Rightarrow \int \left(\frac{1+\cos 2x}{2}\right) \left(\frac{1+\cos 2x}{2}\right) dx \dots \left(\frac{1+\cos 2x}{2} = \cos^2 x\right)$$

$$\Rightarrow \frac{1}{4} \int (1 + \cos 2x)^2 dx$$

$$\Rightarrow \frac{1}{4} \int (1 + \cos^2 2x + 2\cos 2x) dx$$

$$\Rightarrow \frac{1}{4} [\int 1 dx + \int \cos^2 2x dx + \int 2\cos 2x dx]$$

$$\Rightarrow \frac{1}{4} \left[x + \int \frac{(1 + \cos 4x) dx}{2} + 2 \frac{\sin 2x}{2} \right] \dots (1 + \cos 4x = 2\cos^2 2x)$$

$$\Rightarrow \frac{1}{4} \left[x + \frac{1}{2} (\int dx + \int \cos 4x dx) + \sin 2x \right] + c$$

$$\Rightarrow \left[\frac{x}{4} + \frac{1}{2} \times \frac{1}{4} (\int dx + \int \cos 4x dx) + \frac{\sin 2x}{4} \right] + c$$

$$\Rightarrow \left[\frac{x}{4} + \left(\frac{x}{8} + \frac{\sin 4x}{32} \right) + \frac{\sin 2x}{4} \right] + c$$

$$\Rightarrow \frac{3x}{8} + \frac{\sin 4x}{32} + \frac{\sin 2x}{4} + c$$

$$\text{Ans: } \frac{3x}{8} + \frac{\sin 4x}{32} + \frac{\sin 2x}{4} + c$$

14. Question

Evaluate the following integrals:

$$\int \cos 2x \cos 4x \cos 6x dx$$

Answer

$$\Rightarrow \int \cos 2x \cos 4x \cos 6x dx$$

$$\Rightarrow \frac{1}{2} \int (\cos 6x + \cos 2x) \cos 6x dx$$

$$\Rightarrow \frac{1}{2} \int \cos^2 6x dx + \frac{1}{2} \int \cos 2x \cos 6x dx$$

$$\Rightarrow \frac{1}{2} \int \cos^2 6x dx + \frac{1}{4} \int (\cos 8x + \cos 4x) dx$$

$$\Rightarrow \frac{1}{2} \int \cos^2 6x dx + \frac{1}{4} \int \cos 8x dx + \frac{1}{4} \int \cos 4x dx$$

$$\Rightarrow \frac{1}{2} \int \frac{(1 + \cos 12x) dx}{2} + \frac{1}{4} \frac{\sin 8x}{8} + \frac{1}{4} \frac{\sin 4x}{4} + c$$

$$\Rightarrow \frac{1}{4} \left(x + \frac{\sin 12x}{12} \right) + \frac{\sin 8x}{32} + \frac{\sin 4x}{16} + c$$

$$\Rightarrow \frac{x}{4} + \frac{\sin 12x}{48} + \frac{\sin 8x}{32} + \frac{\sin 4x}{16} + c$$

$$\text{Ans: } \frac{x}{4} + \frac{\sin 12x}{48} + \frac{\sin 8x}{32} + \frac{\sin 4x}{16} + c$$

15. Question

Evaluate the following integrals:

$$\int \sin^3 x \cos x dx$$

Answer

Let $\sin x = t$

$$\Rightarrow \cos x dx = dt$$

$$\Rightarrow \int \sin^3 x \cos x dx = \int t^3 dt$$

$$\Rightarrow \frac{t^4}{4} + c$$

Resubstituting the value of $t = \sin x$ we get

$$\Rightarrow \frac{\sin^4 x}{4} + c$$

$$\text{Ans: } \frac{\sin^4 x}{4} + c$$

16. Question

Evaluate the following integrals:

$$\int \sec^4 x \, dx$$

Answer

$$\Rightarrow \int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx$$

$$\Rightarrow \int \sec^2 x (1 + \tan^2 x) \, dx$$

$$\Rightarrow \text{Put } \tan x = t \Rightarrow \sec^2 x \, dx = dt$$

$$\Rightarrow \int (1 + t^2) \, dt$$

$$\Rightarrow t + \frac{t^3}{3} + c$$

Resubstituting the value of $t = \tan x$ we get

$$\Rightarrow \tan x + \frac{\tan^3 x}{3} + c$$

$$\text{Ans: } \tan x + \frac{\tan^3 x}{3} + c$$

17. Question

Evaluate the following integrals:

$$\int \cos^3 x \sin^4 x \, dx$$

Answer

$$\Rightarrow \int \cos^3 x \sin^4 x \, dx$$

$$\Rightarrow \int \cos x \sin^4 x \cos^2 x \, dx$$

$$\Rightarrow \int \cos x \sin^4 x (1 - \sin^2 x) \, dx$$

$$\text{Put } \sin x = t$$

$$\Rightarrow \cos x \, dx = dt$$

$$\Rightarrow \int t^4 (1 - t^2) \, dt$$

$$\Rightarrow \int t^4 \, dt - \int t^6 \, dt$$

$$\Rightarrow \frac{t^5}{5} - \frac{t^7}{7} + c$$

Resubstituting the value of $t = \sin x$ we get,

$$\Rightarrow \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$$

$$\text{Ans: } \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$$

18. Question

Evaluate the following integrals:

$$\int \cos^4 x \sin^3 x \, dx$$

Answer

$$\Rightarrow \int \cos^4 x \sin^3 x \, dx$$

$$\Rightarrow \int \sin x \sin^2 x \cos^4 x \, dx$$

$$\Rightarrow \int \sin x \cos^4 x (1 - \cos^2 x) \, dx$$

Put $\cos x = t$

$$\Rightarrow -\sin x \, dx = dt$$

$$\Rightarrow \int t^4 (t^2 - 1) \, dt$$

$$\Rightarrow \int t^6 \, dt - \int t^4 \, dt$$

$$\Rightarrow \frac{t^7}{7} - \frac{t^5}{5} + c$$

Resubstituting the value of $t = \sin x$ we get,

$$\Rightarrow \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$

$$\text{Ans: } \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$

19. Question

Evaluate the following integrals:

$$\int \sin^{2/3} x \cos^3 x \, dx$$

Answer

$$\Rightarrow \int \cos^3 x \sin^{2/3} x \, dx$$

$$\Rightarrow \int \cos x \cos^2 x \sin^{2/3} x \, dx$$

$$\Rightarrow \int \cos x (1 - \sin^2 x) \sin^{2/3} x \, dx$$

Put $\sin x = t$

$$\Rightarrow \cos x \, dx = dt$$

$$\Rightarrow \int t^{2/3} (1 - t^2) \, dt$$

$$\Rightarrow \int t^{2/3} \, dt - \int t^{8/3} \, dt$$

$$\Rightarrow \frac{t^{5/3}}{5/3} - \frac{t^{11/3}}{11/3} + c$$

Resubstituting the value of $t = \sin x$ we get

$$\Rightarrow \frac{3 \sin^{5/3} x}{5} - \frac{3 \sin^{11/3} x}{11} + c$$

$$\text{Ans: } \frac{3 \sin^{5/3} x}{5} - \frac{3 \sin^{11/3} x}{11} + c$$

20. Question

Evaluate the following integrals:

$$\int \cos^{3/5} x \sin^3 x \, dx$$

Answer

$$\Rightarrow \int \sin^3 x \cos^{3/5} x \, dx$$

$$\Rightarrow \int \sin x \sin^2 x \cos^{3/5} x \, dx$$

$$\Rightarrow \int \sin x (1 - \cos^2 x) \cos^{3/5} x \, dx$$

Put $\cos x = t$

$$\Rightarrow -\sin x \, dx = dt$$

$$\Rightarrow \int t^{3/5} (t^2 - 1) \, dt$$

$$\Rightarrow \int t^{13/5} \, dt - \int t^{3/5} \, dt$$

$$\Rightarrow \frac{t^{18/5}}{18/5} - \frac{t^{8/5}}{8/5} + c$$

Resubstituting the value of $t = \cos x$ we get

$$\Rightarrow \frac{5 \cos^{18/5} x}{18} - \frac{5 \cos^{8/5} x}{8} + c$$

$$\text{Ans: } \frac{5 \cos^{18/5} x}{18} - \frac{5 \cos^{8/5} x}{8} + c$$

21. Question

Evaluate the following integrals:

$$\int \operatorname{cosec}^4 2x \, dx$$

Answer

$$\Rightarrow \int \operatorname{cosec}^4 2x \, dx$$

$$\Rightarrow \int \operatorname{cosec}^2 2x \operatorname{cosec}^2 2x \, dx$$

$$\Rightarrow \int \operatorname{cosec}^2 2x (1 + \cot^2 2x) \, dx$$

$$\Rightarrow \cot 2x = t \Rightarrow -2 \operatorname{cosec}^2 2x \, dx = dt$$

$$\Rightarrow -1/2 \int (1 + t^2) \, dt$$

$$\Rightarrow -1/2 \int dt - 1/2 \int t^2 \, dt$$

$$\Rightarrow -\left(\frac{1}{2}\right)t - \frac{t^3}{6} + c$$

Resubstituting the value of $t = \cot x$ we get

$$\Rightarrow -\frac{\cot x}{2} - \frac{\cot^3 x}{6} + c$$

$$\text{Ans: } -\frac{\cot x}{2} - \frac{\cot^3 x}{6} + c$$

22. Question

Evaluate the following integrals:

$$\int \frac{\cos 2x}{\cos x} dx$$

Answer

$$\Rightarrow \int \frac{\cos 2x}{\cos x} dx = \int \frac{2\cos^2 x - 1}{\cos x} dx$$

$$\Rightarrow \int \frac{2\cos^2 x}{\cos x} dx - \int \frac{1}{\cos x} dx$$

$$\Rightarrow \int 2\cos x dx - \int \sec x dx$$

$$\Rightarrow 2\sin x - \log|\sec x + \tan x| + c$$

$$\text{Ans: } 2\sin x - \log|\sec x + \tan x| + c$$

23. Question

Evaluate the following integrals:

$$\int \frac{\cos x}{\cos(x + \alpha)} dx$$

Answer

$$\Rightarrow \int \frac{\cos x}{\cos(x + \alpha)} dx = \int \frac{\cos((x + \alpha) - \alpha)}{\cos(x + \alpha)} dx$$

$$\Rightarrow \int \frac{\cos(x + \alpha)\cos\alpha + \sin(x + \alpha)\sin\alpha}{\cos(x + \alpha)} dx$$

$$\Rightarrow \int \cos\alpha dx + \int \tan(x + \alpha)\sin\alpha dx$$

Now α is a constant

$$\Rightarrow x\cos\alpha - \sin\alpha \log|\cos(x + \alpha)| + c$$

$$\text{Ans: } x\cos\alpha - \sin\alpha \log|\cos(x + \alpha)| + c$$

24. Question

Evaluate the following integrals:

$$\int \cos^3 x \sin 2x dx$$

Answer

$$\Rightarrow \int \sin 2x \cos^3 x dx$$

$$\Rightarrow \int 2\sin x \cos x \cos^3 x dx$$

$$\Rightarrow \int 2\sin x \cos^4 x dx$$

Now put $\cos x = t$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow -2 \int t^4 dt$$

$$\Rightarrow -2 \times \frac{t^5}{5} + c$$

Resubstituting the value of $t = \cos x$ we get,

$$\Rightarrow \frac{-2\cos^5 x}{5} + c$$

$$\text{Ans: } \frac{-2\cos^5 x}{5} + c$$

25. Question

Evaluate the following integrals:

$$\int \frac{\cos^9 x}{\sin x} dx$$

Answer

$$\Rightarrow \int \frac{\cos^9 x}{\sin x} dx$$

$$\Rightarrow \int \frac{\cos^9 x}{\sin^2 x} \sin x dx$$

$$\Rightarrow \int \frac{\cos^9 x}{1 - \cos^2 x} \sin x dx$$

Put $\cos x = t$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow \int \frac{t^9}{t^2 - 1} dt$$

Now put $t^2 - 1 = a$

$$\Rightarrow 2t dt = da$$

And $t^8 = (a+1)^4$

$$\Rightarrow \frac{1}{2} \int \frac{(a+1)^4}{a} da$$

$$\Rightarrow \frac{1}{2} \int (a^3 + 4a^2 + 6a + \frac{1}{a} + 4) da$$

$$\Rightarrow \frac{1}{2} \left(\frac{a^4}{4} + \frac{4a^3}{3} + \frac{6a^2}{2} + \ln a + 4a \right) + c$$

$$\Rightarrow \left(\frac{a^4}{8} + \frac{2a^3}{3} + \frac{3a^2}{2} + \frac{\ln a}{2} + 2a \right) + c$$

Resubstituting the value of $a = t^2 - 1$ and $t = \cos x \Rightarrow a = \cos^2 x - 1 = -\sin^2 x$ we get

$$\Rightarrow \left(\frac{(-\sin^2 x)^4}{8} + \frac{2(-\sin^2 x)^3}{3} + \frac{3(-\sin^2 x)^2}{2} + \frac{\ln |(-\sin^2 x)|}{2} + 2(-\sin^2 x) \right) + c$$

$$\Rightarrow \left(\frac{\sin^8 x}{8} - \frac{2\sin^6 x}{3} + \frac{3\sin^4 x}{2} + \frac{2 \ln |(-\sin x)|}{2} - 2\sin^2 x \right) + c$$

$$\Rightarrow \left(\frac{\sin^8 x}{8} - \frac{2\sin^6 x}{3} + \frac{3\sin^4 x}{2} + \ln(\sin x) - 2\sin^2 x \right) + c$$

$$\text{Ans: } \left(\frac{\sin^8 x}{8} - \frac{2\sin^6 x}{3} + \frac{3\sin^4 x}{2} + \ln(\sin x) - 2\sin^2 x \right) + c$$

26. Question

Evaluate the following integrals:

$$\int \cos^4 2x \, dx$$

Answer

$$\Rightarrow \int \cos^2 2x \cos^2 2x \, dx$$

$$\Rightarrow \int \left(\frac{1+\cos 4x}{2} \right) \left(\frac{1+\cos 4x}{2} \right) dx \dots \left(\frac{1+\cos 4x}{2} = \cos^2 2x \right)$$

$$\Rightarrow \frac{1}{4} \int (1 + \cos 4x)^2 dx$$

$$\Rightarrow \frac{1}{4} \int (1 + \cos^2 4x + 2\cos 4x) dx$$

$$\Rightarrow \frac{1}{4} \left[\int 1 dx + \int \cos^2 4x dx + \int 2\cos 4x dx \right]$$

$$\Rightarrow \frac{1}{4} \left[x + \int \frac{(1+\cos 8x)dx}{2} + 2 \frac{\sin 4x}{4} \right] \dots (1+\cos 8x = 2\cos^2 4x)$$

$$\Rightarrow \frac{1}{4} \left[x + \frac{1}{2} \left(\int dx + \int \cos 8x dx \right) + \left(\frac{\sin 4x}{2} \right) \right] + c$$

$$\Rightarrow \left[\frac{x}{4} + \frac{1}{2} \times \frac{1}{4} \left(\int dx + \int \cos 8x dx \right) + \frac{\sin 4x}{8} \right] + c$$

$$\Rightarrow \left[\frac{x}{4} + \left(\frac{x}{8} + \frac{\sin 8x}{64} \right) + \frac{\sin 4x}{8} \right] + c$$

$$\Rightarrow \frac{3x}{8} + \frac{\sin 8x}{64} + \frac{\sin 4x}{8} + c$$

$$\text{Ans: } \frac{3x}{8} + \frac{\sin 8x}{64} + \frac{\sin 4x}{8} + c$$

27. Question

Evaluate the following integrals:

$$\int \frac{\sin^2 x}{(1 + \cos x)^2} dx$$

Answer

Doing tangent half angle substitution we get,

$$\Rightarrow \int \frac{\sin^2 x}{(1 + \cos^2 x)} dx = \int \frac{\left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)^2}{\left[1 + \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) \right]^2} dx$$

Substitute $u = \tan(x/2)$

$$\Rightarrow 2du = \sec^2(x/2)dx$$

$$\Rightarrow dx = \frac{2du}{u^2+1}$$

$$\Rightarrow 2 \int \frac{u^2}{1+u^2} du$$

$$\Rightarrow 2 \int \frac{1+u^2}{1+u^2} du - 2 \int \frac{1}{1+u^2} du$$

$$\Rightarrow 2 \int du - \tan^{-1} u + c$$

$$\Rightarrow 2u - \tan^{-1} u + c$$

Resubstituting the values we get,

$$\Rightarrow 2 \tan \frac{x}{2} - \tan^{-1} \tan \frac{x}{2} + c$$

$$\Rightarrow 2 \tan \frac{x}{2} - \frac{x}{2} + c$$

$$\text{Ans: } 2 \tan \frac{x}{2} - \frac{x}{2} + c$$

28. Question

Evaluate the following integrals:

$$\int \frac{dx}{(3\cos x + 4\sin x)}$$

Answer

$$\int \frac{dx}{3\cos x + 4\sin x} = \int \frac{dx}{3 \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + 4 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)}$$

$$\Rightarrow \int \frac{\sec^2 \frac{x}{2} dx}{3 + 8 \tan \frac{x}{2} - 3 \tan^2 \frac{x}{2}}$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\therefore \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$\Rightarrow \int \frac{2dt}{3 + 8t - 3t^2} = \frac{2}{3} \int \frac{dt}{1 + \frac{8}{3}t - t^2} = \frac{2}{3} \int \frac{dt}{1 - \left(t - \frac{4}{3}\right)^2 + \frac{16}{9}}$$

$$\Rightarrow \frac{2}{3} \int \frac{dt}{\frac{25}{9} - \left(t - \frac{4}{3}\right)^2} = \frac{2}{3} \int \frac{dt}{\left(\frac{5}{3}\right)^2 - \left(t - \frac{4}{3}\right)^2}$$

$$\Rightarrow \frac{2}{3} \times \frac{1}{2 \times \frac{5}{3}} \ln \left| \frac{\frac{5}{3} + \left(t - \frac{4}{3}\right)}{\frac{5}{3} - \left(t - \frac{4}{3}\right)} \right| + c = \frac{1}{5} \ln \left| \frac{1 + 3t}{9 - 3t} \right| + c$$

Resubstituting the value of t we get

$$\Rightarrow \frac{1}{5} \ln \left| \frac{1 + 3 \tan \frac{x}{2}}{9 - 3 \tan \frac{x}{2}} \right| + c$$

Ans: $\frac{1}{5} \ln \left| \frac{1+3\tan\frac{x}{2}}{9-3\tan\frac{x}{2}} \right| + c$

29. Question

Evaluate the following integrals:

$$\int \frac{dx}{(a \cos x + b \sin x)^2}, a > 0 \text{ and } b > 0$$

Answer

$$\int \frac{dx}{(a \cos x + b \sin x)^2}$$

Taking $b \cos x$ common from the denominator we get,

$$\int \frac{dx}{b^2 \cos^2 x \left(\frac{a}{b} + \tan x \right)^2}$$

$$\Rightarrow \frac{1}{b^2} \int \frac{\sec^2 x dx}{\left(\frac{a}{b} + \tan x \right)^2}$$

Let $(a/b) + \tan x = t$

$$\therefore \sec^2 x dx = dt$$

$$\Rightarrow \frac{1}{b^2} \int \frac{dt}{t^2} = \frac{-1}{b^2} \times \frac{1}{t} = \frac{-1}{b^2 t} + c$$

Resubstituting the value of $t = (a/b) + \tan x$ we get

$$\Rightarrow \frac{-1}{b^2 \left(\frac{a}{b} + \tan x \right)} + c = \frac{-1}{ab + b^2 \tan x} + c$$

Ans: $\frac{-1}{ab + b^2 \tan x} + c$

30. Question

Evaluate the following integrals:

$$\int \frac{dx}{(\cos x - \sin x)}$$

Answer

$$\int \frac{dx}{\cos x - \sin x} = \int \frac{dx}{\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) - \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)}$$

$$\Rightarrow \int \frac{\sec^2 \frac{x}{2} dx}{1 - 2 \tan \frac{x}{2} - \tan^2 \frac{x}{2}}$$

Let $\tan \frac{x}{2} = t$

$$\therefore \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$\Rightarrow \int \frac{2dt}{1-2t-t^2} = -2 \int \frac{dt}{t^2+2t-1} = -2 \int \frac{dt}{(t+1)^2-2}$$

$$= -2 \int \frac{dt}{(t+1)^2 - (\sqrt{2})^2}$$

$$\Rightarrow -2 \times \frac{1}{2 \times \sqrt{2}} \ln \left| \frac{t+1-\sqrt{2}}{t+1+\sqrt{2}} \right| + c \text{ resubstituting the value of } t \text{ we get}$$

$$\Rightarrow \frac{-1}{\sqrt{2}} \ln \left| \frac{\tan \frac{x}{2} + 1 - \sqrt{2}}{\tan \frac{x}{2} + 1 + \sqrt{2}} \right| + c = \frac{-1}{\sqrt{2}} \ln \left| \tan \left(\frac{\pi}{8} - \frac{x}{2} \right) \right| + c$$

$$\text{Ans: } \frac{-1}{\sqrt{2}} \ln \left| \tan \left(\frac{\pi}{8} - \frac{x}{2} \right) \right| + c$$

31. Question

Evaluate the following integrals:

$$\int (2 \tan x - 3 \cot x)^2 dx$$

Answer

$$\int (2 \tan x - 3 \cot x)^2 dx$$

$$\Rightarrow \int (4 \tan^2 x + 9 \cot^2 x - 12 \tan x \cot x) dx$$

$$\Rightarrow \int (4(\sec^2 x - 1) + 9(\operatorname{cosec}^2 x - 1) - 12) dx$$

$$\Rightarrow \int 4 \sec^2 x dx + \int 9 \operatorname{cosec}^2 x dx - \int 25 dx$$

$$\Rightarrow 4 \tan x - 9 \cot x - 25x + c$$

$$\text{Ans: } 4 \tan x - 9 \cot x - 25x + c$$

32. Question

Evaluate the following integrals:

$$\int \sin x \sin 2x \sin 3x dx$$

Answer

$$\Rightarrow \int \sin x \sin 2x \sin 3x dx$$

Applying the formula: $\sin x \times \sin y = \frac{1}{2}(\cos(y-x) - \cos(y+x))$

$$\Rightarrow \frac{1}{2} \int (\cos 2x - \cos 4x) \sin 2x dx$$

$$\Rightarrow \frac{1}{2} \int \sin 2x \cos 2x dx - \frac{1}{2} \int \sin 2x \cos 4x dx$$

$$\Rightarrow \frac{1}{4} \int \sin 4x dx - \frac{1}{4} \int (\sin 6x - \sin 2x) dx$$

$$\Rightarrow \frac{-\cos 4x}{16} + \frac{\cos 6x}{24} - \frac{\cos 2x}{8} + c$$

$$\text{Ans: } \frac{-\cos 4x}{16} + \frac{\cos 6x}{24} - \frac{\cos 2x}{8} + c$$

33. Question

Evaluate the following integrals:

$$\int \left(\frac{1 - \cot x}{1 + \cot x} \right) dx$$

Answer

$$\begin{aligned} \Rightarrow \int \frac{1 - \cot x}{1 + \cot x} dx &= \int \frac{1 - \frac{\cos x}{\sin x}}{1 + \frac{\cos x}{\sin x}} dx \\ \Rightarrow \int \frac{\sin x - \cos x}{\sin x + \cos x} dx &= - \int \frac{\cos x - \sin x}{\sin x + \cos x} dx \\ \Rightarrow - \int \frac{d(\sin x + \cos x)}{\sin x + \cos x} \\ \Rightarrow -\log|\sin x + \cos x| + c \end{aligned}$$

Ans: $-\log(\sin x + \cos x) + c$

34. Question

Evaluate the following integrals:

$$\int \frac{dx}{(2 \sin x + \cos x + 3)}$$

Answer

$$\begin{aligned} \int \frac{dx}{\cos x + 2 \sin x + 3} &= \int \frac{dx}{\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + 2 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + 3} \\ \Rightarrow \int \frac{\sec^2 \frac{x}{2} dx}{3 + 1 + 3 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} - \tan^2 \frac{x}{2}} \end{aligned}$$

Let $\tan \frac{x}{2} = t$

$$\therefore \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$\Rightarrow \int \frac{2dt}{4 + 4t + 2t^2} = \int \frac{dt}{2 + 2t + t^2} = \frac{2}{3} \int \frac{dt}{(t+1)^2 + 2 - 1}$$

$$\Rightarrow \int \frac{dt}{(t+1)^2 + 1} = \int \frac{dt}{(1)^2 + (t+1)^2}$$

$$\Rightarrow \tan^{-1}(t+1) + c$$

Resubstituting the value of t we get

$$\Rightarrow \tan^{-1}\left(\tan \frac{x}{2} + 1\right) + c$$

Ans: $\tan^{-1}\left(\tan \frac{x}{2} + 1\right) + c$

Exercise 13C

1. Question

Evaluate the following integrals:

$$\int x e^x dx$$

Answer

Using BY PART METHOD.

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\int x.e^x dx = x \int e^x - \int \frac{dx}{dx} \cdot \int e^x dx$$

$$= x e^x - \int 1.e^x dx$$

$$= x e^x - e^x + c$$

$$= e^x (x - 1) + c$$

2. Question

Evaluate the following integrals:

$$\int x \cos x dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and $\cos x$ is the second function.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\Rightarrow \int x \cos x dx = x \int \cos x - \int \left[\frac{dx}{dx} \cdot \int \cos x dx \right] dx$$

$$= x \sin x - \int 1.\sin x dx$$

$$= x \sin x + \cos x + c$$

3. Question

Evaluate the following integrals:

$$\int x e^{2x} dx$$

Answer

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^{2x} is the second function.

Using Integration by part

$$\int a.b.dx = a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx$$

$$\Rightarrow \int x e^{2x} dx = x \int e^{2x} dx - \int \left[\frac{dx}{dx} \cdot \int e^{2x} dx \right] dx$$

$$= x \frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} dx$$

$$= x \frac{e^{2x}}{2} - \frac{e^{2x}}{2 \times 2} + c$$

$$= x \frac{e^{2x}}{2} - \frac{e^{2x}}{4} + c$$

4. Question

Evaluate the following integrals:

$$\int x \sin 3x dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Sin 3x is the second function.

Using Integration by part

$$\int a.b.dx = a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx$$

$$\Rightarrow \int x \sin 3x dx = x \int \sin 3x dx - \int \left[\frac{dx}{dx} \cdot \int \sin 3x dx \right] dx$$

$$= x \left(\frac{-\cos 3x}{3} \right) - \int 1 \cdot \left(\frac{-\cos 3x}{3} \right) dx$$

$$= x \left(\frac{-\cos 3x}{3} \right) + \left(\frac{\sin 3x}{3 \times 3} \right) + c$$

$$= x \left(\frac{-\cos 3x}{3} \right) + \left(\frac{\sin 3x}{9} \right) + c$$

5. Question

Evaluate the following integrals:

$$\int x \cos 2x dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx$$

$$\begin{aligned}
\Rightarrow \int x \cos 2x dx &= x \int \cos 2x dx - \int \left[\frac{dx}{dx} \cdot \int \cos 2x dx \right] dx \\
&= x \left(\frac{\sin 2x}{2} \right) - \int 1 \cdot \left(\frac{\sin 2x}{2} \right) dx \\
&= x \left(\frac{\sin 2x}{2} \right) + \left(\frac{\cos 2x}{2 \times 2} \right) + c \\
&= x \left(\frac{\sin 2x}{2} \right) + \left(\frac{\cos 2x}{4} \right) + c
\end{aligned}$$

6. Question

Evaluate the following integrals:

$$\int x \log 2x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log 2x$ is the first function, and x is the second function.

Using Integration by part

$$\begin{aligned}
\int a.b.dx &= a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx \\
\Rightarrow \int x \log 2x dx &= \log 2x \int x dx - \int \left[\frac{d \log 2x}{dx} \cdot \int x dx \right] dx \\
&= \log 2x \cdot \frac{x^2}{2} - \int \left[\frac{1 \times 2}{2x} \cdot \frac{x^2}{2} \right] dx \\
&= \frac{x^2}{2} \log 2x - \int \frac{x}{2} dx \\
&= \frac{x^2}{2} \log 2x - \frac{x^2}{2 \times 2} + c \\
&= \frac{x^2}{2} \log 2x - \frac{x^2}{4} + c
\end{aligned}$$

7. Question

Evaluate the following integrals:

$$\int x \operatorname{cosec}^2 x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and $\operatorname{cosec}^2 x$ is the second function.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\begin{aligned}
\Rightarrow \int x \operatorname{cosec}^2 x dx &= x \int \operatorname{cosec}^2 x - \int \left[\frac{dx}{dx} \cdot \int \operatorname{cosec}^2 x dx \right] dx \\
&= x(-\cot x) - \int 1.(-\cot x) dx \\
&= -x \cot x + \int \cot x dx \\
&= -x \cot x + \ln |\sin x| + c
\end{aligned}$$

8. Question

Evaluate the following integrals:

$$\int x^2 \cos x dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and $\cos x$ is the second function.

Using Integration by part

$$\begin{aligned}
\int a.b.dx &= a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx \\
\Rightarrow \int x^2 \cos x dx &= x^2 \int \cos x dx - \int \left[\frac{dx^2}{dx} \cdot \int \cos x dx \right] dx \\
&= x^2 \sin x - \int [2x \times \sin x] dx \\
&= x^2 \sin x - 2 \left[\int x \sin x dx \right]
\end{aligned}$$

Again applying by the part method in the second half, we get

$$\begin{aligned}
&x^2 \sin x - 2 \int x \sin x dx \\
&= x^2 \sin x - 2 \left[x \int \sin x dx - \int \left(\frac{dx}{dx} \cdot \int \sin x dx \right) dx \right] \\
&= x^2 \sin x - 2 \left[x(-\cos x) - \int 1.(-\cos x) dx \right] \\
&= x^2 \sin x - 2[-x \cos x + \sin x] + c \\
&= x^2 \sin x + 2x \cos x - 2 \sin x + c
\end{aligned}$$

9. Question

Evaluate the following integrals:

$$\int x \sin^2 x dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

Writing $\sin^2 x = \frac{1 + \cos 2x}{2}$

We have

$$\begin{aligned}\int x \sin^2 x dx &= \int x \left(\frac{1 + \cos 2x}{2} \right) dx \\&= \int \left(\frac{x}{2} + \frac{x \cos 2x}{2} \right) dx \\&= \int \frac{x}{2} dx + \int \frac{x \cos 2x}{2} dx \\&= \frac{x^2}{2 \times 2} + \frac{1}{2} \int x \cos 2x dx\end{aligned}$$

Taking x as first function and $\cos 2x$ as the second function.

$$\begin{aligned}&= \frac{x^2}{4} + \frac{1}{2} \left\{ x \int \cos 2x dx - \int \left(\frac{dx}{dx} \cdot \int \cos 2x dx \right) dx \right\} \\&= \frac{x^2}{4} + \frac{1}{2} \left\{ x \cdot \frac{\sin 2x}{2} - \int \left(1 \cdot \frac{\sin 2x}{2} \right) dx \right\} \\&= \frac{x^2}{4} + \frac{1}{2} \left\{ \frac{x \sin 2x}{2} - \left(\frac{-\cos 2x}{2 \times 2} \right) \right\} + c \\&= \frac{x^2}{4} + \frac{1}{2} \left\{ \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} \right\} + c \\&= \frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + c\end{aligned}$$

10. Question

Evaluate the following integrals:

$$\int x \tan^2 x dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

Writing $\tan^2 x = \sec^2 x - 1$

We have

$$\begin{aligned}\int x \tan^2 x dx &= \int x (\sec^2 x - 1) dx \\&= \int x \sec^2 x dx - \int x dx\end{aligned}$$

Using x as the first function and $\sec^2 x$ as the second function

$$\begin{aligned}
& \int x \sec^2 x dx - \int x dx \\
&= \left\{ x \int \sec^2 x dx - \int \left(\frac{dx}{dx} \cdot \int \sec^2 x dx \right) dx \right\} - \frac{x^2}{2} \\
&= \left\{ x \cdot \tan x - \int 1 \cdot \tan x dx \right\} - \frac{x^2}{2} \\
&= x \tan x - \ln |\sec x| - \frac{x^2}{2} + c \\
&= x \tan x - \ln \left| \frac{1}{\cos x} \right| - \frac{x^2}{2} + c \\
&= x \tan x + \ln |\cos x| - \frac{x^2}{2} + c
\end{aligned}$$

11. Question

Evaluate the following integrals:

$$\int x^2 e^x dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^x is the second function.

Using Integration by part

$$\begin{aligned}
\int a \cdot b \cdot dx &= a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx \\
\int x^2 e^x dx &= \left[x^2 \int e^x dx - \int \left(\frac{dx^2}{dx} \cdot \int e^x dx \right) dx \right] \\
&= x^2 e^x - \int 2x \cdot e^x dx \\
&= x^2 e^x - 2 \int x e^x dx \\
&= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\frac{dx}{dx} \cdot \int e^x dx \right) dx \right] \\
&= x^2 e^x - 2 \left[x e^x - \int 1 \cdot e^x dx \right] \\
&= x^2 e^x - 2 \left[x e^x - e^x \right] + c \\
&= x^2 e^x - 2x e^x + 2e^x + c \\
&= e^x (x^2 - 2x + 2) + c
\end{aligned}$$

12. Question

Evaluate the following integrals:

$$\int x^2 \cos^3 x dx$$

Answer

We know that $\cos 3x = 4\cos^3 x - 3\cos x$

$$\cos^3 x = \frac{\cos 3x + 3\cos x}{4}$$

$$\begin{aligned}\int x^2 \cos^3 x dx &= \int x^2 \left(\frac{\cos 3x + 3\cos x}{4} \right) dx \\ &= \frac{1}{4} \left(\int x^2 \cos 3x dx + 3 \int x^2 \cos x dx \right)\end{aligned}$$

Taking x^2 as the first function and $\cos 3x$ and $\cos x$ as the second function and applying By part method.

$$\begin{aligned}& \frac{1}{4} \left(\int x^2 \cos 3x dx + 3 \int x^2 \cos x dx \right) \\ &= \frac{1}{4} \left\{ \left(x^2 \int \cos 3x dx - \int \left[\frac{dx^2}{dx} \cdot \int \cos 3x dx \right] dx \right) + 3 \left(x^2 \int \cos x dx - \int \left[\frac{dx^2}{dx} \cdot \int \cos x dx \right] dx \right) \right\} \\ &= \frac{1}{4} \left\{ \left(\frac{x^2 \cdot \sin 3x}{3} - \int 2x \cdot \frac{\sin 3x}{3} dx \right) + 3 \left(x^2 \sin x - \int 2x \cdot \sin x dx \right) \right\} \\ &= \frac{1}{4} \left\{ \left(\frac{x^2 \sin 3x}{3} - \frac{2}{3} \int x \sin 3x dx \right) + 3 \left(x^2 \sin x - 2 \int x \sin x dx \right) \right\} \\ &= \frac{1}{4} \left\{ \left(\frac{x^2 \sin 3x}{3} - \frac{2}{3} \left[x \int \sin 3x dx - \int \left(\frac{dx}{dx} \cdot \int \sin 3x dx \right) dx \right] \right) + 3 \left(x^2 \sin x - 2 \left[x \int \sin x dx - \int \left(\frac{dx}{dx} \cdot \int \sin x dx \right) dx \right] \right) \right\} \\ &= \frac{1}{4} \left\{ \left(\frac{x^2 \sin 3x}{3} - \frac{2}{3} \left[x \frac{-\cos 3x}{3} - \int 1 \cdot \frac{-\cos 3x}{3} dx \right] \right) + 3 \left(x^2 \sin x - 2 \left[-x \cos x - \int -\cos x dx \right] \right) \right\} \\ &= \frac{1}{4} \left\{ \left(\frac{x^2 \sin 3x}{3} - \frac{2}{3} \left[\frac{-x \cos 3x}{3} + \frac{\sin 3x}{9} \right] \right) + 3 \left(x^2 \sin x + 2x \cos x - 2 \sin x \right) \right\} + c \\ &= \frac{1}{4} \left\{ \frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + 3x^2 \sin x + 6x \cos x - 6 \sin x \right\} + c \\ &= \frac{x^2 \sin 3x}{12} + \frac{x \cos 3x}{18} - \frac{\sin 3x}{54} + \frac{3x^2 \sin x}{4} + \frac{3x \cos x}{2} - \frac{3}{2} \sin x + c\end{aligned}$$

13. Question

Evaluate the following integrals:

$$\int x^2 e^{3x} dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^{3x} is the second function.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\begin{aligned}
\int x^2 e^{3x} dx &= x^2 \int e^{3x} dx - \int \left(\frac{dx^2}{dx} \cdot \int e^{3x} dx \right) dx \\
&= x^2 \frac{e^{3x}}{3} - \int 2x \cdot \frac{e^{3x}}{3} dx \\
&= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \int x e^{3x} dx \\
&= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \left(x \int e^{3x} dx - \int \left[\frac{dx}{dx} \cdot \int e^{3x} dx \right] dx \right) \\
&= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \left(x \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} dx \right) \\
&= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \left(x \frac{e^{3x}}{3} - \frac{e^{3x}}{9} \right) + c \\
&= x^2 \frac{e^{3x}}{3} - \frac{2x e^{3x}}{9} + \frac{2e^{3x}}{27} + c \\
&= e^{3x} \left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27} \right) + c
\end{aligned}$$

14. Question

Evaluate the following integrals:

$$\int x^2 \sin^2 x dx$$

Answer

$$\text{We can write } \sin^2 x = \frac{1 - \cos 2x}{2}$$

We have

$$\begin{aligned}
\int x^2 \left(\frac{1 - \cos 2x}{2} \right) dx &= \int \frac{x^2}{2} - \frac{x^2 \cos 2x}{2} dx \\
&= \int \frac{x^2}{2} dx - \int \frac{x^2 \cos 2x}{2} dx
\end{aligned}$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and $\cos 2x$ is the second function.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\begin{aligned}
&= \frac{x^3}{3 \times 2} - \frac{1}{2} \int x^2 \cos 2x dx \\
&= \frac{x^3}{6} - \frac{1}{2} \left(x^2 \int \cos 2x dx - \int \left[\frac{dx^2}{dx} \cdot \int \cos 2x dx \right] dx \right) \\
&= \frac{x^3}{6} - \frac{1}{2} \left(x^2 \cdot \frac{\sin 2x}{2} - \int 2x \cdot \frac{\sin 2x}{2} dx \right) \\
&= \frac{x^3}{6} - \frac{1}{2} \left(x^2 \cdot \frac{\sin 2x}{2} - \int x \cdot \sin 2x dx \right) \\
&= \frac{x^3}{6} - \frac{1}{2} \left(x^2 \cdot \frac{\sin 2x}{2} - \left[x \int \sin 2x dx - \int \left(\frac{dx}{dx} \cdot \int \sin 2x dx \right) dx \right] \right) \\
&= \frac{x^3}{6} - \frac{1}{2} \left(x^2 \cdot \frac{\sin 2x}{2} - \left[x \cdot \frac{-\cos 2x}{2} - \int 1 \cdot \frac{-\cos 2x}{2} dx \right] \right) \\
&= \frac{x^3}{6} - \frac{1}{2} \left(x^2 \cdot \frac{\sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4} \right) + c \\
&= \frac{x^3}{6} - \frac{x^2 \sin 2x}{4} - \frac{x \cos 2x}{4} + \frac{\sin 2x}{8} + c
\end{aligned}$$

15. Question

Evaluate the following integrals:

$$\int x^3 \log 2x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log 2x$ is the first function, and x^3 is the second function.

Using Integration by part

$$\begin{aligned}
\int a \cdot b \cdot dx &= a \int b \, dx - \int \left[\frac{da}{dx} \cdot \int b \, dx \right] dx \\
\int x^3 \log 2x \, dx &= \log 2x \int x^3 \, dx - \int \left(\frac{d \log 2x}{dx} \cdot \int x^3 \, dx \right) dx \\
&= \log 2x \cdot \frac{x^4}{4} - \int \frac{1}{2x} \cdot \frac{x^4}{4} \, dx \\
&= \log 2x \cdot \frac{x^4}{4} - \frac{1}{4} \int x^3 \, dx \\
&= \log 2x \cdot \frac{x^4}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + c \\
&= \log 2x \cdot \frac{x^4}{4} - \frac{x^4}{16} + c
\end{aligned}$$

16. Question

Evaluate the following integrals:

$$\int x \cdot \log(x+1) dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(x+1)$ is first function and x is second function.

$$\int a \cdot b \cdot dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\int x \log(x+1) = \log(x+1) \int x dx - \int \left(\frac{d \log(x+1)}{dx} \cdot \int x dx \right) dx$$

$$= \log(x+1) \frac{x^2}{2} - \int \frac{1}{x+1} \times \frac{x^2}{2} dx$$

$$= \log(x+1) \frac{x^2}{2} - \frac{1}{2} \int \frac{x^2 - 1 + 1}{x+1} dx$$

Adding and subtracting 1 in the numerator,

$$= \log(x+1) \frac{x^2}{2} - \frac{1}{2} \left[\int \frac{x^2 - 1}{x+1} + \frac{1}{x+1} dx \right]$$

$$= \log(x+1) \frac{x^2}{2} - \frac{1}{2} \left[\int \frac{(x+1)(x-1)}{x+1} + \frac{1}{x+1} dx \right]$$

$$= \log(x+1) \frac{x^2}{2} - \frac{1}{2} \left[\int (x-1) + \frac{1}{x+1} dx \right]$$

$$= \log(x+1) \frac{x^2}{2} - \frac{1}{2} \left[\frac{x^2}{2} - x + \log(x+1) \right] + c$$

$$= \log(x+1) \frac{x^2}{2} - \frac{x^2}{4} + \frac{x}{2} - \frac{\log(x+1)}{2} + c$$

$$= \log(x+1) \frac{x^2 - 1}{2} - \frac{x^2}{4} + \frac{x}{2} + c$$

17. Question

Evaluate the following integrals:

$$\int \frac{\log x}{x^n} dx$$

Answer

We can write it as $\int x^{-n} \cdot \log x dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log x$ is the first function, and x^{-n} is the second function.

$$\int a \cdot b \cdot dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\begin{aligned}
\Rightarrow \int x^{-n} \log x dx &= \log x \int x^{-n} dx - \int \left(\frac{d \log x}{dx} \cdot \int x^{-n} dx \right) dx \\
&= \log x \left(\frac{x^{-n+1}}{-n+1} \right) - \int \frac{1}{x} \cdot \frac{x^{-n+1}}{-n+1} dx \\
&= \frac{x^{-n+1} \log x}{1-n} + \frac{1}{1-n} \int \frac{x^{-n} \cdot x}{x} dx \\
&= \frac{x^{-n+1} \log x}{1-n} + \frac{1}{1-n} \times \frac{x^{-n+1}}{-n+1} + c \\
&= \frac{x^{-n+1} \log x}{1-n} - \frac{x^{-n+1}}{(1-n)^2} + c
\end{aligned}$$

18. Question

Evaluate the following integrals:

$$\int 2x^3 e^{x^2} dx$$

Answer

We can write it as $\int 2 \cdot x \cdot x^2 \cdot e^{x^2} dx$

Let $x^2 = t$

$2x dx = dt$

Using the relation in the above condition, we get

$$\int 2x \cdot x^2 \cdot e^{x^2} dx = \int t \cdot e^t dt$$

Integrating with respect to t

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function, and e^t is the second function.

$$\int a \cdot b \cdot dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\int t e^t dt = t \int e^t dt - \int \left(\frac{dt}{dt} \cdot \int e^t dt \right) dt$$

$$= t e^t - \int 1 \cdot e^t dt$$

$$= t e^t - e^t + c$$

Replacing t with x^2 , we get

$$x^2 e^{x^2} - e^{x^2} + c$$

$$= e^{x^2} (x^2 - 1) + c$$

19. Question

Evaluate the following integrals:

$$\int x \sin^3 x \, dx$$

Answer

We know that $\sin 3x = 3\sin x - 4\sin^3 x$

$$\sin^3 x = (3\sin x - \sin 3x)/4$$

$$\begin{aligned}\int x \sin^3 x \, dx &= \int x \left(\frac{3\sin x - \sin 3x}{4} \right) dx \\ &= \frac{1}{4} \int 3x \sin x - x \sin 3x \, dx \\ &= \frac{3}{4} \int x \sin x \, dx - \frac{1}{4} \int x \sin 3x \, dx\end{aligned}$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and $\sin x$ and $\sin 3x$ as the second function.

$$\begin{aligned}\int a.b \, dx &= a \int b \, dx - \int \left[\frac{da}{dx} \cdot \int b \, dx \right] dx \\ &= \frac{3}{4} \int x \sin x \, dx - \frac{1}{4} \int x \sin 3x \, dx \\ &= \frac{3}{4} \left(x \int \sin x \, dx - \int \left[\frac{dx}{dx} \cdot \int \sin x \, dx \right] dx \right) - \frac{1}{4} \left(x \int \sin 3x \, dx - \int \left[\frac{dx}{dx} \cdot \int \sin 3x \, dx \right] dx \right) \\ &= \frac{3}{4} \left(-x \cos x + \int \cos x \, dx \right) - \frac{1}{4} \left(\frac{-x \cos 3x}{3} + \int \frac{\cos 3x}{3} \, dx \right) \\ &= \frac{3}{4} (-x \cos x + \sin x) - \frac{1}{4} \left(\frac{-x \cos 3x}{3} + \frac{\sin 3x}{9} \right) + c \\ &= \frac{-3x \cos x}{4} + \frac{3 \sin x}{4} + \frac{x \cos 3x}{12} - \frac{\sin 3x}{36} + c\end{aligned}$$

20. Question

Evaluate the following integrals:

$$\int x \cos^3 x \, dx$$

Answer

We can write $\cos^3 x = (\cos 3x + 3\cos x)/4$, we have

$$\begin{aligned}\int x \cos^3 x \, dx &= \int x \left(\frac{\cos 3x + 3\cos x}{4} \right) dx \\ &= \frac{1}{4} \int x \cos 3x \, dx + \frac{3}{4} \int x \cos x \, dx\end{aligned}$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and $\cos x$ and $\cos 3x$ as the second function.

$$\begin{aligned}
 \int a.b.dx &= a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx \\
 &= \frac{1}{4} \left(x \int \cos 3x dx - \int \left[\frac{dx}{dx} \cdot \int \cos 3x dx \right] dx \right) + \frac{3}{4} \left(x \int \cos x dx - \int \left[\frac{dx}{dx} \cdot \int \cos x dx \right] dx \right) \\
 &= \frac{1}{4} \left(x \frac{\sin 3x}{3} - \int \frac{\sin 3x}{3} dx \right) + \frac{3}{4} \left(x \sin x - \int \sin x dx \right) \\
 &= \frac{1}{4} \left(\frac{x \sin 3x}{3} + \frac{\cos 3x}{9} \right) + \frac{3}{4} (x \sin x + \cos x) + c \\
 &= \frac{x \sin 3x}{12} + \frac{\cos 3x}{36} + \frac{3x \sin x}{4} + \frac{3 \cos x}{4} + c
 \end{aligned}$$

21. Question

Evaluate the following integrals:

$$\int x^3 \cos x^2 dx$$

Answer

We can write it as

$$\int x.x^2 \cos x^2 dx$$

Now let $x^2 = t$

$$2x dx = dt$$

$$x dx = dt/2$$

Now

$$\frac{1}{2} \int t \cos t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and $\cos t$ as the second function.

$$\begin{aligned}
 \int a.b.dx &= a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx \\
 \frac{1}{2} \int t \cos t dt &= \frac{1}{2} \left(t \int \cos t dt - \int \left[\frac{dt}{dt} \cdot \int \cos t dt \right] dt \right) \\
 &= \frac{1}{2} (t \sin t - \int \sin t dt) \\
 &= \frac{1}{2} (t \sin t + \cos t) + c
 \end{aligned}$$

Replacing t with x^2

$$= \frac{1}{2} x^2 \sin x^2 + \frac{1}{2} \cos x^2 + c$$

22. Question

Evaluate the following integrals:

$$\int \sin x \log(\cos x) dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(\cos x)$ is the first function and $\sin x$ as the second function.

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\int \sin x \log(\cos x) dx = \log(\cos x) \int \sin x dx - \int \left(\frac{d \log(\cos x)}{dx} \cdot \int \sin x dx \right) dx$$

$$= -\cos x \log(\cos x) + \int \frac{-\sin x}{\cos x} \cdot \cos x dx$$

$$= -\cos x \log(\cos x) - \int \sin x dx$$

$$= -\cos x \log(\cos x) + \cos x + c$$

23. Question

Evaluate the following integrals:

$$\int x \sin x \cos x dx$$

Answer

We know that $\sin 2x = 2 \sin x \cos x$

$$\int x \sin x \cos x dx = \frac{1}{2} \int x \sin 2x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and $\sin 2x$ as the second function.

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\frac{1}{2} \int x \sin 2x dx = \frac{1}{2} \left(x \int \sin 2x dx - \int \left[\frac{dx}{dx} \cdot \int \sin 2x dx \right] dx \right)$$

$$= \frac{1}{2} \left(x \frac{-\cos 2x}{2} + \int \frac{\cos 2x}{2} dx \right)$$

$$= \frac{1}{2} \left(\frac{-x \cos 2x}{2} + \frac{\sin 2x}{4} \right) + c$$

$$= \frac{-x \cos 2x}{4} + \frac{\sin 2x}{8} + c$$

24. Question

Evaluate the following integrals:

$$\int \cos \sqrt{x} \, dx$$

Answer

Let $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow dx = 2\sqrt{x} dt$$

$$\Rightarrow dx = 2t dt$$

We can write it as

$$\int \cos \sqrt{x} dx = 2 \int t \cos t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is first function and $\cos t$ as the second function.

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\Rightarrow 2 \int t \cos t dt = 2 \left(t \int \cos t dt - \int \left[\frac{dt}{dt} \right] \int \cos t dt \right) dt$$

$$= 2 \left(t \sin t - \int \sin t dt \right)$$

$$= 2t \sin t + 2 \cos t + c$$

Replacing t with $\sqrt{x}$

$$= 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + c$$

$$= 2(\cos \sqrt{x} + \sqrt{x} \sin \sqrt{x}) + c$$

25. Question

Evaluate the following integrals:

$$\int \operatorname{cosec}^3 x \, dx$$

Answer

$$\text{We can write it as } \int \operatorname{cosec}^3 x \, dx = \int \operatorname{cosec} x \cdot \operatorname{cosec}^2 x \, dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\operatorname{cosec} x$ is first function and $\operatorname{cosec}^2 x$ as the second function.

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

$$\begin{aligned}\int \cos \operatorname{cosec} x \cdot \cos \operatorname{cosec}^2 x dx &= \cos \operatorname{cosec} x \int \cos \operatorname{cosec}^2 x dx - \int \left(\frac{d \cos \operatorname{cosec} x}{dx} \cdot \int \cos \operatorname{cosec}^2 x dx \right) dx \\&= \cos \operatorname{cosec} x (-\cot x) - \int (-\cos \operatorname{cosec} x \cdot \cot x)(-\cot x) dx \\&= -\cos \operatorname{cosec} x \cdot \cot x - \int \cos \operatorname{cosec} x \cdot \cot^2 x dx\end{aligned}$$

We know that $\cot^2 x = \operatorname{cosec}^2 x - 1$

$$\begin{aligned}-\cos \operatorname{cosec} x \cdot \cot x &- \int \cos \operatorname{cosec} x (\operatorname{cosec}^2 x - 1) dx \\&= -\cos \operatorname{cosec} x \cdot \cot x - \int \cos \operatorname{cosec}^3 x dx + \int \cos \operatorname{cosec} x dx\end{aligned}$$

We can write $\int \cos \operatorname{cosec}^3 x dx = I$

$$\begin{aligned}\Rightarrow \int \cos \operatorname{cosec}^3 x dx - \cos \operatorname{cosec} x \cdot \cot x &- \int \cos \operatorname{cosec}^3 x dx + \int \cos \operatorname{cosec} x dx \\ \Rightarrow 2 \int \cos \operatorname{cosec}^3 x dx &= -\cos \operatorname{cosec} x \cdot \cot x + \int \cos \operatorname{cosec} x dx \\ \Rightarrow 2 \int \cos \operatorname{cosec}^3 x dx &= -\cos \operatorname{cosec} x \cdot \cot x + \ln |\sec x + \tan x| + c_1 \\ \Rightarrow \int \cos \operatorname{cosec}^3 x dx &= \frac{-\cos \operatorname{cosec} x \cdot \cot x + \ln |\sec x + \tan x|}{2} + c\end{aligned}$$

26. Question

Evaluate the following integrals:

$$\int x \sin^3 x \cos x dx$$

Answer

We can write it as $\int x \sin^2 x \sin x \cos x dx$

We also know that $2 \sin x \cdot \cos x = \sin 2x$

$$\int x \sin^2 x \sin x \cos x dx = \frac{1}{2} \int x \sin^2 x \sin 2x dx$$

We also know that $\sin^2 x = \frac{1 - \cos 2x}{2}$

$$\begin{aligned}\frac{1}{2} \int x \sin^2 x \sin 2x dx &= \frac{1}{2} \int x \cdot \left(\frac{1 - \cos 2x}{2} \right) \sin 2x dx \\&= \frac{1}{2} \left[\left(\int \frac{x \sin 2x}{2} dx - \int \frac{x \cos 2x \sin 2x}{2} dx \right) \right]\end{aligned}$$

Here $\sin 4x = 2 \sin 2x \cdot \cos 2x$

$$= \frac{1}{2} \left[\left(\int \frac{x \sin 2x}{2} dx - \frac{1}{4} \int x \sin 4x dx \right) \right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and Sin2x and sin4x as the second function.

$$\begin{aligned}\int a.b.dx &= a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx \\&= \frac{1}{2} \left[\left(\frac{1}{2} \left\{ x \int \sin 2x dx - \int \left(\frac{dx}{dx} \cdot \int \sin 2x dx \right) dx \right\} \right) - \left(\frac{1}{4} \left\{ x \int \sin 4x - \int \left(\frac{dx}{dx} \cdot \int \sin 4x dx \right) dx \right\} \right) \right] \\&= \frac{1}{2} \left[\left(\frac{1}{2} \left\{ -x \frac{\cos 2x}{2} + \int \frac{\cos 2x}{2} dx \right\} \right) - \left(\frac{1}{4} \left\{ -x \frac{\cos 4x}{4} + \int \frac{\cos 4x}{4} dx \right\} \right) \right] \\&= \frac{1}{2} \left[\left(\frac{1}{2} \left\{ -x \frac{\cos 2x}{2} + \frac{\sin 2x}{4} \right\} \right) - \left(\frac{1}{4} \left\{ -x \frac{\cos 4x}{4} + \frac{\sin 4x}{16} \right\} \right) \right] + c \\&= \frac{-x \cos 2x}{8} + \frac{\sin 2x}{16} + \frac{x \cos 4x}{32} - \frac{\sin 4x}{128} + c\end{aligned}$$

27. Question

Evaluate the following integrals:

$$\int \sin x \log(\cos x) dx$$

Answer

Let $\cos x = t$

$-\sin x dx = dt$

Now the integral we have is

$$\begin{aligned}\int \sin x \log(\cos x) dx &= -\int \log t dt \\&= -\int 1 \cdot \log t dt\end{aligned}$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\begin{aligned}\int a.b.dx &= a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx \\-\int 1 \cdot \log t dt &= \log t \int 1 dt - \int \left(\frac{d \log t}{dt} \cdot \int 1 \cdot dt \right) dt \\&= -\log t \cdot t + \int \frac{1}{t} \cdot t dt \\&= -t \log t + t + c\end{aligned}$$

Replacing t with cosx

$$\begin{aligned}&t(-\log t + 1) + c \\&= \cos x(1 - \log(\cos x)) + c\end{aligned}$$

28. Question

Evaluate the following integrals:

$$\int \frac{\log(\log x)}{x} dx$$

Answer

Let $\log x = t$

$$1/x \, dx = dt$$

$$\int \frac{\log(\log x)}{x} dx = \int \log t \, dt = \int 1 \cdot \log t \, dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log t$ is first function and 1 as the second function.

$$\int a \cdot b \cdot dx = a \int b \, dx - \int \left[\frac{da}{dx} \cdot \int b \, dx \right] dx$$

$$\int 1 \cdot \log t \, dt = \log t \int 1 \, dt - \int \left(\frac{d \log t}{dt} \cdot \int 1 \cdot dt \right) dt$$

$$= t \cdot \log t - \int \frac{1}{t} t \, dt$$

$$= t \log t - t + c$$

Now replacing t with $\log x$

$$\log x \cdot \log(\log x) - \log x + c$$

$$= \log x (\log(\log x) - 1) + c$$

29. Question

Evaluate the following integrals:

$$\int \log(2 + x^2) dx$$

Answer

$$= \int 1 \cdot \log(2 + x^2) dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(2 + x^2)$ is the first function and 1 as the second function.

$$\int a \cdot b \cdot dx = a \int b \, dx - \int \left[\frac{da}{dx} \cdot \int b \, dx \right] dx$$

$$\begin{aligned}
 \int 1 \cdot \log(2+x^2) dx &= \log(2+x^2) \int 1 dx - \int \left(\frac{d \log(2+x^2)}{dx} \cdot \int 1 dx \right) dx \\
 &= \log(2+x^2) \cdot x - \int \frac{1 \cdot 2x}{2+x^2} \cdot x dx \\
 &= x \log(2+x^2) - \int \frac{2x^2}{2+x^2} dx \\
 &= x \log(2+x^2) - 2 \int \frac{x^2+2-2}{2+x^2} dx \\
 &= x \log(2+x^2) - 2 \left[\left(\int 1 dx \right) - \int \frac{2}{2+x^2} dx \right] \\
 &= x \log(2+x^2) - 2 \left[x - \left(2 \int \frac{1}{2+x^2} dx \right) \right] \\
 &= x \log(2+x^2) - 2 \left[x - 2 \left(\frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} \right) \right] + c \\
 &= x \log(2+x^2) - 2x + 2\sqrt{2} \tan^{-1} \frac{x}{\sqrt{2}} + c
 \end{aligned}$$

30. Question

Evaluate the following integrals:

$$\int \frac{x}{(1+\sin x)} dx$$

Answer

$$\begin{aligned}
 \int \frac{x}{1+\sin x} dx &= \int \frac{x(1-\sin x)}{(1+\sin x)(1-\sin x)} dx \\
 &= \int \frac{x(1-\sin x)}{1-\sin^2 x} dx \\
 \text{We can write it as } &= \int \frac{x(1-\sin x)}{\cos^2 x} dx \\
 &= \int x \sec^2 x dx - \int x \tan x \sec x dx
 \end{aligned}$$

Using by part and ILATE

Taking x as first function and $\sec^2 x$ and $\sec x \tan x$ as the second function, we have

$$\begin{aligned}
 \int x \sec^2 x dx - \int x \sec x \tan x dx &= \left(x \int \sec^2 x dx - \int \left(\frac{dx}{dx} \cdot \int \sec^2 x dx \right) dx \right) \\
 &\quad - \left(x \int \sec x \tan x dx - \int \left(\frac{dx}{dx} \cdot \int \sec x \tan x dx \right) dx \right) \\
 &= (x \tan x - \int 1 \cdot \tan x dx) - (x \cdot \sec x - \int 1 \cdot \sec x dx) \\
 &= x \tan x - \ln |\sec x| - x \sec x + \ln |\sec x + \tan x| + c \\
 &= x(\tan x - \sec x) + \ln \left| \frac{\sec x + \tan x}{\sec x} \right| + c \\
 &= x(\tan x - \sec x) + \ln |1 + \sin x| + c
 \end{aligned}$$

31. Question

Evaluate the following integrals:

$$\int \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$$

Answer

Let us assume $\log x = t$

$$x = e^t$$

$$dx = e^t dt$$

Now we have

$$\int \left(\frac{1}{\log x} - \frac{1}{(\log x)^2} \right) dx = \int \left(\frac{1}{t} - \frac{1}{t^2} \right) e^t dt$$

Considering $f(x) = 1/t$; $f'(x) = -1/t^2$

$$\frac{d}{dt} \left(\frac{1}{t} \right) = -\frac{1}{t^2}$$

By the integral property of $\int \{f(x) + f'(x)\} e^x dx = e^x \cdot f(x) + c$

So the solution of the integral is

$$\int \left(\frac{1}{\log x} - \frac{1}{(\log x)^2} \right) dx = e^t \times \frac{1}{t} + c$$

Substituting the value of t as $\log x$

$$= e^{\log x} \times \frac{1}{\log x} + c$$

$$= \frac{x}{\log x} + c$$

32. Question

Evaluate the following integrals:

$$\int e^{-x} \cos 2x \cos 4x dx$$

Answer

$$\cos A \cdot \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\text{We know that } \Rightarrow \cos 4x \cdot \cos 2x = \frac{1}{2} [\cos(4x+2x) + \cos(4x-2x)]$$

$$= \frac{1}{2} [\cos 6x + \cos 2x]$$

Putting in the original equation

$$\int e^{-x} \cos 2x \cdot \cos 4x dx = \int e^{-x} \left(\frac{1}{2} [\cos 6x + \cos 2x] \right) dx$$

$$= \frac{1}{2} \left[\left(\int e^{-x} \cos 6x dx \right) + \left(\int e^{-x} \cos 2x dx \right) \right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\cos 6x$ and $\cos 2x$ is first function and e^{-x} as the second function.

$$\int a \cdot b \cdot dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int b dx \right] dx$$

Solving both parts individually

$$I = \int e^{-x} \cos 6x dx = \cos 6x \int e^{-x} dx - \int \left(\frac{d \cos 6x}{dx} \cdot \int e^{-x} dx \right) dx$$

$$I = \cos 6x \cdot (-e^{-x}) - \int (-6 \sin 6x) \cdot (-e^{-x}) dt$$

$$I = -\cos 6x \cdot e^{-x} - 6 \int \sin 6x \cdot e^{-x} dx$$

$$I = -e^{-x} \cos 6x - 6 \left[\sin 6x \int e^{-x} dx - \int \left(\frac{d \sin 6x}{dx} \cdot \int e^{-x} dx \right) dx \right]$$

$$I = -e^{-x} \cos 6x - 6 \left[\sin 6x (-e^{-x}) - \int (6 \cos 6x) \cdot (-e^{-x}) dt \right]$$

$$I = -e^{-x} \cos 6x - 6 \left[-e^{-x} \sin 6x + 6 \int e^{-x} \cos 6x dx \right]$$

$$I = -e^{-x} \cos 6x - 6 \left[-e^{-x} \sin 6x + 6I \right]$$

$$I = -e^{-x} \cos 6x + 6e^{-x} \sin 6x - 36I$$

$$37I = e^{-x} (6 \sin 6x - \cos 6x)$$

$$I = \frac{e^{-x} (6 \sin 6x - \cos 6x)}{37}$$

Solving the second part,

$$I = \int e^{-x} \cos 2x dx = \cos 2x \int e^{-x} dx - \int \left(\frac{d \cos 2x}{dx} \cdot \int e^{-x} dx \right) dx$$

$$J = \cos 2x.(-e^{-x}) - \int (-2 \sin 2x).(-e^{-x}) dt$$

$$J = -\cos 2x.e^{-x} - 2 \int \sin 2x.e^{-x} dx$$

$$J = -e^{-x} \cos 2x - 2 \left[\sin 2x \int e^{-x} dx - \int \left(\frac{d \sin 2x}{dx} \cdot \int e^{-x} dx \right) dx \right]$$

$$J = -e^{-x} \cos 2x - 2 \left[\sin 2x (-e^{-x}) - \int (2 \cos 2x).(-e^{-x}) dt \right]$$

$$J = -e^{-x} \cos 2x - 2 \left[-e^{-x} \sin 2x + 2 \int e^{-x} \cos 2x dx \right]$$

$$J = -e^{-x} \cos 2x - 2 \left[-e^{-x} \sin 2x + 2J \right]$$

$$J = -e^{-x} \cos 2x + 2e^{-x} \sin 2x - 4J$$

$$5J = e^{-x} (2 \sin 2x - \cos 2x)$$

$$J = \frac{e^{-x} (2 \sin 2x - \cos 2x)}{5}$$

Putting in the obtained equation

$$= \frac{1}{2} \left[\frac{e^{-x} (6 \sin 6x - \cos 6x)}{37} + \frac{e^{-x} (2 \sin 2x - \cos 2x)}{5} \right] + c$$

$$= \frac{e^{-x} (6 \sin 6x - \cos 6x)}{74} + \frac{e^{-x} (2 \sin 2x - \cos 2x)}{10} + c$$

$$= e^{-x} \left(\frac{(6 \sin 6x - \cos 6x)}{74} + \frac{(2 \sin 2x - \cos 2x)}{10} \right) + c$$

33. Question

Evaluate the following integrals:

$$\int e^{\sqrt{x}} dx$$

Answer

Let $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}} dx = dt$$

$$dx = 2\sqrt{x} dt$$

$$\Rightarrow dx = 2t dt$$

Replacing in the original equation, we get

$$\int e^{\sqrt{x}} dx = \int e^t \cdot 2t dt$$

$$= 2 \int t e^t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx$$

$$2 \int te^t dt = 2 \left[t \int e^t dt - \int \left(\frac{dt}{dt} \cdot \int e^t dt \right) dt \right]$$

$$= 2 \left[te^t - \int 1.e^t dt \right]$$

$$= 2 \left[te^t - e^t \right] + c$$

$$= 2e^t (t - 1) + c$$

Replacing t with $\sqrt{x}$

$$= 2e^{\sqrt{x}}(\sqrt{x} - 1) + c$$

34. Question

Evaluate the following integrals:

$$\int e^{\sin x} \sin 2x \, dx$$

Answer

We can write $\sin 2x = 2 \sin x \cos x$

$$\int e^{\sin x} \sin 2x \, dx = 2 \int e^{\sin x} \cdot \sin x \cos x \, dx$$

Let $\sin x = t$

$\cos x \, dx = dt$

$$2 \int e^{\sin x} \sin x \cos x \, dx = 2 \int e^t \cdot t \cdot dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx$$

$$2 \int e^t \cdot t \, dt = 2 \left[t \int e^t dt - \int \left(\frac{dt}{dt} \cdot \int e^t dt \right) dt \right]$$

$$= 2 \left[t.e^t - \int 1.e^t dt \right]$$

$$= 2 \left[t.e^t - e^t \right] + c$$

$$= 2e^t (t - 1) + c$$

Replacing t with $\sin x$

$$= 2e^{\sin x}(\sin x - 1) + c$$

35. Question

Evaluate the following integrals:

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx$$

Answer

Let $\sin^{-1}x = t$

$x = \sin t$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

Putting this in the original equation, we get

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \int t \cdot \sin t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and $\sin t$ as the second function.

$$\int a \cdot b \cdot dx = a \int b dx - \int \left[\frac{da}{dx} \cdot b \right] dx$$

$$\int t \cdot \sin t dt = t \int \sin t dt - \int \left(\frac{dt}{dt} \cdot \sin t \right) dt$$

$$= t(-\cos t) - \int 1 \cdot (-\cos t) dt$$

$$= -t \cos t + \sin t + c$$

We can write $\cos t = \sqrt{1 - \sin^2 t}$

$$= -t(\sqrt{1 - \sin^2 t}) + \sin t + c$$

Now replacing $\sin^{-1}x = t$

$$= -\sin^{-1}x(\sqrt{1 - x^2}) + x + c$$

36. Question

Evaluate the following integrals:

$$\int \frac{x^2 \tan^{-1} x}{(1+x^2)} dx$$

Answer

Let $\tan^{-1} x = t$ and $x = \tan t$

Differentiating both sides, we get

$$\frac{1}{1+x^2} dx = dt$$

Now we have

$$\int \frac{x^2 \tan^{-1} x}{(1+x^2)} dx = \int \tan^2 t \cdot t dt$$

$$\int t \cdot \tan^2 t dt = \int t(\sec^2 t - 1) dt$$

$$= \int t \sec^2 t dt - \int t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and $\sec^2 t$ as the second function.

$$\int a.b.dx = a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx$$

$$\int t \sec^2 t dt - \int t dt = t \int \sec^2 t dt - \int \left(\frac{dt}{dt} \cdot \int \sec^2 t dt \right) dt - \frac{t^2}{2}$$

$$= t \cdot \tan t - \int \tan t dt - \frac{t^2}{2}$$

$$= t \cdot \tan t - \ln |\sec t| - \frac{t^2}{2} + c$$

We know that $\sec t = \sqrt{\tan^2 t + 1}$

$$= \tan^{-1} x \cdot x - \ln |\sqrt{\tan^2 t + 1}| - \frac{\tan^2 x}{2} + c$$

$$= x \tan^{-1} x - \ln |\sqrt{x^2 + 1}| - \frac{\tan^2 x}{2} + c$$

37. Question

Evaluate the following integrals:

$$\int \frac{\log(x+2)}{(x+2)^2} dx$$

Answer

$$\text{We can write it as } \int \log(x+2) \cdot \frac{1}{(x+2)^2} dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(x+2)$ is first function and $(x+2)^{-2}$ as second function.

$$\int a.b.dx = a \int bdx - \int \left[\frac{da}{dx} \cdot \int bdx \right] dx$$

$$\int \log(x+2) \cdot \frac{1}{(x+2)^2} dx = \log(x+2)$$

$$\begin{aligned} & \int \frac{1}{(x+2)^2} dx - \int \left(\frac{d \log(x+2)}{dx} \cdot \int \frac{1}{(x+2)^2} dx \right) dx \\ &= \log(x+2) \cdot \frac{-1}{(x+2)} - \int \frac{1}{x+2} \cdot \frac{-1}{(x+2)} dx \\ &= -\log(x+2) \frac{1}{(x+2)} + \int \frac{1}{(x+2)^2} dx \\ &= -\log(x+2) \frac{1}{(x+2)} - \frac{1}{(x+2)} + c \end{aligned}$$

38. Question

Evaluate the following integrals:

$$\int x \sin^{-1} x \, dx$$

Answer

Let $x = \sin t$; $t = \sin^{-1} x$

$$dx = \cos t \, dt$$

$$\begin{aligned} \Rightarrow \int x \sin^{-1} x \, dx &= \int \sin t \cdot \sin^{-1}(\sin t) \cos t \, dt \\ &= \int \sin t \cdot t \cdot \cos t \, dt \end{aligned}$$

We know that $\sin 2t = 2 \sin t \cos t$

$$\text{We have } \int t \cos t \sin t \, dt = \frac{1}{2} \int t \sin 2t \, dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and $\sin 2t$ as the second function.

$$\begin{aligned} \int a \cdot b \cdot dx &= a \int b \, dx - \int \left[\frac{da}{dx} \cdot \int b \, dx \right] dx \\ \frac{1}{2} \int t \sin 2t \, dt &= \frac{1}{2} \left(t \int \sin 2t \, dt - \int \left[\frac{dt}{dt} \cdot \int \sin 2t \, dt \right] dt \right) \\ &= \frac{1}{2} \left(t \cdot \frac{-\cos 2t}{2} + \int \frac{\cos 2t}{2} dt \right) \\ &= \frac{1}{2} \left(\frac{-t \cos 2t}{2} + \frac{\sin 2t}{4} \right) + c \\ &= \frac{-t \cos 2t}{4} + \frac{\sin 2t}{8} + c \end{aligned}$$

We know that $\cos 2t = 1 - 2\sin^2 t$, $\sin 2t = 2 \sin t \cos t$ and $\cos t = \sqrt{1 - \sin^2 t}$

Replacing in above equation

$$\begin{aligned}
&= \frac{-t(1-2\sin^2 t)}{4} + \frac{2\sin t \times \cos t}{8} + c \\
&= \frac{-t(1-2\sin^2 t)}{4} + \frac{\sqrt{1-\sin^2 t}}{4} \cdot \sin t + c \\
&= \frac{-\sin^{-1} x (1-2x^2)}{4} + \frac{x\sqrt{1-x^2}}{4} + c \\
&= \frac{1}{2}x^2 \sin^{-1} x - \frac{\sin^{-1} x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c \\
&= \frac{1}{2}x^2 \sin^{-1} x - \frac{\sin^{-1} x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c
\end{aligned}$$

39. Question

Evaluate the following integrals:

$$\int x \cos^{-1} x \, dx$$

Answer

Let $x = \cos t$; $t = \cos^{-1} x$

$$dx = -\sin t \, dt$$

$$\begin{aligned}
\int x \cos^{-1} x \, dx &= -\int \cos t \cdot \cos^{-1}(\cos t) \sin t \, dt \\
&= -\int \cos t \cdot t \cdot \sin t \, dt
\end{aligned}$$

We know that $\sin 2t = 2 \sin t \cos t$

$$\text{We have } -\int t \cos t \sin t \, dt = \frac{-1}{2} \int t \sin 2t \, dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking first function to the one which comes first in the list.

Here t is first function and $\sin 2t$ as second function.

$$\begin{aligned}
\int a \cdot b \cdot dx &= a \int b \, dx - \int \left[\frac{da}{dx} \cdot \int b \, dx \right] dx \\
\frac{-1}{2} \int t \sin 2t \, dt &= \frac{-1}{2} \left(t \int \sin 2t \, dt - \int \left[\frac{dt}{dt} \cdot \int \sin 2t \, dt \right] dt \right) \\
&= \frac{-1}{2} \left(t \cdot \frac{-\cos 2t}{2} + \int \frac{\cos 2t}{2} dt \right) \\
&= \frac{-1}{2} \left(\frac{-t \cos 2t}{2} + \frac{\sin 2t}{4} \right) + c \\
&= \frac{t \cos 2t}{4} - \frac{\sin 2t}{8} + c
\end{aligned}$$

We know that $\cos 2t = 2\cos^2 t - 1$ and $\sin 2t = 2\sin t \cos t$ and $\sin t = \sqrt{1 - \cos^2 t}$

Replacing in above equation

$$\begin{aligned}
&= \frac{t(2\cos^2 t - 1)}{4} - \frac{2\sin t \times \cos t}{8} + c \\
&= \frac{t(2\cos^2 t - 1)}{4} - \frac{\sqrt{1 - \cos^2 t}}{4} \cdot \cos t + c \\
&= \frac{\cos^{-1} x (2x^2 - 1)}{4} - \frac{x\sqrt{1 - x^2}}{4} + c \\
&= \frac{1}{2}x^2 \cos^{-1} x - \frac{\cos^{-1} x}{4} - \frac{1}{4}x\sqrt{1 - x^2} + c \\
&= \frac{1}{2}x^2 \cos^{-1} x + \frac{\sin^{-1} x}{4} - \frac{1}{4}x\sqrt{1 - x^2} + c
\end{aligned}$$

40. Question

Evaluate the following integrals:

$$\int \cot^{-1} x \, dx$$

Answer

We can write it as $\int \cot^{-1} x \cdot 1 \, dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\cot^{-1} x$ is first function and 1 as the second function.

$$\begin{aligned}
\int a \cdot b \cdot dx &= a \int b \, dx - \int \left[\frac{da}{dx} \cdot \int b \, dx \right] dx \\
\int \cot^{-1} x \cdot 1 \, dx &= \cot^{-1} x \int 1 \, dx - \int \left(\frac{d \cot^{-1} x}{dx} \cdot \int 1 \, dx \right) dx \\
&= \cot^{-1} x \cdot x - \int \frac{-1}{1+x^2} \cdot x \cdot dx \\
&= x \cot^{-1} x + \int \frac{x}{1+x^2} \, dx
\end{aligned}$$

$$\text{Let } 1 + x^2 = t$$

$$2x \, dx = dt$$

$$x \, dx = dt/2$$

$$\begin{aligned}
\Rightarrow \int \cot^{-1} x \, dx &= x \cot^{-1} x + \int \frac{dt}{2t} \\
&= x \cot^{-1} x + \frac{\log t}{2} + c
\end{aligned}$$

Now replacing t with $1 + x^2$

$$= x \cot^{-1} x + \log(1 + x^2)/2 + c$$

41. Question

Evaluate the following integrals:

$$\int x \cot^{-1} x \, dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x$,

$$\begin{aligned} \therefore \int x \cot^{-1} x \, dx &= \cot^{-1} x \int x \, dx - \int \left\{ \frac{d}{dx} (\cot^{-1} x) \int x \, dx \right\} dx \\ &= \frac{x^2 \cot^{-1} x}{2} - \int \frac{1}{(1+x^2)} \times \frac{x^2}{2} dx \\ &= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{(1+x^2)} dx \\ &= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int \frac{1+x^2-x^2}{(1+x^2)} dx \\ &= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int 1 - \frac{1}{(1+x^2)} dx \\ &= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} [x - \tan^{-1} x] + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

42. Question

Evaluate the following integrals:

$$\int x^2 \cot^{-1} x \, dx$$

[CBSE 2006C]

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x^2$,

$$\begin{aligned} \therefore \int x^2 \cot^{-1} x \, dx &= \cot^{-1} x \int x^2 \, dx - \int \left\{ \frac{d}{dx} (\cot^{-1} x) \int x^2 \, dx \right\} dx \\ &= \frac{x^3 \cot^{-1} x}{3} - \int \frac{1}{(1+x^2)} \times \frac{x^3}{3} dx \\ &= \frac{x^3 \cot^{-1} x}{3} - \frac{1}{3} \int \frac{x^3}{(1+x^2)} dx \end{aligned}$$

Taking $(1+x^2)=a$,

$2xdx=da$ i.e. $xdx=da/2$

Again, $x^2 = a - 1$

$$\begin{aligned} &\therefore \frac{1}{3} \int \frac{x^2 \times x dx}{(1 + x^2)} \\ &= \frac{1}{3} \int \frac{(a - 1) da}{2a} \\ &= \frac{1}{6} \int \left(1 - \frac{1}{a}\right) da \\ &= \frac{1}{6} (a - \ln a) \end{aligned}$$

Replacing the value of a, we get,

$$\begin{aligned} &\therefore \frac{1}{6} (a - \ln a) \\ &= \frac{1}{6} [(1 + x^2) - \ln|x^2 + 1|] + c_1 \\ &= \frac{x^2}{6} - \frac{\ln|x^2 + 1|}{6} + \left(c_1 + \frac{1}{6}\right) \\ &= \frac{x^2}{6} - \frac{\ln|x^2 + 1|}{6} + c \end{aligned}$$

The total integration yields as

$$= \frac{x^3 \cot^{-1} x}{3} + \frac{x^2}{6} - \frac{\ln|x^2 + 1|}{6} + c, \text{ where } c \text{ is the integrating constant}$$

43. Question

Evaluate the following integrals:

$$\int \sin^{-1} \sqrt{x} \, dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \sin^{-1} \sqrt{x}$ and $f_2(x) = 1$,

$$\begin{aligned} &\therefore \int \sin^{-1} \sqrt{x} \, dx \\ &= \sin^{-1} \sqrt{x} \int dx - \int \left\{ \frac{d}{dx} (\sin^{-1} \sqrt{x}) \int dx \right\} dx \\ &= x \sin^{-1} \sqrt{x} - \int \frac{1}{2\sqrt{x}\sqrt{1-x}} \times x dx \\ &= x \sin^{-1} \sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx \end{aligned}$$

Taking $(1-x) = a^2$,

$$-dx = 2ada \text{ i.e. } dx = -2ada$$

Again, $x = 1 - a^2$

$$\begin{aligned}
 &\therefore \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx \\
 &= \frac{1}{2} \int \frac{\sqrt{1-a^2}}{a} (-2ada) \\
 &= - \int \sqrt{1-a^2} da \\
 &= - \left[\frac{1}{2} a \sqrt{1-a^2} + \frac{1}{2} \sin^{-1} a \right]
 \end{aligned}$$

Replacing the value of a, we get,

$$\begin{aligned}
 &\therefore - \left[\frac{1}{2} a \sqrt{1-a^2} + \frac{1}{2} \sin^{-1} a \right] \\
 &= - \left[\frac{1}{2} x \sqrt{1-x} + \frac{1}{2} \sin^{-1} \sqrt{1-x} \right] + c
 \end{aligned}$$

The total integration yields as

$$= x \sin^{-1} \sqrt{x} + \left[\frac{1}{2} x \sqrt{1-x} + \frac{1}{2} \sin^{-1} \sqrt{1-x} \right] + c, \text{ where } c \text{ is the integrating constant}$$

44. Question

Evaluate the following integrals:

$$\int \cos^{-1} \sqrt{x} \, dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \cos^{-1} \sqrt{x}$ and $f_2(x) = 1$,

$$\begin{aligned}
 &\therefore \int \cos^{-1} \sqrt{x} dx \\
 &= \cos^{-1} \sqrt{x} \int dx - \int \left\{ \frac{d}{dx} (\cos^{-1} \sqrt{x}) \int dx \right\} dx \\
 &= x \cos^{-1} \sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times x dx \\
 &= x \cos^{-1} \sqrt{x} + \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx
 \end{aligned}$$

Taking $(1-x)=a^2$,

$$-dx=2ada \text{ i.e. } dx=-2ada$$

Again, $x=1-a^2$

$$\begin{aligned}
 &\therefore \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx \\
 &= \frac{1}{2} \int \frac{\sqrt{1-a^2}}{a} (-2ada) \\
 &= - \int \sqrt{1-a^2} da
 \end{aligned}$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$$= x \cos^{-1}\sqrt{x} - \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c, \text{ where } c \text{ is the integrating constant}$$

45. Question

Evaluate the following integrals:

$$\int \cos^{-1}(4x^3 - 3x) dx$$

Answer

Formula to be used - We know, $\cos 3x = 4\cos^3x - 3\cos x$

$$\therefore \int \cos^{-1}(4x^3 - 3x) dx$$

Assuming $x = \cos a$, $4\cos^3a - 3\cos a = \cos 3a$

And, $dx = -\sin a da$

Hence, $a = \cos^{-1}x$

Again, $\sin a = \sqrt{1-x^2}$

$$\therefore \int \cos^{-1}(4x^3 - 3x) dx$$

$$= \int \cos^{-1}(\cos 3a) \{-\sin a da\}$$

$$= -3 \int a \sin a da$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = a$ and $f_2(x) = \sin a$,

$$\therefore -3 \int a \sin a da$$

$$= -3 \left[a \int \sin a da - \int \left\{ \frac{d}{dx} a \int \sin a da \right\} da \right]$$

$$= 3a \cos a - \int \cos a da$$

$$= 3a \cos a - \sin a + c$$

Replacing the value of a we get,

$$\therefore 3a \cos a - \sin a + c$$

$$= 3x \cos^{-1} x - \sqrt{1-x^2} + c, \text{ where } c \text{ is the integrating constant}$$

46. Question

Evaluate the following integrals:

$$\int \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$ and $f_2(x) = 1$,

$$\begin{aligned} & \int \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx \\ &= \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \int dx - \int \left[\frac{d}{dx} \left\{ \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right\} \int dx \right] dx \\ &= x \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) + \int \left[\frac{\frac{(1+x^2)(-2x) - (1-x^2)(2x)}{(1+x^2)^2}}{\sqrt{1 - \left(\frac{1-x^2}{1+x^2} \right)^2}} \right] dx \\ &= x \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) + \int \frac{-4x^2 dx}{(1+x^2)^2 \times \frac{1}{1+x^2} \times 2x} \\ &= x \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) - \int \frac{2x dx}{1+x^2} \end{aligned}$$

Now,

$$\begin{aligned} & \int \frac{2x dx}{1+x^2} \\ &= \int \frac{d(1+x^2)}{1+x^2} \\ &= \ln(1+x^2) + c \end{aligned}$$

Again, we know,

$$\begin{aligned} \cos 2x &= \frac{1 - \tan^2 x}{1 + \tan^2 x} \\ \Rightarrow 2x &= \cos^{-1} \left(\frac{1 - \tan^2 x}{1 + \tan^2 x} \right) \end{aligned}$$

Replacing x by $\tan x$, it is obtained that,

$$2 \tan x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

So, the final integral yielded is

$2x \tan x - \ln(1+x^2) + c$, where c is the integrating constant

47. Question

Evaluate the following integrals:

$$\int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$$

Answer

Formula to be used - We know, $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$$

Assuming $x = \tan a$,

$$\frac{2 \tan a}{1 - \tan^2 a} = \tan 2a$$

And, $dx = \sec^2 a da$

Hence, $a = \tan^{-1} x$

Now, $\sec^2 a - \tan^2 a = 1$, so, $\sec a = \sqrt{1+x^2}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$$

$$= \int \tan^{-1}(\tan 2a) \{\sec^2 a da\}$$

$$= 2 \int a \sec^2 a da$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = a$ and $f_2(x) = \sec^2 a$,

$$\therefore 2 \int a \sec^2 a da$$

$$= 2 \left[a \int \sec^2 a da - \int \left\{ \frac{d}{dx} a \int \sec^2 a da \right\} da \right]$$

$$= 2a \tan a - \int \tan a da$$

$$= 2a \tan a - \ln |\sec a| + c$$

Replacing the value of a we get,

$$\therefore 2a \tan a - \ln |\sec a| + c$$

$$= 2x \tan^{-1} x - \ln \sqrt{1+x^2} + c, \text{ where } c \text{ is the integrating constant}$$

48. Question

Evaluate the following integrals:

$$\int \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) dx$$

Answer

Formula to be used - We know, $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$$\therefore \int \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) dx$$

Assuming $x = \tan a$,

$$\frac{3\tan a - \tan^3 a}{1 - 3\tan^2 a} = \tan 3a$$

And, $dx = \sec^2 a da$

Hence, $a = \tan^{-1} x$

Now, $\sec^2 a - \tan^2 a = 1$, so, $\sec a = \sqrt{1 + x^2}$

$$\begin{aligned} &\therefore \int \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) dx \\ &= \int \tan^{-1}(\tan 3a) \{ \sec^2 a da \} \\ &= 3 \int a \sec^2 a da \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = a$ and $f_2(x) = \sec^2 a$,

$$\begin{aligned} &\therefore 3 \int a \sec^2 a da \\ &= 3 \left[a \int \sec^2 a da - \int \left\{ \frac{d}{dx} a \int \sec^2 a da \right\} da \right] \\ &= 3a \tan a - \frac{3}{2} \int \tan a da \\ &= 3a \tan a - \frac{3}{2} \ln |\sec a| + c \end{aligned}$$

Replacing the value of a we get,

$$\begin{aligned} &\therefore 3a \tan a - \frac{3}{2} \ln |\sec a| + c \\ &= 3x \tan^{-1} x - \frac{3}{2} \ln \sqrt{1 + x^2} + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

49. Question

Evaluate the following integrals:

$$\int \frac{\sin^{-1} x}{x^2} dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = 1/x^2$,

$$\begin{aligned} &\therefore \int \frac{\sin^{-1}x}{x^2} dx \\ &= \sin^{-1}x \int \frac{1}{x^2} dx - \int \left\{ \frac{d}{dx} (\sin^{-1}x) \int \frac{1}{x^2} dx \right\} dx \\ &= \frac{-\sin^{-1}x}{x} - \int \frac{1}{\sqrt{1-x^2}} \times \left(-\frac{1}{x}\right) dx \\ &= \frac{-\sin^{-1}x}{x} + \int \frac{1}{x\sqrt{1-x^2}} dx \end{aligned}$$

Taking $x = \sin a$, $dx = \cos a da$

Hence, $\operatorname{cosec} a = 1/x$

Now, $\operatorname{cosec}^2 a - \cot^2 a = 1$ so $\cot a = \sqrt{(1-x^2)}/x$

$$\begin{aligned} &\therefore \int \frac{1}{x\sqrt{1-x^2}} dx \\ &= \int \frac{1}{\sin a \cos a} (\cos a da) \\ &= \int \operatorname{cosec} a da \\ &= \ln |\operatorname{cosec} a - \cot a| + c \end{aligned}$$

Replacing the value of a , we get,

$$\begin{aligned} &\therefore \ln |\operatorname{cosec} a - \cot a| + c \\ &= \ln \left| \frac{1}{x} - \frac{\sqrt{1-x^2}}{x} \right| + c \end{aligned}$$

The total integration yields as

$$= \frac{-\sin^{-1}x}{x} + \ln \left| \frac{1}{x} - \frac{\sqrt{1-x^2}}{x} \right| + c, \text{ where } c \text{ is the integrating constant}$$

50. Question

Evaluate the following integrals:

$$\int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$$

Answer

Say, $\tan x = a$

Hence, $\sec^2 x dx = da$

$$\therefore \int \frac{\tan x \sec^2 x}{1 - \tan^2 x} dx$$

$$= \int \frac{ada}{1-a^2}$$

Now, taking $1-a^2 = k$, $-2ada = dk$ i.e. $ada = -dk/2$

$$\therefore \int \frac{ada}{1-a^2}$$

$$= \int \frac{-dk}{2k}$$

$$= -\frac{1}{2} \ln|k| + c$$

Replacing the value of k ,

$$-\frac{1}{2} \ln|k| + c$$

$$= -\frac{1}{2} \ln|1-a^2| + c$$

Replacing the value of a ,

$$-\frac{1}{2} \ln|1-a^2| + c$$

$$= -\frac{1}{2} \ln|1-\tan^2 x| + c, \text{ where } c \text{ is the integrating constant}$$

51. Question

Evaluate the following integrals:

$$\int e^{3x} \sin 4x \, dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin 4x$ and $f_2(x) = e^{3x}$,

$$\therefore \int e^{3x} \sin 4x \, dx$$

$$= \sin 4x \int e^{3x} dx - \int \left\{ \frac{d}{dx} (\sin 4x) \int e^{3x} dx \right\} dx$$

$$= \frac{e^{3x} \sin 4x}{3} - \int 4 \cos 4x \times \frac{e^{3x}}{3} dx$$

$$= \frac{e^{3x} \sin 4x}{3} - \frac{4}{3} \int e^{3x} \cos 4x \, dx$$

$$= \frac{e^{3x} \sin 4x}{3} - \frac{4}{3} \left[\cos 4x \int e^{3x} dx - \int \left\{ \frac{d}{dx} (\cos 4x) \int e^{3x} dx \right\} dx \right]$$

$$= \frac{e^{3x} \sin 4x}{3} - \frac{4e^{3x} \cos 4x}{9} - \frac{4}{3} \int 4 \sin 4x \times \frac{e^{3x}}{3} dx$$

$$= \frac{e^{3x} \sin 4x}{3} - \frac{4e^{3x} \cos 4x}{9} - \frac{16}{9} \int e^{3x} \sin 4x \, dx$$

$$\therefore \left(1 + \frac{16}{9}\right) \int e^{3x} \sin 4x dx = \frac{e^{3x} \sin 4x}{3} - \frac{4e^{3x} \cos 4x}{9} + c_1$$

$$\Rightarrow \frac{25}{9} \int e^{3x} \sin 4x dx = \frac{3e^{3x} \sin 4x - 4e^{3x} \cos 4x}{9} + c_1$$

$$\Rightarrow \int e^{3x} \sin 4x dx = \frac{e^{3x}}{25} (3 \sin 4x - 4 \cos 4x) + c, \text{ where } c \text{ is the integrating constant}$$

52. Question

Evaluate the following integrals:

$$\int e^{2x} \sin x \, dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x} \sin x dx$$

$$= \sin x \int e^{2x} dx - \int \left\{ \frac{d}{dx} (\sin x) \int e^{2x} dx \right\} dx$$

$$= \frac{e^{2x} \sin x}{2} - \int \cos x \times \frac{e^{2x}}{2} dx$$

$$= \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x dx$$

$$= \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} dx - \int \left\{ \frac{d}{dx} (\cos x) \int e^{2x} dx \right\} dx \right]$$

$$= \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{2} \int \sin x \times \frac{e^{2x}}{2} dx$$

$$= \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} \int e^{2x} \sin x dx$$

$$\therefore \left(1 + \frac{1}{4}\right) \int e^{2x} \sin x dx = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} + c_1$$

$$\Rightarrow \frac{5}{4} \int e^{2x} \sin x dx = \frac{2e^{2x} \sin x - e^{2x} \cos x}{4} + c_1$$

$$\Rightarrow \int e^{2x} \sin x dx = \frac{e^{2x}}{5} (2 \sin x - \cos x) + c, \text{ where } c \text{ is the integrating constant}$$

53. Question

Evaluate the following integrals:

$$\int e^{2x} \sin x \cos x \, dx$$

Answer

$$\int e^{2x} \sin x \cos x dx$$

$$= \frac{1}{2} \int e^{2x} \times 2 \sin x \cos x dx$$

$$= \frac{1}{2} \int e^{2x} \sin 2x dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin 2x$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x} \sin 2x dx$$

$$= \sin 2x \int e^{2x} dx - \int \left\{ \frac{d}{dx} (\sin 2x) \int e^{2x} dx \right\} dx$$

$$= \frac{e^{2x} \sin 2x}{2} - \int 2 \cos 2x \times \frac{e^{2x}}{2} dx$$

$$= \frac{e^{2x} \sin 2x}{2} - \int e^{2x} \cos 2x dx$$

$$= \frac{e^{2x} \sin 2x}{2} - \left[\cos 2x \int e^{2x} dx - \int \left\{ \frac{d}{dx} (\cos 2x) \int e^{2x} dx \right\} dx \right]$$

$$= \frac{e^{2x} \sin 2x}{2} - \frac{e^{2x} \cos 2x}{2} - \int 2 \sin 2x \times \frac{e^{2x}}{2} dx$$

$$= \frac{e^{2x} \sin 2x}{2} - \frac{e^{2x} \cos 2x}{2} - \int e^{2x} \sin x dx$$

$$\therefore (1 + 1) \int e^{2x} \sin 2x dx = \frac{e^{2x} \sin 2x}{2} - \frac{e^{2x} \cos 2x}{2} + c_1$$

$$\Rightarrow 2 \int e^{2x} \sin 2x dx = \frac{e^{2x} \sin 2x - e^{2x} \cos 2x}{2} + c_1$$

$$\Rightarrow \int e^{2x} \sin 2x dx = \frac{e^{2x}}{4} (\sin 2x - \cos 2x) + c'$$

$$\therefore \frac{1}{2} \int e^{2x} \sin 2x dx$$

$$= \frac{1}{2} \times \left[\frac{e^{2x}}{4} (\sin 2x - \cos 2x) + c' \right]$$

$$= \frac{e^{2x}}{8} (\sin 2x - \cos 2x) + c, \text{ where } c \text{ is the integrating constant}$$

54. Question

Evaluate the following integrals:

$$\int e^{2x} \cos(3x + 4) dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos(3x+4)$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x} \cos(3x+4) dx$$

$$= \cos(3x+4) \int e^{2x} dx - \int \left\{ \frac{d}{dx} \cos(3x+4) \int e^{2x} dx \right\} dx$$

$$= \frac{e^{2x} \cos(3x+4)}{2} + \int 3 \sin(3x+4) \times \frac{e^{2x}}{2} dx$$

$$= \frac{e^{2x} \cos(3x+4)}{2} + \frac{3}{2} \int e^{2x} \sin(3x+4) dx$$

$$= \frac{e^{2x} \cos(3x+4)}{2} + \frac{3}{2} \left[\sin(3x+4) \int e^{2x} dx - \int \left\{ \frac{d}{dx} \sin(3x+4) \int e^{2x} dx \right\} dx \right]$$

$$= \frac{e^{2x} \cos(3x+4)}{2} + \frac{3e^{2x} \sin(3x+4)}{4} - \frac{3}{2} \int 3 \cos(3x+4) \times \frac{e^{2x}}{2} dx$$

$$= \frac{e^{2x} \cos(3x+4)}{2} + \frac{3e^{2x} \sin(3x+4)}{4} - \frac{9}{4} \int e^{2x} \cos(3x+4) dx$$

$$\therefore \left(1 + \frac{9}{4}\right) \int e^{2x} \cos(3x+4) dx = \frac{e^{2x} \cos(3x+4)}{2} + \frac{3e^{2x} \sin(3x+4)}{4} + c_1$$

$$\Rightarrow \frac{13}{4} \int e^{2x} \cos(3x+4) dx = \frac{2e^{2x} \cos(3x+4) + 3e^{2x} \sin(3x+4)}{4} + c_1$$

$$\Rightarrow \int e^{2x} \cos(3x+4) dx = \frac{e^{2x}}{13} (2 \cos(3x+4) + 3 \sin(3x+4)) + c, \text{ where } c \text{ is the integrating constant}$$

55. Question

Evaluate the following integrals:

$$\int e^{-x} \cos x dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \cos x$ and $f_2(x) = e^{-x}$,

$$\therefore \int e^{-x} \cos x dx$$

$$= \cos x \int e^{-x} dx - \int \left\{ \frac{d}{dx} \cos x \int e^{-x} dx \right\} dx$$

$$= -e^{-x} \cos x - \int e^{-x} \sin x dx$$

$$= -e^{-x} \cos x - \left[\sin x \int e^{-x} dx - \int \left\{ \frac{d}{dx} \sin x \int e^{-x} dx \right\} dx \right]$$

$$= -e^{-x} \cos x - \left[-e^{-x} \sin x + \int e^{-x} \cos x dx \right]$$

$$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x dx$$

$$\therefore (1 + 1) \int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$$

$$\Rightarrow 2 \int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$$

$$\Rightarrow \int e^{-x}\cos x dx = \frac{e^{-x}}{2} (\sin x - \cos x) + c, \text{ where } c \text{ is the integrating constant}$$

56. Question

Evaluate the following integrals:

$$\int e^x (\sin x + \cos x) dx$$

Answer

$$\int e^x (\sin x + \cos x) dx$$

$$= \int e^x \sin x dx + \int e^x \cos x dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \sin x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \sin x dx + \int e^x \cos x dx$$

$$= \sin x \int e^x dx - \int \left[\frac{d}{dx} (\sin x) \int e^x dx \right] dx + \int e^x \cos x dx$$

$$= e^x \sin x - \int e^x \cos x dx + \int e^x \cos x dx + c$$

$$= e^x \sin x + c, \text{ where } c \text{ is the integrating constant}$$

57. Question

Evaluate the following integrals:

$$\int e^x (\cot x - \operatorname{cosec}^2 x) dx$$

Answer

$$\int e^x (\cot x - \operatorname{cosec}^2 x) dx$$

$$= \int e^x \cot x dx + \int e^x \operatorname{cosec}^2 x dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned}
& \int e^x \cot x dx + \int e^x \operatorname{cosec}^2 x dx \\
&= \cot x \int e^x dx - \int \left[\frac{d}{dx} (\cot x) \int e^x dx \right] dx + \int e^x \operatorname{cosec}^2 x dx \\
&= e^x \cot x - \int e^x \operatorname{cosec}^2 x dx + \int e^x \operatorname{cosec}^2 x dx + c \\
&= e^x \cot x + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

58. Question

Evaluate the following integrals:

$$\int e^x \sec x (1 + \tan x) dx$$

Answer

$$\begin{aligned}
& \int e^x \sec x (1 + \tan x) dx \\
&= \int e^x \sec x dx + \int e^x \sec x \tan x dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned}
& \int e^x \sec x dx + \int e^x \sec x \tan x dx \\
&= \sec x \int e^x dx - \int \left[\frac{d}{dx} (\sec x) \int e^x dx \right] dx + \int e^x \sec x \tan x dx \\
&= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c \\
&= e^x \sec x + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

59. Question

Evaluate the following integrals:

$$\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx$$

Answer

$$\begin{aligned}
& \int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx \\
&= \int e^x \tan^{-1} x dx + \int \frac{e^x}{1+x^2} dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \tan^{-1} x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned}
& \int e^x \tan^{-1} x \, dx + \int \frac{e^x}{1+x^2} \, dx \\
&= \tan^{-1} x \int e^x \, dx - \int \left[\frac{d}{dx} (\tan^{-1} x) \int e^x \, dx \right] dx + \int \frac{e^x}{1+x^2} \, dx \\
&= e^x \tan^{-1} x - \int \frac{e^x}{1+x^2} \, dx + \int \frac{e^x}{1+x^2} \, dx + c \\
&= e^x \tan^{-1} x + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

60. Question

Evaluate the following integrals:

$$\int e^x (\cot x + \log \sin x) \, dx$$

Answer

$$\begin{aligned}
& \int e^x (\cot x + \log \sin x) \, dx \\
&= \int e^x \cot x \, dx + \int e^x \log \sin x \, dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x) \, dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) \, dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) \, dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \log \sin x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\begin{aligned}
& \int e^x \cot x \, dx + \int e^x \log \sin x \, dx \\
&= \int e^x \cot x \, dx + \log \sin x \int e^x \, dx - \int \left[\frac{d}{dx} (\log \sin x) \int e^x \, dx \right] \\
&= \int e^x \cot x \, dx + e^x \log \sin x - \int e^x \cot x \, dx + c \\
&= e^x \log |\sin x| + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

61. Question

Evaluate the following integrals:

$$\int e^x (\tan x - \log \cos x) \, dx$$

Answer

$$\begin{aligned}
& \int e^x (\tan x + \log \cos x) \, dx \\
&= \int e^x \tan x \, dx + \int e^x \log \cos x \, dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x) \, dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) \, dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) \, dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \log \cos x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \tan x \, dx - \int e^x \log \cos x \, dx$$

$$\begin{aligned}
&= \int e^x \tan x \, dx - \log \cos x \int e^x dx + \int \left[\frac{d}{dx} (\log \cos x) \int e^x dx \right] \\
&= \int e^x \tan x \, dx - e^x \log \cos x - \int e^x \tan x \, dx + c \\
&= \mathbf{e^x \log |\sec x| + c}, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

62. Question

Evaluate the following integrals:

$$\int e^x [\sec x + \log(\sec x + \tan x)] dx$$

Answer

$$\begin{aligned}
&\int e^x [\sec x + \log(\sec x + \tan x)] dx \\
&= \int e^x \sec x \, dx + \int e^x \log(\sec x + \tan x) dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \log \cos x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\begin{aligned}
&\int e^x \sec x \, dx + \int e^x \log(\sec x + \tan x) dx \\
&= \int e^x \sec x dx + \log(\sec x + \tan x) \int e^x dx \\
&\quad - \int \left[\frac{d}{dx} (\log(\sec x + \tan x)) \int e^x dx \right] \\
&= \int e^x \sec x \, dx + e^x \log(\sec x + \tan x) \\
&\quad - \int \frac{e^x \tan x \times (\sec^2 x + \sec x \tan x) dx}{\sec x + \tan x} + c \\
&= \int e^x \sec x \, dx + e^x \log(\sec x + \tan x) - \int e^x \sec x \, dx + c \\
&= \mathbf{e^x \log |\sec x + \tan x| + c}, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

63. Question

Evaluate the following integrals:

$$\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx$$

Answer

$$\begin{aligned}
&\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx \\
&= \int e^x (\sec^2 x + \tan x) dx \\
&= \int e^x \sec^2 x dx + \int e^x \tan x dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY

PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\begin{aligned} & \int e^x \sec^2 x dx + \int e^x \tan x dx \\ &= \int e^x \sec^2 x dx + \tan x \int e^x dx - \int \left[\frac{d}{dx} (\tan x) \int e^x dx \right] \\ &= \int e^x \sec^2 x dx + e^x \tan x - \int e^x \sec^2 x dx + c \\ &= e^x \tan x + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

64. Question

Evaluate the following integrals:

$$\int e^x \left(\frac{\sin x \cos x - 1}{\sin^2 x} \right) dx$$

Answer

$$\begin{aligned} & \int e^x \left(\frac{\sin x \cos x - 1}{\sin^2 x} \right) dx \\ &= \int e^x (\cot x - \operatorname{cosec}^2 x) dx \\ &= \int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned} & \int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx \\ &= \cot x \int e^x dx - \int \left\{ \frac{d}{dx} (\cot x) \int e^x dx \right\} dx - \int e^x \operatorname{cosec}^2 x dx \\ &= e^x \cot x + \int e^x \operatorname{cosec}^2 x dx - \int e^x \operatorname{cosec}^2 x dx + c \\ &= e^x \cot x + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

65. Question

Evaluate the following integrals:

$$\int e^x \left(\frac{\cos x + \sin x}{\cos^2 x} \right) dx$$

Answer

$$\int e^x \left(\frac{\cos x + \sin x}{\cos^2 x} \right) dx$$

$$= \int e^x(\sec x + \sec x \tan x) dx$$

$$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned} & \int e^x \sec x dx + \int e^x \sec x \tan x dx \\ &= \sec x \int e^x dx - \int \left[\frac{d}{dx} (\sec x) \int e^x dx \right] dx + \int e^x \sec x \tan x dx \\ &= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c \\ &= e^x \sec x + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

66. Question

Evaluate the following integrals:

$$\int e^x \left(\frac{2 - \sin 2x}{1 - \cos 2x} \right) dx$$

Answer

$$\begin{aligned} & \int e^x \left(\frac{2 - \sin 2x}{1 - \cos 2x} \right) dx \\ &= \int e^x \left(\frac{1 - \sin x \cos x}{\sin^2 x} \right) dx \\ &= \int e^x (\operatorname{cosec}^2 x - \cot x) dx \\ &= \int e^x \operatorname{cosec}^2 x dx - \int e^x \cot x dx \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\begin{aligned} & \int e^x \operatorname{cosec}^2 x dx - \int e^x \cot x dx \\ &= \int e^x \operatorname{cosec}^2 x dx - \cot x \int e^x dx + \int \left\{ \frac{d}{dx} (\cot x) \int e^x dx \right\} dx \\ &= \int e^x \operatorname{cosec}^2 x dx - e^x \cot x - \int e^x \operatorname{cosec}^2 x dx \\ &= -e^x \cot x + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

67. Question

Evaluate the following integrals:

$$\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$$

Answer

$$\left(\frac{1 + \sin x}{1 + \cos x} \right)$$

$$= \left(\frac{1 + \frac{2 \tan^{x/2}}{1 + \tan^2(x/2)}}{1 + \frac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)}} \right)$$

$$= \frac{(1 + \tan^{x/2})^2}{2}$$

$$\therefore \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$$

$$= \int e^x \times \frac{(1 + \tan^{x/2})^2}{2}$$

$$= \int \frac{e^x (1 + \tan^2 x/2 + 2 \tan^{x/2})}{2} dx$$

$$= \int \frac{e^x (\sec^2 x/2 + 2 \tan^{x/2})}{2} dx$$

$$= \int \frac{e^x \sec^2 x/2 dx}{2} + \int e^x \tan^{x/2} dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan(x/2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x \sec^2 x/2 dx}{2} + \int e^x \tan^{x/2} dx$$

$$= \int \frac{e^x \sec^2 x/2 dx}{2} + \tan^{x/2} \int e^x dx - \int \left[\frac{d}{dx} (\tan^{x/2}) \int e^x dx \right] dx$$

$$= \int \frac{e^x \sec^2 x/2 dx}{2} + e^x \tan^{x/2} - \int \frac{e^x \sec^2 x/2 dx}{2} + c$$

$$= e^x \tan^{x/2} + c, \text{ where } c \text{ is the integrating constant}$$

68. Question

Evaluate the following integrals:

$$\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x} \right) dx$$

Answer

$$\int e^x \left(\frac{\sin 4x - 1}{1 - \cos 4x} \right) dx$$

$$\begin{aligned}
&= \int e^x \left(\frac{2\sin 2x \cos 2x - 4}{2\sin^2 2x} \right) dx \\
&= \int e^x (\cot 2x - 2\operatorname{cosec}^2 2x) dx \\
&= \int e^x \cot 2x dx - \int 2e^x \operatorname{cosec}^2 2x dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot 2x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned}
&\int e^x \cot 2x dx - \int 2e^x \operatorname{cosec}^2 2x dx \\
&= \cot 2x \int e^x dx - \int \left\{ \frac{d}{dx} (\cot 2x) \int e^x dx \right\} dx - \int 2e^x \operatorname{cosec}^2 2x dx \\
&= e^x \cot 2x + \int 2e^x \operatorname{cosec}^2 2x dx - \int 2e^x \operatorname{cosec}^2 2x dx + c \\
&= e^x \cot 2x + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

69. Question

Evaluate the following integrals:

$$\int \frac{e^x [\sqrt{1-x^2} \sin^{-1} x + 1]}{\sqrt{1-x^2}} dx$$

Answer

$$\begin{aligned}
&\int \frac{e^x [\sqrt{1-x^2} \sin^{-1} x + 1]}{\sqrt{1-x^2}} dx \\
&= \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right) dx \\
&= \int e^x \sin^{-1} x dx + \int \frac{e^x}{\sqrt{1-x^2}} dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1} x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned}
&\int e^x \sin^{-1} x dx + \int \frac{e^x}{\sqrt{1-x^2}} dx \\
&= \sin^{-1} x \int e^x dx - \int \left\{ \frac{d}{dx} (\sin^{-1} x) \int e^x dx \right\} dx + \int \frac{e^x}{\sqrt{1-x^2}} dx \\
&= e^x \sin^{-1} x - \int \frac{e^x}{\sqrt{1-x^2}} dx + \int \frac{e^x}{\sqrt{1-x^2}} dx + c \\
&= e^x \sin^{-1} x + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

70. Question

Evaluate the following integrals:

$$\int e^x \left(\frac{1+x \log x}{x} \right) dx$$

Answer

$$\begin{aligned} & \int e^x \left(\frac{1+x \log x}{x} \right) dx \\ &= \int e^x \left(\frac{1}{x} + \log x \right) dx \\ &= \int \frac{e^x}{x} dx + \int e^x \log x dx \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \log x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\begin{aligned} & \int \frac{e^x}{x} dx + \int e^x \log x dx \\ &= \int \frac{e^x}{x} dx + \log x \int e^x dx - \int \left[\frac{d}{dx} (\log x) \int e^x dx \right] dx \\ &= \int \frac{e^x}{x} dx + e^x \log x - \int \frac{e^x}{x} dx + c \\ &= e^x \log x + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

71. Question

Evaluate the following integrals:

$$\int e^x \cdot \frac{x}{(1+x)^2} dx$$

Answer

$$\frac{x}{(1+x)^2} = \frac{A}{(1+x)} + \frac{B}{(1+x)^2}$$

$$\Rightarrow x = A(1+x) + B$$

For $x=-1$, equation: $-1 = B$ i.e. $B = -1$

For $x=0$, equation: $0 = A-1$ i.e. $A = 1$

$$\therefore \frac{x}{(1+x)^2}$$

$$= \frac{1}{(1+x)} - \frac{1}{(1+x)^2}$$

The given equation becomes

$$\begin{aligned} & \int e^x \left[\frac{1}{(1+x)} - \frac{1}{(1+x)^2} \right] dx \\ &= \int e^x \times \frac{1}{(1+x)} dx - \int e^x \times \frac{1}{(1+x)^2} dx \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned} & \int \frac{e^x}{(1+x)} dx - \int \frac{e^x}{(1+x)^2} dx \\ &= \frac{1}{(1+x)} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{1+x} \right) \int e^x dx \right] dx - \int \frac{e^x}{(1+x)^2} dx \\ &= \frac{e^x}{(1+x)} + \int \frac{e^x}{(1+x)^2} dx - \int \frac{e^x}{(1+x)^2} dx + c \\ &= \frac{e^x}{(1+x)} + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

72. Question

Evaluate the following integrals:

$$\int e^x \frac{(x-1)}{(x+1)^3} dx$$

Answer

$$\frac{x-1}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$$\Rightarrow x-1 = A(x+1)^2 + B(x+1) + C$$

For $x=-1$, equation: $-2 = C$ i.e. $C = -2$

For $x=0$, equation: $-1 = A+B-2$ i.e. $A+B = 1$

For $x=1$, equation: $0 = 4A+2B-2$

i.e. $2(A+B+A) = 2$

$$\Rightarrow 1+A = 1$$

$$\Rightarrow A = 0$$

And, $B = 1$

$$\begin{aligned} & \therefore \frac{x-1}{(x+1)^3} \\ &= \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \end{aligned}$$

The given equation becomes

$$\begin{aligned} & \int e^x \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right] dx \\ &= \int e^x \times \frac{1}{(x+1)^2} dx - \int e^x \times \frac{2}{(x+1)^3} dx \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned} & \int \frac{e^x}{(x+1)^2} dx - \int \frac{2e^x}{(x+1)^3} dx \\ &= \frac{1}{(x+1)^2} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{(x+1)^2} \right) \int e^x dx \right] dx - \int \frac{2e^x}{(x+1)^3} dx \\ &= \frac{e^x}{(x+1)^2} + \int \frac{2e^x}{(x+1)^3} dx - \int \frac{2e^x}{(x+1)^3} dx + c \\ &= \frac{e^x}{(x+1)^2} + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

73. Question

Evaluate the following integrals:

$$\int e^x \frac{(2-x)}{(1-x)^2} dx$$

Answer

$$\frac{2-x}{(1-x)^2} = \frac{A}{(1-x)} + \frac{B}{(1-x)^2}$$

$$\Rightarrow 2-x = A(1-x) + B$$

For $x=1$, equation: $1 = B$ i.e. $B = 1$

For $x=2$, equation: $0 = -A+1$ i.e. $A = 1$

$$\begin{aligned} & \therefore \frac{2-x}{(1-x)^2} \\ &= \frac{1}{(1-x)} + \frac{1}{(1-x)^2} \end{aligned}$$

The given equation becomes

$$\begin{aligned} & \int e^x \left[\frac{1}{(1-x)} + \frac{1}{(1-x)^2} \right] dx \\ &= \int e^x \times \frac{1}{(1-x)^2} dx + \int e^x \times \frac{1}{1-x} dx \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\begin{aligned} & \int \frac{e^x}{(1-x)^2} dx + \int \frac{e^x}{1-x} dx \\ &= \int \frac{e^x}{(1-x)^2} dx + \frac{1}{1-x} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{1-x} \right) \int e^x dx \right] dx \\ &= \int \frac{e^x}{(1-x)^2} dx + \frac{e^x}{1-x} - \int \frac{e^x}{(1-x)^2} dx + c \\ &= \frac{e^x}{1-x} + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

74. Question

Evaluate the following integrals:

$$\int e^x \cdot \frac{(x-3)}{(x-1)^3} dx$$

Answer

$$\frac{x-3}{(x-1)^3} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$$\Rightarrow x-3 = A(x-1)^2 + B(x-1) + C$$

For $x=1$, equation: $-2 = C$ i.e. $C = -2$

For $x=0$, equation: $-3 = A-B-2$ i.e. $B = A+1$

For $x=3$, equation: $0 = 4A+2B-2$

i.e. $2(A+B+A) = 2$

$$\Rightarrow 1+3A = 1$$

$$\Rightarrow A = 0$$

And, $B = 1$

$$\begin{aligned} \therefore \frac{x-3}{(x-1)^3} \\ = \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \end{aligned}$$

The given equation becomes

$$\begin{aligned} \int e^x \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right] dx \\ = \int e^x \times \frac{1}{(x-1)^2} dx - \int e^x \times \frac{2}{(x-1)^3} dx \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned} \int \frac{e^x}{(x-1)^2} dx - \int \frac{2e^x}{(x-1)^3} dx \\ = \frac{1}{(x-1)^2} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{(x-1)^2} \right) \int e^x dx \right] dx - \int \frac{2e^x}{(x-1)^3} dx \\ = \frac{e^x}{(x-1)^2} + \int \frac{2e^x}{(x-1)^3} dx - \int \frac{2e^x}{(x-1)^3} dx + c \\ = \frac{e^x}{(x-1)^2} + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

75. Question

Evaluate the following integrals:

$$\int e^{3x} \left(\frac{3x-1}{9x^2} \right) dx$$

Answer

$$\int e^{3x} \left(\frac{3x-1}{9x^2} \right) dx$$

$$= \int \frac{e^{3x}}{3x} dx - \int \frac{e^{3x}}{9x^2} dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = 1/3x$ and $f_2(x) = e^{3x}$ in the first integral and keeping the second integral intact,

$$\int \frac{e^{3x}}{3x} dx - \int \frac{e^{3x}}{9x^2} dx$$

$$= \frac{1}{3x} \int e^{3x} dx - \int \left[\frac{d}{dx} \left(\frac{1}{3x} \right) \int e^{3x} dx \right] dx - \int \frac{e^{3x}}{9x^2} dx$$

$$= \frac{e^{3x}}{9x} + \int \frac{e^{3x}}{9x^2} dx - \int \frac{e^{3x}}{9x^2} dx + c$$

$$= \frac{e^{3x}}{9x} + c, \text{ where } c \text{ is the integrating constant}$$

76. Question

Evaluate the following integrals:

$$\int \frac{(x+1)}{(x+2)^2} e^x dx$$

Answer

$$\frac{x+1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$$

$$\Rightarrow x+1 = A(x+2) + B$$

For $x=-2$, equation: $-1 = B$ i.e. $B = -1$

For $x=-1$, equation: $0 = A-1$ i.e. $A = 1$

$$\therefore \frac{x+1}{(x+2)^2}$$

$$= \frac{1}{(x+2)} - \frac{1}{(x+2)^2}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(x+2)} - \frac{1}{(x+2)^2} \right] dx$$

$$= \int e^x \times \frac{1}{x+2} dx - \int e^x \times \frac{1}{(x+2)^2} dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = 1/(x+2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\begin{aligned}
& \int \frac{e^x}{x+2} dx - \int \frac{e^x}{(x+2)^2} dx \\
&= \frac{1}{x+2} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{x+2} \right) \int e^x dx \right] dx - \int \frac{e^x}{(x+2)^2} dx \\
&= \frac{e^x}{x+2} + \int \frac{e^x}{(x+2)^2} dx - \int \frac{e^x}{(x+2)^2} dx + c \\
&= \frac{e^x}{x+2} + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

77. Question

Evaluate the following integrals:

$$\int \frac{x e^{2x}}{(1+2x)^2} dx$$

Answer

$$\frac{x}{(1+2x)^2} = \frac{A}{(1+2x)} + \frac{B}{(1+2x)^2}$$

$$\Rightarrow x = A(1+2x) + B$$

For $x = -1/2$, equation: $-1/2 = B$ i.e. $B = -1/2$

For $x = 0$, equation: $0 = A - 1/2$ i.e. $A = 1/2$

$$\begin{aligned}
& \therefore \frac{x}{(1+2x)^2} \\
&= \frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}
\end{aligned}$$

The given equation becomes

$$\begin{aligned}
& \int e^{2x} \left[\frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2} \right] dx \\
&= \int e^{2x} \times \frac{1}{2(1+2x)} dx - \int e^{2x} \times \frac{1}{2(1+2x)^2} dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = 1/(1+2x)$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\begin{aligned}
& \int e^{2x} \times \frac{1}{2(1+2x)} dx - \int e^{2x} \times \frac{1}{2(1+2x)^2} dx \\
&= \frac{1}{2} \left[\frac{1}{1+2x} \int e^{2x} dx - \int \left[\frac{d}{dx} \left(\frac{1}{1+2x} \right) \int e^{2x} dx \right] dx - \int \frac{e^{2x}}{(1+2x)^2} dx \right] \\
&= \frac{1}{2} \left[\frac{e^{2x}}{2(2x+1)} + \int \frac{e^{2x}}{(2x+1)^2} dx - \int \frac{e^{2x}}{(2x+1)^2} dx \right] \\
&= \frac{e^{2x}}{4(2x+1)} + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

78. Question

Evaluate the following integrals:

$$\int e^{2x} \left(\frac{2x-1}{4x^2} \right) dx$$

Answer

$$\begin{aligned} & \int e^{2x} \left(\frac{2x-1}{4x^2} \right) dx \\ &= \int \frac{e^{2x}}{2x} dx - \int \frac{e^{2x}}{4x^2} dx \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = 1/2x$ and $f_2(x) = e^{2x}$ in the first integral and keeping the second integral intact,

$$\begin{aligned} & \int \frac{e^{2x}}{2x} dx - \int \frac{e^{2x}}{4x^2} dx \\ &= \frac{1}{2x} \int e^{2x} dx - \int \left[\frac{d}{dx} \left(\frac{1}{2x} \right) \int e^{2x} dx \right] dx - \int \frac{e^{2x}}{4x^2} dx \\ &= \frac{e^{2x}}{4x} + \int \frac{e^{2x}}{4x^2} dx - \int \frac{e^{2x}}{4x^2} dx + c \\ &= \frac{e^{2x}}{4x} + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

79. Question

Evaluate the following integrals:

$$\int e^x \left(\log x + \frac{1}{x^2} \right) dx$$

Answer

$$\begin{aligned} & \int e^x \left(\log x + \frac{1}{x^2} \right) dx \\ &= \int e^x \log x dx - \int \frac{e^x}{x^2} dx \end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \log x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\begin{aligned} & \int e^x \log x dx - \int \frac{e^x}{x^2} dx \\ &= \log x \int e^x dx - \int \left[\frac{d}{dx} (\log x) \int e^x dx \right] dx - \int \frac{e^x}{x^2} dx \\ &= e^x \log x - \int \frac{e^x}{x} dx - \int \frac{e^x}{x^2} dx \\ &= e^x \log x - \left[\frac{1}{x} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{x} \right) \int e^x dx \right] dx \right] - \int \frac{e^x}{x^2} dx \end{aligned}$$

$$= e^x \log x - \frac{e^x}{x} + \int \frac{e^x}{x^2} dx - \int \frac{e^x}{x^2} dx + c$$

$$= e^x \left(\log x - \frac{1}{x} \right) + c, \text{ where } c \text{ is the integrating constant}$$

80. Question

Evaluate the following integrals:

$$\int \frac{\log x}{(1 + \log x)^2} dx$$

Answer

$$\frac{\log x}{(1 + \log x)^2} = \frac{A}{(1 + \log x)} + \frac{B}{(1 + \log x)^2}$$

$$\Rightarrow \log x = A(1 + \log x) + B$$

$$\text{For } x=1, \text{ equation: } 0 = A+B$$

$$\text{For } x=1/e, \text{ equation: } -1 = B \text{ i.e. } B = -1$$

$$\text{So, } A = 1$$

$$\therefore \frac{\log x}{(1 + \log x)^2}$$

$$= \frac{1}{(1 + \log x)} - \frac{1}{(1 + \log x)^2}$$

The given equation becomes

$$\int \left[\frac{1}{(1 + \log x)} - \frac{1}{(1 + \log x)^2} \right] dx$$

$$= \int \frac{1}{(1 + \log x)} dx - \int \frac{1}{(1 + \log x)^2} dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = 1/(1+\log x)$ and $f_2(x) = 1$ in the second integral and keeping the first integral intact,

$$\int \frac{1}{(1 + \log x)} dx - \int \frac{1}{(1 + \log x)^2} dx$$

$$= \frac{1}{(1 + \log x)} \int dx - \int \left[\frac{d}{dx} \left(\frac{1}{(1 + \log x)} \right) \int dx \right] dx - \int \frac{1}{(1 + \log x)^2} dx$$

$$= \frac{x}{(1 + \log x)} + \int \frac{1}{(1 + \log x)^2} dx - \int \frac{1}{(1 + \log x)^2} dx + c$$

$$= \frac{x}{(1 + \log x)} + c, \text{ where } c \text{ is the integrating constant}$$

81. Question

Evaluate the following integrals:

$$\int \{ \sin(\log x) + \cos(\log x) \} dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin(\log x)$ and $f_2(x) = 1$ in the first integral and keeping the second integral intact,

$$\begin{aligned} & \int \sin(\log x) dx + \int \cos(\log x) dx \\ &= \sin(\log x) \int dx - \int \left[\frac{d}{dx} (\sin(\log x)) \int dx \right] dx + \int \cos(\log x) dx \\ &= x \sin(\log x) - \int \cos(\log x) dx + \int \cos(\log x) dx + c \\ &= e^{\log x} \sin(\log x) + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

82. Question

Evaluate the following integrals:

$$\int \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(\log x)$ and $f_2(x) = 1$ in the first integral and keeping the second integral intact,

$$\begin{aligned} & \int \frac{1}{\log x} dx - \int \frac{1}{(\log x)^2} dx \\ &= \frac{1}{\log x} \int dx - \int \left[\frac{d}{dx} \left(\frac{1}{\log x} \right) \int dx \right] dx - \int \frac{1}{(\log x)^2} dx \\ &= \frac{x}{\log x} + \int \frac{1}{(\log x)^2} dx - \int \frac{1}{(\log x)^2} dx + c \\ &= \frac{x}{\log x} + c, \text{ where } c \text{ is the integrating constant} \end{aligned}$$

83. Question

Evaluate the following integrals:

$$\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log(\log x)$ and $f_2(x) = 1$ in the first integral and keeping the second integral intact,

$$\int \log(\log x) dx + \int \frac{1}{(\log x)^2} dx$$

$$\begin{aligned}
&= \log(\log x) \int dx - \int \left[\frac{d}{dx} (\log(\log x)) \int dx \right] dx + \int \frac{1}{(\log x)^2} dx \\
&= x \log(\log x) - \int \frac{1}{\log x} dx + \int \frac{1}{(\log x)^2} dx \\
&= x \log(\log x) - \left[\frac{1}{\log x} \int dx - \int \left[\frac{d}{dx} \left(\frac{1}{\log x} \right) \int dx \right] dx \right] + \int \frac{1}{(\log x)^2} dx \\
&= x \log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx + c \\
&= x \left[\log(\log x) - \frac{1}{\log x} \right] + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

84. Question

Evaluate the following integrals:

$$\int \left(\frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} \right) dx$$

Answer

It is known that $\sin^{-1} x + \cos^{-1} x = \pi/2$

$$\begin{aligned}
&\therefore \left(\frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} \right) \\
&= \frac{2}{\pi} (\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x})
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Now, for the first term,

Taking $f_1(x) = \sin^{-1} \sqrt{x}$ and $f_2(x) = 1$,

$$\begin{aligned}
&\therefore \int \sin^{-1} \sqrt{x} dx \\
&= \sin^{-1} \sqrt{x} \int dx - \int \left\{ \frac{d}{dx} (\sin^{-1} \sqrt{x}) \int dx \right\} dx \\
&= x \sin^{-1} \sqrt{x} - \int \frac{1}{2\sqrt{x}\sqrt{1-x}} \times x dx \\
&= x \sin^{-1} \sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx
\end{aligned}$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\begin{aligned}
&\therefore \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx \\
&= \frac{1}{2} \int \frac{\sqrt{1-a^2}}{a} (-2ada)
\end{aligned}$$

$$= - \int \sqrt{1-a^2} da$$

$$= - \left[\frac{1}{2} a \sqrt{1-a^2} + \frac{1}{2} \sin^{-1} a \right]$$

Replacing the value of a, we get,

$$\therefore - \left[\frac{1}{2} a \sqrt{1-a^2} + \frac{1}{2} \sin^{-1} a \right]$$

$$= - \left[\frac{1}{2} x \sqrt{1-x} + \frac{1}{2} \sin^{-1} \sqrt{1-x} \right] + c$$

The total integration yields as

$$= x \sin^{-1} \sqrt{x} + \left[\frac{1}{2} x \sqrt{1-x} + \frac{1}{2} \sin^{-1} \sqrt{1-x} \right] + c', \text{ where } c' \text{ is the integrating constant}$$

For the second term,

Taking $f_1(x) = \cos^{-1} \sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \cos^{-1} \sqrt{x} dx$$

$$= \cos^{-1} \sqrt{x} \int dx - \int \left\{ \frac{d}{dx} (\cos^{-1} \sqrt{x}) \int dx \right\} dx$$

$$= x \cos^{-1} \sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times x dx$$

$$= x \cos^{-1} \sqrt{x} + \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

Taking $(1-x)=a^2$,

$$-dx=2ada \text{ i.e. } dx=-2ada$$

Again, $x=1-a^2$

$$\therefore \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

$$= \frac{1}{2} \int \frac{\sqrt{1-a^2}}{a} (-2ada)$$

$$= - \int \sqrt{1-a^2} da$$

$$= - \left[\frac{1}{2} a \sqrt{1-a^2} + \frac{1}{2} \sin^{-1} a \right]$$

Replacing the value of a, we get,

$$\therefore - \left[\frac{1}{2} a \sqrt{1-a^2} + \frac{1}{2} \sin^{-1} a \right]$$

$$= - \left[\frac{1}{2} x \sqrt{1-x} + \frac{1}{2} \sin^{-1} \sqrt{1-x} \right] + c$$

The total integration yields as

$$= x \cos^{-1} \sqrt{x} - \left[\frac{1}{2} x \sqrt{1-x} + \frac{1}{2} \sin^{-1} \sqrt{1-x} \right] + c'', \text{ where } c'' \text{ is the integrating constant}$$

$$\therefore \int \left(\frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} \right) dx$$

$$= \frac{2}{\pi} \int (\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}) dx$$

$$= \frac{2}{\pi} \left[x \sin^{-1} \sqrt{x} + \left[\frac{1}{2} x \sqrt{1-x} + \frac{1}{2} \sin^{-1} \sqrt{1-x} \right] - x \cos^{-1} \sqrt{x} + \left[\frac{1}{2} x \sqrt{1-x} + \frac{1}{2} \sin^{-1} \sqrt{1-x} \right] \right] + c$$

$$= \frac{2}{\pi} [\sqrt{x-x^2} + x(\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}) + \sin^{-1} \sqrt{1-x}] + c \text{ where } c \text{ is the integrating constant}$$

85. Question

Evaluate the following integrals:

$$\int 5^{5^x} \cdot 5^{5^x} \cdot 5^x dx$$

Answer

Tip - 5^x is to be replaced by a

$$\therefore 5^x = a$$

$$\Rightarrow 5^x \log 5 dx = da$$

$$\Rightarrow 5^x dx = \frac{da}{\log 5}$$

The equation becomes as follows:

$$\int 5^{5^a} \times 5^a \times \frac{1}{\log 5} da$$

Tip - 5^a is to be replaced by k

$$\therefore 5^a = k$$

$$\Rightarrow 5^a \log 5 da = dk$$

$$\Rightarrow 5^a da = \frac{dk}{\log 5}$$

The equation becomes as follows:

$$\int 5^k \times \frac{1}{(\log 5)^2} dk$$

$$= \frac{1}{(\log 5)^2} \int 5^k dk$$

$$= \frac{5^k}{(\log 5)^3} + c$$

Re-replacing the value of k,

$$\frac{5^{5^a}}{(\log 5)^3} + c$$

Re-replacing the value of a,

$$\frac{5^{5^{5^x}}}{(\log 5)^3} + c, \text{ where } c \text{ is the integrating constant}$$

86. Question

Evaluate the following integrals:

$$\int e^{2x} \left(\frac{1 + \sin 2x}{1 + \cos 2x} \right) dx$$

Answer

$$\left(\frac{1 + \sin 2x}{1 + \cos 2x} \right)$$

$$= \left(\frac{1 + \frac{2 \tan x}{1 + \tan^2 x}}{1 + \frac{1 - \tan^2 x}{1 + \tan^2 x}} \right)$$

$$= \frac{(1 + \tan x)^2}{2}$$

$$\therefore \int e^{2x} \left(\frac{1 + \sin 2x}{1 + \cos 2x} \right) dx$$

$$= \int e^{2x} \times \frac{(1 + \tan x)^2}{2} dx$$

$$= \int \frac{e^{2x}(1 + \tan^2 x + 2 \tan x)}{2} dx$$

$$= \int \frac{e^{2x}(\sec^2 x + 2 \tan x)}{2} dx$$

$$= \int \frac{e^{2x} \sec^2 x dx}{2} + \int e^{2x} \tan x dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx \text{ where } f_1(x) \text{ and } f_2(x) \text{ are the first and second functions respectively.}$$

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int \frac{e^{2x} \sec^2 x dx}{2} + \int e^{2x} \tan x dx$$

$$= \int \frac{e^{2x} \sec^2 x dx}{2} + \tan x \int e^{2x} dx - \int \left[\frac{d}{dx} (\tan x) \int e^{2x} dx \right] dx$$

$$= \int \frac{e^{2x} \sec^2 x dx}{2} + \frac{1}{2} e^{2x} \tan x - \int \frac{e^{2x} \sec^2 x dx}{2} + c$$

$$= \frac{1}{2} e^x \tan x / 2 + c, \text{ where } c \text{ is the integrating constant}$$

87. Question

Evaluate the following integrals:

$$\int e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

Answer

$$\left(\frac{1 - \sin 2x}{1 - \cos 2x} \right)$$

$$\begin{aligned}
&= \left(\frac{1 - \frac{2\tan x}{1 + \tan^2 x}}{1 - \frac{1 - \tan^2 x}{1 + \tan^2 x}} \right) \\
&= \frac{(1 - \tan x)^2}{2} \\
&\therefore \int e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx \\
&= \int e^{2x} \times \frac{(1 - \tan x)^2}{2} \\
&= \int \frac{e^{2x}(1 + \tan^2 x - 2\tan x)}{2} dx \\
&= \int \frac{e^{2x}(\sec^2 x - 2\tan x)}{2} dx \\
&= \int \frac{e^{2x}\sec^2 x dx}{2} - \int e^{2x}\tan x dx
\end{aligned}$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\begin{aligned}
&\int \frac{e^{2x}\sec^2 x dx}{2} - \int e^{2x}\tan x dx \\
&= \int \frac{e^{2x}\sec^2 x dx}{2} - \tan x \int e^{2x} dx + \int \left[\frac{d}{dx} (\tan x) \int e^{2x} dx \right] dx \\
&= \int \frac{e^{2x}\sec^2 x dx}{2} - \frac{1}{2} e^{2x}\tan x + \int \frac{e^{2x}\sec^2 x dx}{2} + c \\
&= -\frac{1}{2} e^{2x}\tan x + c, \text{ where } c \text{ is the integrating constant}
\end{aligned}$$

Objective Questions II

1. Question

Mark (✓) against the correct answer in each of the following:

$$\int x e^x dx = ?$$

- A. $e^x (1 - x) + C$
- B. $e^x (x - 1) + C$
- C. $e^x (x - 1) + C$
- D. none of these

Answer

To find: Value of $\int x e^x dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x e^x dx \dots (i)$

$$I = \int x e^x dx$$

$$\Rightarrow x \int e^x dx - \int \left[\frac{d(x)}{dx} \int e^x dx \right] dx$$

$$\Rightarrow I = x e^x - \int 1 \cdot e^x dx$$

$$\Rightarrow I = x e^x - e^x + c$$

$$\therefore I = e^x (x-1) + c$$

$$\text{Ans) } c e^x (x-1) + c$$

2. Question

Mark (✓) against the correct answer in each of the following:

$$\int x e^{2x} dx = ?$$

A. $\frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C$

B. $\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$

C. $2x e^{2x} + 4e^{2x} + C$

D. none of these

Answer

To find: Value of $\int x e^{2x} dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x e^{2x} dx \dots (i)$

$$I = \int x e^{2x} dx$$

$$\Rightarrow x \int e^{2x} dx - \int \left[\frac{d(x)}{dx} \int e^{2x} dx \right] dx$$

$$\Rightarrow I = x \frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = x \frac{e^{2x}}{2} - \frac{1}{2} \int \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = \frac{x}{2} e^{2x} - \frac{1}{2} \int e^{2x} dx$$

$$\Rightarrow I = \frac{x}{2} e^{2x} - \frac{1}{2} \frac{e^{2x}}{2} + c$$

$$\Rightarrow I = \frac{x}{2} e^{2x} - \frac{e^{2x}}{4} + c$$

$$\therefore I = \frac{x}{2} e^{2x} - \frac{e^{2x}}{4} + c$$

Ans) B $\frac{x}{2} e^{2x} - \frac{e^{2x}}{4} + c$

3. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \cos 2x \, dx = ?$$

A. $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + C$

B. $\frac{1}{2} x \sin 2x - \frac{1}{4} \cos 2x + C$

C. $2x \sin 2x + 4 \cos 2x + C$

D. none of these

Answer

To find: Value of $\int x \cos 2x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \cos 2x \, dx$... (i)

Let $2x = t$

$$\Rightarrow x = \frac{t}{2}$$

$$\Rightarrow 2 = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{dt}{2}$$

$$I = \int \frac{t}{2} \cos t \frac{dt}{2}$$

$$I = \frac{1}{4} \int t \cos t \, dt$$

Taking 1st function as t and second function as $\cos t$

$$\Rightarrow I = \frac{1}{4} \left[t \int \cos t \, dt - \int \left(\frac{dt}{dt} \int \cos t \, dt \right) dt \right]$$

$$\Rightarrow I = \frac{1}{4} \left[t(\sin t) - \int (1(\sin t)) dt \right]$$

$$\Rightarrow I = \frac{1}{4} [t(\sin t) - (-\cos t)] + c$$

$$\Rightarrow I = \frac{1}{4} [t \sin t + \cos t] + c$$

$$\Rightarrow I = \frac{1}{4} [2x \sin 2x + \cos 2x] + c$$

$$\Rightarrow I = \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$$

Ans) A $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$

4. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \sec^2 x \, dx = ?$$

- A. $x \tan x - \log |\cos x| + C$
- B. $x \tan x + \log |\cos x| + C$
- C. $x \tan x + \log |\sec x| + C$
- D. none of these

Answer

To find: Value of $\int x \sec^2 x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \sec^2 x \, dx \dots (i)$

Taking 1st function as x and second function as $\sec^2 x$

$$\Rightarrow I = \left[x \int \sec^2 x \, dx - \int \left(\frac{dx}{dx} \int \sec^2 x \, dx \right) dx \right]$$

$$\Rightarrow I = \left[x \tan x - \int (1 \tan x) dx \right]$$

$$\Rightarrow I = \left[x \tan x - \int \tan x dx \right]$$

$$\Rightarrow I = [x \tan x - (-\log |\cos x|)] + c$$

$$\Rightarrow I = x \tan x + \log |\cos x| + c$$

Ans) B $x \tan x + \log |\cos x| + c$

5. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \sin 2x \, dx = ?$$

- A. $\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x + C$
- B. $-\frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x + C$

C. $-\frac{1}{2}x \cos 2x + \frac{1}{4} \sin 2x + C$

D. none of these

Answer

To find: Value of $\int x \sin 2x dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \sin 2x dx \dots (i)$

Let $2x = t$

$$\Rightarrow x = \frac{t}{2}$$

$$\Rightarrow 2 = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{dt}{2}$$

$$I = \int \frac{t}{2} \sin t \frac{dt}{2}$$

$$I = \frac{1}{4} \int t \sin t dt$$

Taking 1st function as t and second function as $\sin t$

$$\Rightarrow I = \frac{1}{4} \left[t \int \sin t dt - \int \left(\frac{dt}{dt} \int \sin t dt \right) dt \right]$$

$$\Rightarrow I = \frac{1}{4} \left[t(-\cos t) - \int (1(-\cos t)) dt \right]$$

$$\Rightarrow I = \frac{1}{4} \left[-t \cos t - \int -\cos t dt \right]$$

$$\Rightarrow I = \frac{1}{4} [-t \cos t + \sin t] + c$$

$$\Rightarrow I = \frac{1}{4} [-2x \cos 2x + \sin 2x] + c$$

$$\Rightarrow I = -\frac{1}{2}x \cos 2x + \frac{1}{4} \sin 2x + c$$

Ans) C $-\frac{1}{2}x \cos 2x + \frac{1}{4} \sin 2x + c$

6. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \log x dx = ?$$

A. $x \log x + \frac{1}{2}x^2 + C$

B. $\frac{1}{2}x^2 \log x + \frac{1}{4}x^2 + C$

C. $\frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C$

D. none of these

Answer

To find: Value of $\int x \log x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \log x \, dx \dots (i)$

Taking 1st function as $\log x$ and second function as x

$$\Rightarrow I = \left[\log x \int x \, dx - \int \left(\frac{d \log x}{dx} \int x \, dx \right) dx \right]$$

$$\Rightarrow I = \left[\log x \frac{x^2}{2} - \int \left(\frac{1}{x} \frac{x^2}{2} \right) dx \right]$$

$$\Rightarrow I = \left[\log x \frac{x^2}{2} - \int \left(\frac{x}{2} \right) dx \right]$$

$$\Rightarrow I = \left[\log x \frac{x^2}{2} - \frac{1}{2} \int x \, dx \right]$$

$$\Rightarrow I = \left[\log x \frac{x^2}{2} - \frac{1}{2} \frac{x^2}{2} \right] + c$$

$$\Rightarrow I = \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + c$$

Ans) C $\frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + c$

7. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \operatorname{cosec}^2 x \, dx = ?$$

A. $x \cot x - \log |\sin x| + C$

B. $-\cot x + \log |\sin x| + C$

C. $x \tan x - \log |\sec x| + C$

D. none of these

Answer

To find: Value of $\int x \operatorname{cosec}^2 x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \operatorname{cosec}^2 x dx \dots (i)$

$$I = \int x \operatorname{cosec}^2 x dx$$

$$\Rightarrow x \int \operatorname{cosec}^2 x dx - \int \left[\frac{d(x)}{x} \int \operatorname{cosec}^2 x dx \right] dx$$

$$\Rightarrow I = x(-\cot x) - \int 1.(-\cot x) dx$$

$$\Rightarrow I = -x(\cot x) + \log|\sin x| + c$$

Ans) D None of these

8. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \sin x \cos x dx = ?$$

A. $-\frac{1}{4} x \sin 2x + \frac{1}{8} \cos 2x + C$

B. $\frac{1}{4} x \cos 2x - \frac{1}{8} \sin 2x + C$

C. $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + C$

D. $-\frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + C$

Answer

To find: Value of $\int x \sin x \cos x dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \sin x \cos x dx \dots (i)$

$$I = \frac{1}{2} \int x 2 \sin x \cos x dx$$

$$I = \frac{1}{2} \int x \sin 2x dx$$

$$\Rightarrow \frac{1}{2} \left[x \int \sin 2x dx - \int \left[\frac{d(x)}{x} \int \sin 2x dx \right] dx \right]$$

$$\Rightarrow \frac{1}{2} \left[\frac{-x \cos 2x}{2} - \int \left[1 \cdot \frac{-\cos 2x}{2} \right] dx \right]$$

$$\Rightarrow \frac{1}{2} \left[\frac{-x \cos 2x}{2} + \frac{\sin 2x}{4} \right] + c$$

$$\Rightarrow \frac{-x \cos 2x}{4} + \frac{\sin 2x}{8} + c$$

Ans) D $\frac{-x \cos 2x}{4} + \frac{\sin 2x}{8} + c$

9. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \cos^2 x \, dx = ?$$

A. $\frac{x^2}{4} - \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + C$

B. $\frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + C$

C. $\frac{x^2}{4} + \frac{x \sin 2x}{4} - \frac{\cos 2x}{8} + C$

D. none of these

Answer

To find: Value of $\int x \cos^2 x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \cos^2 x \, dx \dots (i)$

$$I = \int x \frac{1}{2} (1 + \cos 2x) dx$$

$$I = \frac{1}{2} \int x \, dx + \frac{1}{2} \int x \cos 2x \, dx$$

$$I = \frac{1}{2} \frac{x^2}{2} + \frac{1}{2} \left[x \int \cos 2x \, dx - \int \left[\frac{d(x)}{dx} \int \cos 2x \, dx \right] dx \right]$$

$$I = \frac{1}{2} \frac{x^2}{2} + \frac{1}{2} \left[x \frac{\sin 2x}{2} - \int 1 \cdot \frac{\sin 2x}{2} dx \right]$$

$$I = \frac{1}{2} \frac{x^2}{2} + \frac{1}{2} \left[x \frac{\sin 2x}{2} - \frac{1}{2} \int \sin 2x \, dx \right]$$

$$I = \frac{1}{2} \frac{x^2}{2} + \frac{1}{2} \left[x \frac{\sin 2x}{2} - \frac{1}{2} \left(-\frac{\cos 2x}{2} \right) + c \right]$$

$$I = \frac{1}{2} \frac{x^2}{2} + \frac{1}{2} \left[\frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + c \right]$$

$$I = \frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + c$$

Ans) D $\frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + c$

10. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\log x}{x^2} dx = ?$$

A. $-\frac{1}{x}(\log x + 1) + C$

B. $\frac{1}{x}(\log x - 1) + C$

C. $\frac{1}{x}(\log x + 1) + C$

D. none of these

Answer

To find: Value of $\int \frac{\log x}{x^2} dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \frac{\log x}{x^2} dx \dots (i)$

$$I = \int x^{-2} \log x dx$$

$$\Rightarrow \log x \int x^{-2} dx - \int \left[\frac{d(\log x)}{x} \int x^{-2} dx \right] dx$$

$$\Rightarrow \log x \frac{x^{-1}}{-1} - \int \left(\frac{1}{-x^2} \right) dx$$

$$\Rightarrow -\frac{\log x}{x} + \left(-\frac{1}{x} \right) + c$$

$$\Rightarrow -\frac{1}{x}(\log x + 1) + c$$

Ans) A - $\frac{1}{x}(\log x + 1) + c$

11. Question

Mark (✓) against the correct answer in each of the following:

$$\int \log x dx = ?$$

A. $\frac{1}{x} + C$

B. $\frac{1}{2}(\log x)^2 + C$

C. $x(\log x + 1) + C$

D. $x(\log x - 1) + C$

Answer

To find: Value of $\int \log x dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \log x \cdot 1 \cdot dx \dots (i)$

Taking 1st function as $\log x$ and second function as 1

$$\Rightarrow I = \left[\log x \int 1 dx - \int \left(\frac{d \log x}{dx} \int 1 dx \right) dx \right]$$

$$\Rightarrow I = \left[\log x \cdot x - \int \left(\frac{1}{x} \int 1 dx \right) dx \right]$$

$$\Rightarrow I = \left[\log x \cdot x - \int \left(\frac{1}{x} x \right) dx \right]$$

$$\Rightarrow I = \left[\log x \cdot x - \int 1 dx \right]$$

$$\Rightarrow I = [\log x \cdot x - x] + c$$

$$\Rightarrow I = x(\log x - 1) + c$$

Ans) D $x(\log x - 1) + c$

12. Question

Mark (✓) against the correct answer in each of the following:

$$\int \log_{10} x dx = ?$$

A. $\frac{1}{x} \log_e 10 + C$

B. $\frac{1}{x} \log_{10} e + C$

C. $x (\log x - 1) \log_e 10 + C$

D. $x (\log x - 1) \log_{10} e + C$

Answer

To find: Value of $\int \log_{10} x dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \log_{10} x dx \dots (i)$

$$I = \int \log_{10} x dx = \int \frac{\log x}{\log 10} dx$$

$$I = \frac{1}{\log_e 10} \int \log x \cdot 1 dx$$

Taking 1st function as $\log x$ and second function as 1

$$\Rightarrow I = \frac{1}{\log_e 10} \left[\log x \int 1 \, dx - \int \left(\frac{d \log x}{dx} \int 1 \, dx \right) dx \right]$$

$$\Rightarrow I = \frac{1}{\log_e 10} \left[\log x \cdot x - \int \left(\frac{1}{x} \int 1 \, dx \right) dx \right]$$

$$\Rightarrow I = \frac{1}{\log_e 10} \left[\log x \cdot x - \int \left(\frac{1}{x} x \right) dx \right]$$

$$\Rightarrow I = \frac{1}{\log_e 10} \left[\log x \cdot x - \int 1 \, dx \right]$$

$$\Rightarrow I = \frac{1}{\log_e 10} [\log x \cdot x - x] + c$$

$$\Rightarrow I = x(\log x - 1) \log_{10} e + c$$

$$\text{Ans) } D \, x(\log x - 1) \log_{10} e + c$$

13. Question

Mark (✓) against the correct answer in each of the following:

$$\int (\log x)^2 \, dx = ?$$

A. $\frac{2 \log x}{x} + C$

B. $\frac{1}{3} (\log x)^3 + C$

C. $x (\log x)^2 - 2x \log x + 2x + C$

D. $x (\log x)^2 + 2x \log x - 2x + C$

Answer

To find: Value of $\int (\log x)^2 \, dx$

Formula used:

$$(i) \int f(x)g(x) \, dx = f(x) \int g(x) \, dx - \int \left[f'(x) \int g(x) \, dx \right] dx$$

$$\text{We have, } I = \int (\log x)^2 \cdot 1 \cdot dx \dots (i)$$

Taking 1st function as $(\log x)^2$ and second function as 1

$$\Rightarrow I = \left[(\log x)^2 \int 1 \, dx - \int \left(\frac{d(\log x)^2}{dx} \int 1 \, dx \right) dx \right]$$

$$\Rightarrow I = \left[(\log x)^2 \int 1 \, dx - \int \left(\frac{2(\log x)}{x} \int 1 \, dx \right) dx \right]$$

$$\Rightarrow I = \left[(\log x)^2 \cdot x - 2 \int \log x \, dx \right]$$

$$\Rightarrow I = [(\log x)^2 \cdot x - 2(x \log x - x)] + c$$

$$\Rightarrow I = [(\log x)^2 \cdot x - 2x \log x + 2x] + c$$

$$\Rightarrow I = x(\log x)^2 - 2x \log x + 2x + c$$

Ans) $C x(\log x)^2 - 2x \log x + 2x + c$

14. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^{\sqrt{x}} dx = ?$$

A. $e^{\sqrt{x}} + \sqrt{x} + C$

B. $\frac{1}{2} e^{\sqrt{x}} (\sqrt{x} + 1) + C$

C. $2e^{\sqrt{x}} (\sqrt{x} - 1) + C$

D. none of these

Answer

To find: Value of $\int e^{\sqrt{x}} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int e^{\sqrt{x}} dx \dots (i)$

Putting $\sqrt{x} = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\Rightarrow dx = 2\sqrt{x} dt$$

$$\Rightarrow dx = 2t dt$$

$$\Rightarrow I = 2 \int t \cdot e^t dt$$

$$\Rightarrow I = 2 \left[t \int e^t dt - \int \left[\frac{d(t)}{dt} \int e^t dt \right] dt \right]$$

$$\Rightarrow I = 2 \left[te^t - \int [1 \cdot e^t] dt \right]$$

$$\Rightarrow I = 2[te^t - e^t]$$

$$\Rightarrow I = e^t \cdot 2(t-1) + c$$

$$\therefore I = 2 e^{\sqrt{x}} (\sqrt{x} - 1) + c$$

Ans) $C 2 e^{\sqrt{x}} (\sqrt{x} - 1) + c$

15. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cos \sqrt{x} dx = ?$$

A. $\sin \sqrt{x} + \cos \sqrt{x} + C$

B. $\frac{1}{2}(\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}) + C$

C. $2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + C$

D. none of these

Answer

To find: Value of $\int \cos \sqrt{x} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x) \int g(x)dx - \int [f'(x) \int g(x)dx]dx$

We have, $I = \int \cos \sqrt{x} dx \dots (i)$

Putting $\sqrt{x}=t$

$$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\Rightarrow dx = 2\sqrt{x} dt$$

$$\Rightarrow dx = 2t dt$$

$$\Rightarrow I = \int \cos t \cdot 2t dt$$

$$\Rightarrow I = 2 \int t \cdot \cos t dt$$

$$\Rightarrow I = 2 \left[t \int \cos t dt - \int \left[\frac{d(t)}{dt} \int \cos t dt \right] dt \right]$$

$$\Rightarrow I = 2 \left[te^t - \int [1 e^t] dt \right]$$

$$\Rightarrow I = 2[te^t - e^t]$$

$$\Rightarrow I = e^t \cdot 2(t-1) + c$$

$$\therefore I = 2 e^{\sqrt{x}} (\sqrt{x}-1) + c$$

Ans) C $2 e^{\sqrt{x}} (\sqrt{x}-1) + c$

16. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cos(\log x) dx = ?$$

A. $\frac{x}{2} [\cos(\log x) - \sin(\log x)] + C$

B. $\frac{x}{2} [\cos(\log x) + \sin(\log x)] + C$

C. $2x [\cos(\log x) + \sin(\log x)] + C$

D. $2x [\cos(\log x) - \sin(\log x)] + C$

Answer

To find: Value of $\int \cos(\log x) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \cos(\log x) dx \dots (i)$

$$I = \int 1 \cdot \cos(\log x) dx$$

Taking $\cos(\log x)$ as first function and 1 as second function.

$$\Rightarrow I = \left[\cos(\log x) \int 1 dx - \int \left[\frac{d[\cos(\log x)]}{dx} \int 1 dx \right] dx \right]$$

$$\Rightarrow I = \left[x \cdot \cos(\log x) - \int \left[-\sin(\log x) \frac{1}{x} x \right] dx \right]$$

$$\Rightarrow I = \left[x \cdot \cos(\log x) + \int [\sin(\log x)] dx \right]$$

$$\Rightarrow I = \left[x \cdot \cos(\log x) + \int [1 \cdot \sin(\log x)] dx \right]$$

$$\Rightarrow I = \left[x \cdot \cos(\log x) + \left\{ \sin(\log x) \int 1 dx - \left(\frac{d \sin(\log x)}{dx} \int 1 \cdot dx \right) dx \right\} \right]$$

$$\Rightarrow I = \left[x \cdot \cos(\log x) + \left\{ x \cdot \sin(\log x) - \left(\cos(\log x) \frac{1}{x} x \right) dx \right\} \right]$$

$$\Rightarrow I = \left[x \cdot \cos(\log x) + \left\{ x \cdot \sin(\log x) - \left(\cos(\log x) \frac{1}{x} x \right) dx \right\} \right]$$

$$\Rightarrow I = [x \cdot \cos(\log x) + \{x \cdot \sin(\log x) - (\cos(\log x)) dx\}]$$

$$\Rightarrow I = [x \cdot \cos(\log x) + x \cdot \sin(\log x) - I]$$

$$\Rightarrow 2I = [x \cdot \cos(\log x) + x \cdot \sin(\log x)]$$

$$\Rightarrow I = \frac{x}{2} [\cos(\log x) + \sin(\log x)] + c$$

$$\text{Ans) } B \frac{x}{2} [\cos(\log x) + \sin(\log x)] + c$$

17. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sec^3 x dx = ?$$

A. $\frac{1}{2} \{ \sec x \tan x - \log |\sec x + \tan x| \} + C$

B. $\frac{1}{2} \{ \sec x \tan x + \log |\sec x + \tan x| \} + C$

C. $2 \{ \sec x \tan x + \log |\sec x + \tan x| \} + C$

D. none of these

Answer

To find: Value of $\int \sec^3 x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \sec^3 x \, dx \dots (i)$

$$I = \int \sec x \sec^2 x \, dx$$

Taking $\sec x$ as first function and $\sec^2 x$ as second function.

$$\Rightarrow I = \left[\sec x \int \sec^2 x \, dx - \int \left[\frac{d[\sec x]}{dx} \int \sec^2 x \, dx \right] dx \right]$$

$$\Rightarrow I = \left[\sec x \tan x - \int [\sec x \tan x \tan x] dx \right]$$

$$\Rightarrow I = \left[\sec x \tan x - \int [\sec x \tan^2 x] dx \right]$$

$$\Rightarrow I = \left[\sec x \tan x - \int [\sec x (\sec^2 x - 1)] dx \right]$$

$$\Rightarrow I = \left[\sec x \tan x - \int (\sec^3 x - \sec x) dx \right]$$

$$\Rightarrow I = \left[\sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx \right]$$

$$\Rightarrow I = [\sec x \tan x - I + \log|\sec x + \tan x| + c]$$

$$\Rightarrow 2I = [\sec x \tan x + \log|\sec x + \tan x| + c]$$

$$\Rightarrow I = \frac{1}{2} [\sec x \tan x + \log|\sec x + \tan x| + c]$$

$$\text{Ans) B } \frac{1}{2} [\sec x \tan x + \log|\sec x + \tan x| + c]$$

18. Question

Mark (✓) against the correct answer in each of the following:

$$\int \left\{ \frac{1}{(\log x)} - \frac{1}{(\log x)^2} \right\} dx = ?$$

A. $x \log x + C$

B. $\frac{x}{\log x} + C$

C. $x + \frac{1}{\log x} + C$

D. none of these

Answer

To find: Value of $\int \left\{ \frac{1}{(\log x)} - \frac{1}{(\log x)^2} \right\} dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \left\{ \frac{1}{(\log x)} - \frac{1}{(\log x)^2} \right\} dx \dots (i)$

Put $t = \log x$

$$e^t = e^{\log x} = x$$

$$\frac{dx}{dt} = e^t$$

$$\Rightarrow dx = e^t dt$$

$$\Rightarrow I = \int \left\{ \frac{1}{t} - \frac{1}{t^2} \right\} dx$$

We know $\int e^x (f(x) + f'(x)) dx = e^x f(x)$

$$\Rightarrow I = \int \left\{ \frac{1}{t} - \frac{1}{t^2} \right\} dx = e^t \frac{1}{t}$$

$$\Rightarrow \frac{x}{\log x} + c$$

Ans) B $\frac{x}{\log x} + c$

19. Question

Mark (✓) against the correct answer in each of the following:

$$\int 2x^3 e^{x^2} dx = ?$$

A. $e^{x^2} (x^2 - 1) + C$

B. $e^{x^2} (x^2 + 1) + C$

C. $e^{x^2} (x + 1) + C$

D. none of these

Answer

To find: Value of $\int \left\{ \frac{1}{(\log x)} - \frac{1}{(\log x)^2} \right\} dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \left\{ \frac{1}{(\log x)} - \frac{1}{(\log x)^2} \right\} dx \dots (i)$

Put $t = \log x$

$$e^t = e^{\log x} = x$$

$$\frac{dx}{dt} = e^t$$

$$\Rightarrow dx = e^t dt$$

$$\Rightarrow I = \int \left\{ \frac{1}{t} - \frac{1}{t^2} \right\} dx$$

$$\text{We know } \int e^x (f(x) + f'(x)) dx = e^x f(x)$$

$$\Rightarrow I = \int \left\{ \frac{1}{t} - \frac{1}{t^2} \right\} dx = e^t \frac{1}{t}$$

$$\Rightarrow \frac{x}{\log x} + c$$

$$\text{Ans) B } \frac{x}{\log x} + c$$

20. Question

Mark (✓) against the correct answer in each of the following:

$$\int (x 2^x) dx = ?$$

$$\text{A. } \frac{2^x}{(\log 2)} (x + \log 2) + C$$

$$\text{B. } \frac{2^x}{(\log 2)^2} (x + \log 2 - 1) + C$$

$$\text{C. } \frac{x \cdot 2^x}{(\log 2)} + \frac{2^x}{(\log 2)^2} + C$$

D. none of these

Answer

To find: Value of $\int (x 2^x) dx$

Formula used:

$$\text{(i) } \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int (x 2^x) dx \dots \text{(i)}$

$$\Rightarrow I = x \int 2^x dx - \int \left(\frac{dx}{dx} \int 2^x dx \right) dx$$

$$\Rightarrow I = x \frac{2^x}{\log 2} - \int \left(\frac{2^x}{\log 2} \right) dx$$

$$\Rightarrow I = x \frac{2^x}{\log 2} - \frac{1}{\log 2} \int 2^x dx$$

$$\Rightarrow I = x \frac{2^x}{\log 2} - \frac{1}{\log 2} \frac{2^x}{\log 2}$$

$$\Rightarrow I = \frac{x \cdot 2^x}{\log 2} - \frac{2^x}{(\log 2)^2} + c$$

$$\Rightarrow I = \frac{2^x}{(\log 2)^2} (x \log 2 - 1) + c$$

Ans) D

21. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \cot^2 x \, dx = ?$$

A. $-x \cot x + \frac{x^2}{2} + \log |\sin x| + C$

B. $-x \cot x - \frac{x^2}{2} + \log |\sin x| + C$

C. $-x \cot x + \frac{x^2}{2} - \log |\sin x| + C$

D. none of these

Answer

To find: Value of $\int x \cot^2 x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \cot^2 x \, dx \dots (i)$

$$\Rightarrow I = x \int \cot^2 x \, dx - \int \left(\frac{dx}{dx} \int \cot^2 x \, dx \right) dx$$

$$\Rightarrow I = x \int (\operatorname{cosec}^2 x - 1) \, dx - \int \left(1 \cdot \int (\operatorname{cosec}^2 x - 1) \, dx \right) dx$$

$$\Rightarrow I = x(-\cot x - x) - \int (-\cot x - x) \, dx$$

$$\Rightarrow I = -x \cot x - x^2 + \log |\sin x| + \frac{x^2}{2}$$

$$\Rightarrow I = -x \cot x - \frac{x^2}{2} + \log |\sin x| + c$$

Ans) B $-x \cot x - \frac{x^2}{2} + \log |\sin x| + c$

22. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin \sqrt{x} \, dx = ?$$

- A. $-\sqrt{x} \cos \sqrt{x} + C$
- B. $-\sqrt{x} \cos \sqrt{x} - 2 \sin \sqrt{x} - C$
- C. $-2\sqrt{x} \cos \sqrt{x} + 2 \sin \sqrt{x} - C$
- D. none of these

Answer

To find: Value of $\int \sin \sqrt{x} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \sin \sqrt{x} dx \dots (i)$

$$\sqrt{x} = t$$

$$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\Rightarrow dx = 2\sqrt{x} dt$$

$$\Rightarrow dx = 2t dt$$

$$I = \int \sin t \cdot 2t dt$$

$$I = 2 \int t \cdot \sin t dt$$

$$\Rightarrow I = 2t \int \sin t dt - \int \left(\frac{dt}{dt} \int \sin t dt \right) dt$$

$$\Rightarrow I = 2t (-\cos t) - \int 1 (-\cos t) dt$$

$$\Rightarrow I = 2t (-\cos t) + \int \cos t dt$$

$$\Rightarrow I = 2t (-\cos t) + \sin t + c$$

$$\Rightarrow I = -2\sqrt{x} \cos \sqrt{x} + \sin \sqrt{x} + c$$

Ans) C $-2\sqrt{x} \cos \sqrt{x} + \sin \sqrt{x} + c$

23. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^{\sin x} \sin 2x dx = ?$$

- A. $(2 \sin x) e^{\sin x} + C$
- B. $(2 \cos x) e^{\sin x} + C$
- C. $2e^{\sin x} (\sin x + 1) + C$
- D. $2e^{\sin x} (\sin x - 1) + C$

Answer

To find: Value of $\int e^{\sin x} \sin 2x dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^{\sin x} \sin 2x \, dx \dots (i)$

$$I = \int e^{\sin x} 2 \sin x \cos x \, dx$$

Put $\sin x = t$

$$\cos x = \frac{dt}{dx}$$

$$\Rightarrow \cos x \, dx = dt$$

$$I = 2 \int e^t \cdot t \cdot dt$$

$$\Rightarrow I = 2 \left[t \int e^t \, dt - \int \left(\frac{dt}{dt} \int e^t \, dt \right) dt \right]$$

$$\Rightarrow I = 2 \left[t e^t - \int 1 \cdot e^t \, dt \right]$$

$$\Rightarrow I = 2t e^t - 2e^t + c$$

$$\Rightarrow I = 2e^t (t-1) + c$$

$$\Rightarrow I = 2e^{\sin x} (\sin x - 1) + c$$

$$\text{Ans) D } 2e^{\sin x} (\sin x - 1) + c$$

24. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx = ?$$

A. $\frac{\sin^{-1} x}{\sqrt{1-x^2}} - \frac{1}{2} \log |1-x^2| + C$

B. $x \sin^{-1} x + \frac{1}{2} \log |1-x^2| + C$

C. $\frac{\sin^{-1} x}{\sqrt{1-x^2}} + \frac{1}{2} \log |1-x^2| + C$

D. none of these

Answer

To find: Value of $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx \dots (i)$

$$I = \int \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$$

$$I = \int \frac{\sin^{-1} x}{\sqrt{1-x^2} (1-x^2)} dx$$

Putting $\sin^{-1} x = t$, $x = \sin t$

$$\Rightarrow \cos t = \sqrt{1-x^2}$$

$$\Rightarrow \tan t = \frac{x}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$I = \int \frac{t}{(1-\sin^2 t)} dt$$

$$I = \int \frac{t}{\cos^2 t} dt$$

$$I = \int t \cdot \sec^2 t \, dt$$

$$\Rightarrow I = \left[t \int \sec^2 t \, dt - \int \left(\frac{dt}{dt} \int \sec^2 t \, dt \right) dt \right]$$

$$\Rightarrow I = \left[t \tan t - \int 1 \tan t \, dt \right]$$

$$\Rightarrow I = \left[\sin^{-1} x \frac{x}{\sqrt{1-x^2}} - \log |\sqrt{1-x^2}| + c \right]$$

$$\Rightarrow I = [t \tan t - \log |\cos t| + c]$$

$$\Rightarrow I = 2t e^t - 2e^t + c$$

$$\Rightarrow I = 2e^t (t-1) + c$$

$$\Rightarrow I = 2e^{\sin x} (\sin x - 1) + c$$

Ans) D $2e^{\sin x} (\sin x - 1) + c$

25. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x \tan^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx = ?$$

A. $\frac{\tan^{-1} x}{\sqrt{1+x^2}} - \frac{x}{\sqrt{1+x^2}} + C$

B. $\frac{-\tan^{-1}x}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + C$

C. $\frac{x \tan^{-1}x}{\sqrt{1+x^2}} + \frac{1}{2} \log \left| \frac{x}{\sqrt{1+x^2}} \right| + C$

D. none of these

Answer

To find: Value of $\int \frac{x \tan^{-1}x}{(1+x^2)^{\frac{3}{2}}} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{x \tan^{-1}x}{(1+x^2)^{\frac{3}{2}}} dx \dots (i)$

$$I = \int \frac{x \tan^{-1}x}{\sqrt{1+x^2} (1+x^2)} dx$$

Putting $\tan^{-1}x = t$, $x = \tan t$

$$dx = \sec^2 t dt$$

When $x = \tan t$

$$\Rightarrow 1+x^2 = 1+\tan^2 t$$

$$\Rightarrow 1+x^2 = \sec^2 t$$

$$\Rightarrow \sqrt{1+x^2} = \sec t$$

$$\Rightarrow \sqrt{1+x^2} = \sec t$$

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \cos t$$

$$\Rightarrow \frac{1}{1+x^2} = \cos^2 t$$

$$\Rightarrow 1 - \frac{1}{1+x^2} = 1 - \cos^2 t$$

$$\Rightarrow \frac{1+x^2-1}{1+x^2} = \sin^2 t$$

$$\Rightarrow \frac{x}{\sqrt{1+x^2}} = \sin t$$

$$I = \int \frac{\tan t}{\sec t \sec^2 t} \sec^2 t dt$$

$$I = \int t \sin t dt$$

Taking 1st function as **t** and second function as **sint**

$$\Rightarrow I = \left[t \int \sin t \, dt - \int \left(\frac{dt}{dt} \int \sin t \, dt \right) dt \right]$$

$$\Rightarrow I = \left[t(-\cos t) - \int (1(-\cos t)) dt \right]$$

$$\Rightarrow I = \left[t(-\cos t) + \int \cos t \, dt \right]$$

$$\Rightarrow I = -t \cos t + \sin t + c$$

$$\Rightarrow I = -\tan^{-1} x \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

$$\Rightarrow I = \frac{-\tan^{-1} x \cdot 1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

Ans) B $\frac{-\tan^{-1} x \cdot 1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$

26. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \tan^{-1} x \, dx = ?$$

A. $\frac{1}{2} \tan^{-1} x + \log(1+x^2) - \frac{1}{2} x + C$

B. $\frac{1}{2} x^2 \tan^{-1} x + \frac{1}{2} x + C$

C. $\frac{1}{2} (1+x^2) \tan^{-1} x - \frac{1}{2} x + C$

D. none of these

Answer

To find: Value of $\int x \tan^{-1} x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \tan^{-1} x \, dx \dots (i)$

Taking 1st function as $\tan^{-1} x$ and second function as x

$$\Rightarrow I = \left[\tan^{-1} x \int x \, dx - \int \left(\frac{d(\tan^{-1} x)}{dx} \int x \, dx \right) dx \right]$$

$$\Rightarrow I = \left[\tan^{-1} x \frac{x^2}{2} - \int \left(\frac{1}{1+x^2} \frac{x^2}{2} \right) dx \right]$$

$$\Rightarrow I = \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(\frac{x^2+1-1}{1+x^2} \right) dx \right]$$

$$\Rightarrow I = \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \left[\int 1 dx - \int \frac{1}{1+x^2} dx \right] \right]$$

$$\Rightarrow I = \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} [x - \tan^{-1} x] \right] + c$$

$$\Rightarrow I = \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x \right] + c$$

$$\Rightarrow I = \frac{1}{2} (1+x^2) \tan^{-1} x - \frac{1}{2} x + c$$

$$\text{Ans) } C \frac{1}{2} (1+x^2) \tan^{-1} x - \frac{1}{2} x + c$$

27. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1} \sqrt{x} dx = ?$$

A. $(x-1) \tan^{-1} \sqrt{x} + \sqrt{x} + C$

B. $(x+1) \tan^{-1} \sqrt{x} - \sqrt{x} + C$

C. $\frac{1}{2} \sqrt{x} \tan^{-1} \sqrt{x} - \frac{1}{2} \sqrt{x} + C$

D. none of these

Answer

To find: Value of $\int \tan^{-1} \sqrt{x} dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \tan^{-1} \sqrt{x} dx \dots (i)$

Let $\sqrt{x} = t$,

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow dx = 2t dt$$

$$I = \int \tan^{-1} \sqrt{x} dx$$

$$\Rightarrow I = \int \tan^{-1} t \cdot 2t dt$$

$$\Rightarrow I = 2 \int \tan^{-1} t \cdot t dt$$

Taking 1st function as $\tan^{-1} t$ and second function as t

$$\Rightarrow I = 2 \left[\tan^{-1} t \int t \, dt - \int \left(\frac{d(\tan^{-1} t)}{dt} \int t \, dt \right) dt \right]$$

$$\Rightarrow I = 2 \left[\tan^{-1} t \frac{t^2}{2} - \int \left(\frac{1}{1+t^2} \frac{t^2}{2} \right) dt \right]$$

$$\Rightarrow I = 2 \left[\frac{t^2}{2} \tan^{-1} t - \frac{1}{2} \int \left(\frac{t^2+1-1}{1+t^2} \right) dt \right]$$

$$\Rightarrow I = 2 \left[\frac{t^2}{2} \tan^{-1} t - \frac{1}{2} \left[\int 1 \, dt - \int \frac{1}{1+t^2} dt \right] \right]$$

$$\Rightarrow I = 2 \left[\frac{t^2}{2} \tan^{-1} t - \frac{1}{2} [t - \tan^{-1} t] \right] + c$$

$$\Rightarrow I = 2 \left[\frac{x}{2} \tan^{-1} \sqrt{x} - \frac{1}{2} \sqrt{x} + \frac{1}{2} \tan^{-1} \sqrt{x} \right] + c$$

$$\Rightarrow I = x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + c$$

$$\Rightarrow I = (x+1) \tan^{-1} \sqrt{x} - \sqrt{x} + c$$

Ans) B $(x+1) \tan^{-1} \sqrt{x} - \sqrt{x} + c$

28. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cos^{-1} x \, dx = ?$$

A. $x \cos^{-1} x - \sqrt{1-x^2} + C$

B. $x \cos^{-1} x + \sqrt{1-x^2} + C$

C. $x \sin^{-1} x - \sqrt{1-x^2} + C$

D. none of these

Answer

To find: Value of $\int \cos^{-1} x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \cos^{-1} x \, dx \dots (i)$

Let $\cos^{-1} x = \theta$, $\Rightarrow x = \cos \theta$

$$\Rightarrow dx = -\sin \theta \, d\theta$$

If $x = \cos \theta$,

Then $\sqrt{1-x^2} = \sin \theta$

$$I = \int \cos^{-1} x \, dx$$

$$\Rightarrow I = - \int \theta \sin \theta \, d\theta$$

Taking 1st function as θ and second function as $\sin \theta$

$$\Rightarrow I = - \left[\theta \int \sin \theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sin \theta \, d\theta \right) d\theta \right]$$

$$\Rightarrow I = - \left[\theta(-\cos \theta) - \int (-\cos \theta) d\theta \right] + c$$

$$\Rightarrow I = -[\theta(-\cos \theta) - (-\sin \theta)] + c$$

$$\Rightarrow I = -[\theta(-\cos \theta) + \sin \theta] + c$$

$$\Rightarrow I = \theta \cos \theta - \sin \theta + c$$

$$\Rightarrow I = x \cdot \cos^{-1} x - \sqrt{1-x^2} + c$$

$$\text{Ans) } A \, x \cdot \cos^{-1} x - \sqrt{1-x^2} + c$$

29. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1} x \, dx = ?$$

A. $x \tan^{-1} x + \frac{1}{2} \log |1+x^2| + C$

B. $x \tan^{-1} x - \frac{1}{2} \log |1+x^2| + C$

C. $-x \tan^{-1} x + \frac{1}{2} \log |1+x^2| + C$

D. none of these

Answer

To find: Value of $\int \tan^{-1} x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \tan^{-1} x \, dx \dots (i)$

Let $\tan^{-1} x = \theta$, $\Rightarrow x = \tan \theta$

$$\Rightarrow dx = \sec^2 \theta \, d\theta$$

If $x = \tan \theta$,

$$\text{Then } 1 + x^2 = \sec^2 \theta$$

$$\Rightarrow \theta = \sec^{-1} \sqrt{1+x^2}$$

$$I = \int \tan^{-1} x \, dx$$

$$\Rightarrow I = \int \theta \sec^2 \theta \, d\theta$$

Taking 1st function as θ and second function as $\sec^2 \theta$

$$\Rightarrow I = \left[\theta \int \sec^2 \theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec^2 \theta \, d\theta \right) d\theta \right]$$

$$\Rightarrow I = \left[\theta(\tan\theta) - \int (1(\tan\theta)) d\theta \right] + c$$

$$\Rightarrow I = [\theta(\tan\theta) - (\log|\sec\theta|)] + c$$

$$\Rightarrow I = [\tan^{-1}x(x) - \log|\sec(\sec^{-1}\sqrt{1+x^2})|] + c$$

$$\Rightarrow I = [x \cdot \tan^{-1}x - (\log|\sqrt{1+x^2}|)] + c$$

$$\Rightarrow I = x \cdot \tan^{-1}x - \frac{1}{2} \log|1+x^2| + c$$

$$\text{Ans) } B \, x \cdot \tan^{-1}x - \frac{1}{2} \log|1+x^2| + c$$

30. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sec^{-1}x \, dx = ?$$

A. $x \sec^{-1}x + \log|x + \sqrt{x^2 - 1}| + C$

B. $x \sec^{-1}x - \log|x + \sqrt{x^2 - 1}| + C$

C. $x \sec^{-1}x + \log|x - \sqrt{x^2 - 1}| + C$

D. none of these

Answer

To find: Value of $\int \sec^{-1}x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

$$\text{We have, } I = \int \sec^{-1}x \, dx \dots (i)$$

$$\text{Let } \sec^{-1}x = \theta, \Rightarrow x = \sec\theta$$

$$\Rightarrow dx = \sec\theta \tan\theta \, d\theta$$

$$\text{If } x = \sec\theta,$$

$$\text{Then } \sqrt{x^2 - 1} = \tan\theta$$

$$I = \int \sec^{-1}x \, dx$$

$$\Rightarrow I = \int \theta \sec\theta \tan\theta \, d\theta$$

Taking 1st function as θ and second function as $\sec\theta \tan\theta$

$$\Rightarrow I = \left[\theta \int \sec\theta \tan\theta d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec\theta \tan\theta d\theta \right) d\theta \right]$$

$$\Rightarrow I = \left[\theta(\sec\theta) - \int (1(\sec\theta)) d\theta \right] + c$$

$$\Rightarrow I = [\theta(\sec\theta) - (\log|\sec\theta + \tan\theta|)] + c$$

$$\Rightarrow I = [\sec^{-1}x(x) - (\log|x + \sqrt{x^2 - 1}|)] + c$$

$$\Rightarrow I = x \cdot \sec^{-1}x - \log|x + \sqrt{x^2 - 1}| + c$$

$$\text{Ans) } B \ x \cdot \sec^{-1}x - \log|x + \sqrt{x^2 - 1}| + c$$

31. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin^{-1}(3x - 4x^3) dx = ?$$

A. $3 \left[x \sin^{-1}x + \sqrt{1 - x^2} \right] + C$

B. $3 \left[x \sin^{-1}x - \sqrt{1 - x^2} \right] + C$

C. $\frac{3x^2}{2} + C$

D. none of these

Answer

To find: Value of $\int \sin^{-1}(3x - 4x^3) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

$$\text{We have, } I = \int \sin^{-1}(3x - 4x^3) dx \dots (i)$$

$$\text{Let } x = \sin\theta, \Rightarrow \theta = \sin^{-1}x$$

$$\Rightarrow dx = \cos\theta d\theta$$

$$\text{If } x = \sin\theta,$$

$$\text{Then } \sqrt{1 - x^2} = \cos\theta$$

$$I = \int \sin^{-1}(3x - 4x^3) dx$$

$$\Rightarrow I = \int \sin^{-1}(3\sin\theta - 4\sin^3\theta) \cos\theta d\theta$$

$$\Rightarrow I = \int \sin^{-1}(\sin 3\theta) \cos\theta d\theta$$

$$\Rightarrow I = \int 3\theta \cos \theta \, d\theta$$

$$\Rightarrow I = 3 \int \theta \cos \theta \, d\theta$$

Taking 1st function as θ and second function as $\cos \theta$

$$\Rightarrow I = 3 \left[\theta \int \cos \theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \cos \theta \, d\theta \right) d\theta \right]$$

$$\Rightarrow I = 3 \left[\theta (\sin \theta) - \int (1 (\sin \theta)) d\theta \right]$$

$$\Rightarrow I = 3[\theta(\sin \theta) - (-\cos \theta)] + c$$

$$\Rightarrow I = 3[\theta(\sin \theta) + \cos \theta] + c$$

$$\Rightarrow I = 3 \sin^{-1} x (x) + 3\sqrt{1-x^2} + c$$

$$\Rightarrow I = 3 x \sin^{-1} x + 3\sqrt{1-x^2} + c$$

$$\Rightarrow I = 3 \left[x \sin^{-1} x + \sqrt{1-x^2} \right] + c$$

$$\text{Ans) A } 3 \left[x \sin^{-1} x + \sqrt{1-x^2} \right] + c$$

32. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx = ?$$

A. $2x \tan^{-1} x + \log |1 + x^2| + C$

B. $2x \tan^{-1} x - \log |1 + x^2| + C$

C. $2x \sin^{-1} x + \log |1 + x^2| + C$

D. none of these

Answer

To find: Value of $\int \sin^{-1} \frac{2x}{1+x^2} dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \sin^{-1} \frac{2x}{1+x^2} dx \dots (i)$

Let $x = \tan \theta$, $\Rightarrow \theta = \tan^{-1} x$

$$\Rightarrow dx = \sec^2 \theta \, d\theta$$

If $x = \tan \theta$,

Then $1 + x^2 = \sec^2 \theta$

$$\Rightarrow \theta = \sec^{-1} \sqrt{1+x^2}$$

$$I = \int \sin^{-1} \frac{2x}{1+x^2} dx$$

$$\Rightarrow I = \int \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \sec^2 \theta d\theta$$

$$\Rightarrow I = \int \sin^{-1}(\sin 2\theta) \sec^2 \theta d\theta$$

$$\Rightarrow I = \int 2\theta \sec^2 \theta d\theta$$

$$\Rightarrow I = 2 \int \theta \sec^2 \theta d\theta$$

Taking 1st function as θ and second function as $\sec^2 \theta$

$$\Rightarrow I = 2 \left[\theta \int \sec^2 \theta d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec^2 \theta d\theta \right) d\theta \right]$$

$$\Rightarrow I = 2 \left[\theta(\tan \theta) - \int (1(\tan \theta)) d\theta \right]$$

$$\Rightarrow I = 2[\theta(\tan \theta) - (\log(\sec \theta))] + c$$

$$\Rightarrow I = 2 \left[\tan^{-1} x(x) - (\log(\sec(\sec^{-1} \sqrt{1+x^2}))) \right] + c$$

$$\Rightarrow I = 2 \left[\tan^{-1} x(x) - (\log \sqrt{1+x^2}) \right] + c$$

$$\Rightarrow I = 2 \left[x \cdot \tan^{-1} x - \frac{1}{2}(\log 1+x^2) \right] + c$$

$$\Rightarrow I = 2x \cdot \tan^{-1} x - (\log 1+x^2) + c$$

$$\text{Ans) } 2x \cdot \tan^{-1} x - (\log 1+x^2) + c$$

33. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx = ?$$

A. $\frac{1}{2}x(\cos^{-1} x) + \frac{1}{2}\sqrt{1-x^2} + C$

B. $\frac{1}{2}x(\sin^{-1} x) + \frac{1}{2}\sqrt{1-x^2} + C$

C. $\frac{1}{2}x(\cos^{-1} x) - \frac{1}{2}\sqrt{1-x^2} + C$

D. none of these

Answer

To find: Value of $\int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx \dots (i)$

Let $x = \cos\theta$, $\Rightarrow \theta = \cos^{-1}x$

$\Rightarrow dx = -\sin\theta d\theta$

If $x = \cos\theta$,

Then $\sqrt{1-x^2} = \sin\theta$

$$I = \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$$

$$\Rightarrow I = \int \tan^{-1} \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \cdot -\sin\theta d\theta$$

$$\Rightarrow I = \int \tan^{-1} \sqrt{\frac{2\sin^2 \frac{\theta}{2}}{2\cos^2 \frac{\theta}{2}}} \cdot -\sin\theta d\theta$$

$$\Rightarrow I = \int \tan^{-1} \sqrt{\tan^2 \frac{\theta}{2}} \cdot -\sin\theta d\theta$$

$$\Rightarrow I = \int \tan^{-1} \left(\tan \frac{\theta}{2} \right) \cdot -\sin\theta d\theta$$

$$\Rightarrow I = \int \frac{\theta}{2} \cdot -\sin\theta d\theta$$

$$\Rightarrow I = -\frac{1}{2} \int \theta \cdot \sin\theta d\theta$$

Taking 1st function as θ and second function as $\sin\theta$

$$\Rightarrow I = -\frac{1}{2} \left[\theta \int \sin\theta d\theta - \int \left(\frac{d\theta}{d\theta} \int \sin\theta d\theta \right) d\theta \right]$$

$$\Rightarrow I = -\frac{1}{2} \left[\theta(-\cos\theta) - \int (1(-\cos\theta)) d\theta \right]$$

$$\Rightarrow I = -\frac{1}{2} \left[\theta(-\cos\theta) + \int (\cos\theta) d\theta \right]$$

$$\Rightarrow I = -\frac{1}{2} [\theta(-\cos\theta) + \sin\theta] + c$$

$$\Rightarrow I = \frac{1}{2} \cos^{-1} x (x) - \frac{1}{2} \sqrt{1-x^2} + c$$

$$\Rightarrow I = \frac{1}{2} x \cdot \cos^{-1} x - \frac{1}{2} \sqrt{1-x^2} + c$$

Ans) C $\frac{1}{2}x \cdot \cos^{-1} x - \frac{1}{2}\sqrt{1-x^2} + c$

34. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) dx = ?$$

A. $3x \tan^{-1} x + \frac{3}{2} \log(1 + x^2) + C$

B. $3x \tan^{-1} x - \frac{3}{2} \log(1 + x^2) + C$

C. $3x \cos^{-1} x - \frac{3}{2} \sqrt{1 - x^2} + C$

D. $3x \sin^{-1} x + \frac{3}{2} \sqrt{1 - x^2} + C$

Answer

To find: Value of $\int \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) dx \dots (i)$

Let $x = \tan \theta$, $\Rightarrow \theta = \tan^{-1} x$

$$\Rightarrow dx = \sec^2 \theta d\theta$$

If $x = \tan \theta$,

$$\text{Then } 1 + x^2 = \sec^2 \theta$$

$$\Rightarrow \theta = \sec^{-1} \sqrt{1 + x^2}$$

$$I = \int \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) dx$$

$$\Rightarrow I = \int \tan^{-1} \left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) \sec^2 \theta d\theta$$

$$\Rightarrow I = \int \tan^{-1}(\tan 3\theta) \sec^2 \theta d\theta$$

$$\Rightarrow I = \int 3\theta \sec^2 \theta d\theta$$

$$\Rightarrow I = 3 \int \theta \sec^2 \theta d\theta$$

Taking 1st function as θ and second function as $\sec^2 \theta$

$$\Rightarrow I = 3 \left[\theta \int \sec^2 \theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec^2 \theta \, d\theta \right) d\theta \right]$$

$$\Rightarrow I = 3 \left[\theta \tan \theta - \int (\tan \theta) d\theta \right]$$

$$\Rightarrow I = 3[\theta \tan \theta - (\log \sec \theta)] + c$$

$$\Rightarrow I = 3\theta \tan \theta - 3 \log(\sec \theta) + c$$

$$\Rightarrow I = 3 \tan^{-1} x \tan(\tan^{-1} x) - 3 \log \{ \sec(\sec^{-1} \sqrt{1+x^2}) \} + c$$

$$\Rightarrow I = 3x \cdot \tan^{-1} x - 3 \log \{ \sqrt{1+x^2} \} + c$$

$$\Rightarrow I = 3x \cdot \tan^{-1} x - \frac{3}{2} \log \{ 1+x^2 \} + c$$

$$\text{Ans) B } 3x \cdot \tan^{-1} x - \frac{3}{2} \log \{ 1+x^2 \} + c$$

35. Question

Mark (✓) against the correct answer in each of the following:

$$\int x^2 \cos x \, dx = ?$$

A. $x^2 \sin x + 2x \cos x - 2 \sin x + C$

B. $2x \cos x - x \sin x + 2 \sin x + C$

C. $x^2 \sin x - 2x \sin x + 2 \sin x + C$

D. none of these

Answer

To find: Value of $\int x^2 \cos x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

$$\text{We have, } I = \int x^2 \cos x \, dx \dots (i)$$

Taking 1st function as x^2 and second function as $\cos x$

$$\Rightarrow I = \left[x^2 \int \cos x \, dx - \int \left(\frac{dx^2}{dx} \int \cos x \, dx \right) dx \right]$$

$$\Rightarrow I = \left[x^2 \sin x - \int (2x \sin x) dx \right]$$

$$\Rightarrow I = \left[x^2 \sin x - 2 \int (x \sin x) dx \right]$$

Taking 1st function as x and second function as $\sin x$

$$\Rightarrow I = x^2 \sin x - 2 \left[x \int \sin x \, dx - \int \left(\frac{dx}{dx} \int \sin x \, dx \right) dx \right]$$

$$\Rightarrow I = x^2 \sin x - 2 \left[x(-\cos x) - \int (1(-\cos x) dx) \right]$$

$$\Rightarrow I = x^2 \sin x - 2[x(-\cos x) - (-\sin x)] + c$$

$$\Rightarrow I = x^2 \sin x - 2[x(-\cos x) + \sin x] + c$$

$$\Rightarrow I = x^2 \sin x + 2x \cos x - 2 \sin x + c$$

$$\text{Ans) A } x^2 \sin x + 2x \cos x - 2 \sin x + c$$

36. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin x \log(\cos x) dx = ?$$

A. $\cos x \log(\cos x) - \cos x + C$

B. $-\cos x \log(\cos x) + \cos x + C$

C. $\cos x \log(\cos x) + \cos x + C$

D. none of these

Answer

To find: Value of $\int \sin x \log(\cos x) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \sin x \log(\cos x) dx \dots (i)$

Let $\cos x = t$

$-\sin x dx = dt$

$$I = \int \sin x \log(\cos x) dx$$

$$I = - \int \log t dt$$

$$I = - \int \log t \cdot 1 \cdot dt$$

Taking 1st function as $\log t$ and second function as 1

$$\Rightarrow I = - \left[\log t \int 1 dt - \int \left(\frac{d \log t}{dt} \int 1 dt \right) dt \right]$$

$$\Rightarrow I = - \left[\log t \cdot t - \int \left(\frac{1}{t} t \right) dt \right]$$

$$\Rightarrow I = - \left[\log t \cdot t - \int 1 dt \right]$$

$$\Rightarrow I = -[\log t \cdot t - t] + c$$

$$\Rightarrow I = -\log t \cdot t + t + c$$

$$\Rightarrow I = -\cos x \cdot \log(\cos x) + \cos x + c$$

$$\text{Ans) B } -\cos x \cdot \log(\cos x) + \cos x + c$$

37. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \sin x \cos x \, dx = ?$$

A. $-\frac{1}{4}x \cos 2x + \frac{1}{8} \sin 2x + C$

B. $\frac{1}{4}x \cos 2x + \frac{1}{8} \sin 2x + C$

C. $\frac{1}{4}x \cos 2x - \frac{1}{8} \sin 2x + C$

D. none of these

Answer

To find: Value of $\int x \sin x \cos x \, dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int x \sin x \cos x \, dx \dots (i)$

$$I = \frac{1}{2} \int x 2 \sin x \cos x \, dx$$

$$I = \frac{1}{2} \int x \sin 2x \, dx$$

Let $2x = t$

$$2dx = dt$$

$$dx = \frac{dt}{2}$$

$$I = \frac{1}{2} \int \frac{t}{2} \sin t \frac{dt}{2}$$

$$I = \frac{1}{8} \int t \sin t \, dt$$

Taking 1st function as t and second function as $\sin t$

$$\Rightarrow I = \frac{1}{8} \left[t \int \sin t \, dt - \int \left(\frac{dt}{dt} \int \sin t \, dt \right) dt \right]$$

$$\Rightarrow I = \frac{1}{8} \left[t \cdot (-\cos t) - \int (-\cos t) \, dt \right]$$

$$\Rightarrow I = \frac{1}{8} [-t \cdot \cos t - (-\sin t)] + c$$

$$\Rightarrow I = \frac{1}{8} [-t \cdot \cos t + \sin t] + c$$

$$\Rightarrow I = -\frac{1}{8} 2x \cdot \cos 2x + \frac{1}{8} \sin 2x + c$$

$$\Rightarrow I = -\frac{1}{4} x \cdot \cos 2x + \frac{1}{8} \sin 2x + c$$

Ans) $A -\frac{1}{4}x \cdot \cos 2x + \frac{1}{8} \sin 2x + c$

38. Question

Mark (✓) against the correct answer in each of the following:

$$\int x^3 \cos x^2 dx = ?$$

A. $x^2 \sin x^2 + \cos x^2 + C$

B. $\frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + C$

C. $-\frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + C$

D. none of these

Answer

To find: Value of $\int x^3 \cos x^2 dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x^3 \cos x^2 dx \dots (i)$

Let $x^2 = t$

$$\Rightarrow x dx = \frac{1}{2} dt$$

$$I = \int x^3 \cos x^2 dx$$

$$I = \int x \cdot x^2 \cos x^2 dx$$

$$I = \int t \cos t \cdot \frac{1}{2} dt$$

$$I = \frac{1}{2} \int t \cos t dt$$

Taking 1st function as t and second function as $\cos t$

$$\Rightarrow I = \frac{1}{2} \left[t \int \cos t dt - \int \left(\frac{dt}{dt} \int \cos t dt \right) dt \right]$$

$$\Rightarrow I = \frac{1}{2} \left[t \cdot \sin t - \int \sin t dt \right]$$

$$\Rightarrow I = \frac{1}{2} [t \cdot \sin t - (-\cos t) + c]$$

$$\Rightarrow I = \frac{1}{2} [t \cdot \sin t + \cos t + c]$$

$$\Rightarrow I = \frac{1}{2} x^2 \cdot \sin x^2 + \frac{1}{2} \cos x^2 + c$$

Ans) B $\frac{1}{2}x^2 \cdot \sin x^2 + \frac{1}{2}\cos x^2 + c$

39. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx = ?$$

- A. $2x \tan^{-1} x + \log(1 + x^2) + C$
- B. $-2x \tan^{-1} x - 2 \log(1 + x^2) + C$
- C. $2x \tan^{-1} x - \log(1 + x^2) + C$
- D. none of these

Answer

To find: Value of $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx \dots (i)$

Let $x = \tan t$, $t = \tan^{-1}x$

$\Rightarrow dx = \sec^2 t dt$

If $\tan t = x$,

$\sec t = 1 + x^2$

$$I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$$

$$I = \int \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right) \sec^2 t dt$$

$$I = \int \cos^{-1}(\cos 2t) \sec^2 t dt$$

$$I = \int 2t \sec^2 t dt$$

$$I = 2 \int t \sec^2 t dt$$

Taking 1st function as t and second function as $\sec^2 t$

$$\Rightarrow I = 2 \left[t \int \sec^2 t dt - \int \left(\frac{dt}{dt} \int \sec^2 t dt \right) dt \right]$$

$$\Rightarrow I = 2 \left[t \tan t - \int \tan t dt \right]$$

$$\Rightarrow I = 2[t \tan t - \log|\sec t| + c]$$

$$\Rightarrow I = 2[\tan^{-1} x \cdot x - \log|1+x^2|] + c$$

$$\Rightarrow I = 2x \tan^{-1} x - 2 \log|1+x^2| + c$$

Ans) D None of these

40. Question

Mark (✓) against the correct answer in each of the following:

$$\int x \tan^{-1} x \, dx = ?$$

A. $\frac{1}{2}(x^2 + 1) \tan^{-1} x - \frac{1}{2}x + C$

B. $\frac{1}{2}(x^2 - 1) \tan^{-1} x - \frac{1}{2}x + C$

C. $\frac{1}{2}(x^2 + 1) \tan^{-1} x + \frac{1}{2}x + C$

D. none of these

Answer

To find: Value of $\int x \tan^{-1} x \, dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int x \tan^{-1} x \, dx \dots (i)$

Taking 1st function as $\tan^{-1} x$ and second function as x

$$\Rightarrow I = \left[\tan^{-1} x \int x \, dx - \int \left(\frac{d \tan^{-1} x}{dx} \int x \, dx \right) dx \right]$$

$$\Rightarrow I = \left[\tan^{-1} x \frac{x^2}{2} - \int \left(\frac{1}{(1+x^2)} \frac{x^2}{2} \right) dx \right]$$

$$\Rightarrow I = \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(\frac{x^2}{(1+x^2)} \right) dx \right]$$

$$\Rightarrow I = \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{(1+x^2)} \right) dx \right]$$

$$\Rightarrow I = \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int 1 \cdot dx + \frac{1}{2} \int \frac{1}{(1+x^2)} \cdot dx \right]$$

$$\Rightarrow I = \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2}x + \frac{1}{2} \tan^{-1} x \right] + c$$

$$\Rightarrow I = \left[\frac{1}{2}(x^2 + 1) \tan^{-1} x - \frac{1}{2}x \right] + c$$

$$\Rightarrow I = \frac{1}{2}(x^2+1) \tan^{-1} x - \frac{1}{2}x + c$$

Ans) A $\frac{1}{2}(x^2+1) \tan^{-1} x - \frac{1}{2}x + c$

41. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin(\log x) dx = ?$$

A. $\frac{1}{2}x \sin \log x + \frac{1}{2}x \cos(\log x) + C$

B. $\frac{1}{2}x \sin \log x - \frac{1}{2}x \cos(\log x) + C$

C. $-\frac{1}{2}x \sin \log x + \frac{1}{2}x \cos(\log x) + C$

D. none of these

Answer

To find: Value of $\int \sin(\log x) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int \sin(\log x) dx \dots (i)$

$$I = \int \sin(\log x) \cdot 1 \cdot dx$$

Taking 1st function as $\sin(\log x)$ and second function as 1

$$\Rightarrow I = \left[\sin(\log x) \int 1 dx - \int \left(\frac{d \sin(\log x)}{dx} \int 1 dx \right) dx \right]$$

$$\Rightarrow I = \left[\sin(\log x) \cdot x - \int \frac{\cos(\log x) \cdot x}{x} dx \right]$$

$$\Rightarrow I = \left[\sin(\log x) \cdot x - \int \cos(\log x) dx \right]$$

Taking 1st function as $\cos(\log x)$ and second function as 1

$$\Rightarrow I = \sin(\log x) \cdot x - \left[\cos(\log x) \int 1 dx - \int \left(\frac{d \cos(\log x)}{dx} \int 1 dx \right) dx \right]$$

$$\Rightarrow I = \sin(\log x) \cdot x - \left[\cos(\log x) \cdot x - \int -\frac{\sin(\log x) \cdot x}{x} dx \right]$$

$$\Rightarrow I = \sin(\log x) \cdot x - \left[\cos(\log x) \cdot x + \int \sin(\log x) dx \right]$$

$$\Rightarrow I = \sin(\log x) \cdot x - [\cos(\log x) \cdot x + I] + c$$

$$\Rightarrow I = \sin(\log x) \cdot x - \cos(\log x) \cdot x - I + c$$

$$\Rightarrow 2I = \sin(\log x) \cdot x - \cos(\log x) \cdot x + c$$

$$\Rightarrow I = \frac{\sin(\log x) \cdot x - \cos(\log x) \cdot x}{2} + c$$

$$\Rightarrow I = \frac{1}{2} x \cdot \sin(\log x) - x \cdot \frac{1}{2} \cos(\log x) + c$$

Ans) B $\frac{1}{2} x \cdot \sin(\log x) - x \cdot \frac{1}{2} \cos(\log x) + c$

42. Question

Mark (✓) against the correct answer in each of the following:

$$\int (\sin^{-1} x)^2 dx = ?$$

A. $\frac{2 \sin^{-1} x}{\sqrt{1-x^2}} + C$

B. $\frac{1}{3} (\sin^{-1} x)^3 + \frac{1}{\sqrt{1-x^2}} + C$

C. $x (\sin^{-1} x)^2 + (\sin^{-1} x) \sqrt{1-x^2} + 2x + C$

D. $x (\sin^{-1} x)^2 + 2 (\sin^{-1} x) \sqrt{1-x^2} - 2x + C$

Answer

To find: Value of $\int (\sin^{-1} x)^2 dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int (\sin^{-1} x)^2 dx \dots (i)$

Putting $\sin t = x$, $\Rightarrow t = \sin^{-1} x$

$$\Rightarrow dx = \cos t dt$$

When $x = \sin t$ then $\sqrt{1-x^2} = \cos t$

$$I = \int (\sin^{-1} x)^2 dx$$

$$\Rightarrow I = \int (\sin^{-1}(\sin t))^2 \cos t dt$$

$$\Rightarrow I = \int t^2 \cos t dt$$

Taking 1st function as t^2 and second function as $\cos t$

$$\Rightarrow I = \left[t^2 \int \cos t dt - \int \left(\frac{dt^2}{dt} \int \cos t dt \right) dt \right]$$

$$\Rightarrow I = \left[t^2 \sin t - \int (2t \sin t) dt \right]$$

$$\Rightarrow I = \left[t^2 \sin t - 2 \int (t \sin t) dt \right]$$

Taking 1st function as t and second function as $\sin t$

$$\Rightarrow I = t^2 \sin t - 2 \left[\int (t \sin t) dt \right]$$

$$\Rightarrow I = t^2 \sin t - 2 \left[t \int \sin t \, dt - \int \left(\frac{dt}{dt} \int \sin t \, dt \right) dt \right]$$

$$\Rightarrow I = t^2 \sin t - 2 \left[t(-\cos t) - \int (-\cos t) dt \right]$$

$$\Rightarrow I = t^2 \sin t - 2[-t \cos t - (-\sin t) + c]$$

$$\Rightarrow I = t^2 \sin t + 2t \cos t - 2 \sin t + c$$

$$\Rightarrow I = x (\sin^{-1} x)^2 + 2 \sin^{-1} x \sqrt{1-x^2} - 2x + c$$

$$\text{Ans) } D \times (\sin^{-1} x)^2 + 2 \sin^{-1} x \sqrt{1-x^2} - 2x + c$$

43. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \left\{ \frac{1}{x} - \frac{1}{x^2} \right\} dx = ?$$

A. $e^x \left\{ \log x + \frac{1}{x} \right\} + C$

B. $xe^x - e^x + C$

C. $e^x \cdot \frac{1}{x} + C$

D. none of these

Answer

To find: Value of $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx \dots (i)$

Here $f(x) = \frac{1}{x}$

$$\Rightarrow f'(x) = -\frac{1}{x^2}$$

$$\Rightarrow I = \int e^x \left(f(x) + f'(x) \right) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \frac{1}{x} + c$$

Ans) $C e^x \frac{1}{x} + c$

44. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx = ?$$

A. $\frac{-e^x}{x^2} + C$

B. $\frac{e^x}{x^2} + C$

C. $e^x \left(\frac{-1}{x} + \frac{1}{x^2} \right) + C$

D. none of these

Answer

To find: Value of $\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx \dots (i)$

Here $f(x) = \frac{1}{x^2}$

$$\Rightarrow f'(x) = -\frac{2}{x^3}$$

$$\Rightarrow I = \int e^x \left(f(x) + f'(x) \right) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \frac{1}{x^2} + c$$

Ans) $B e^x \frac{1}{x^2} + c$

45. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \left\{ \sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right\} dx = ?$$

A. $e^x \cdot \frac{1}{\sqrt{1-x^2}} + C$

B. $e^x \sin^{-1} x + C$

C. $\frac{-e^x}{\sin^{-1} x} + C$

D. none of these

Answer

To find: Value of $\int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right) dx \dots (i)$

Here $f(x) = \sin^{-1} x$

$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x) \right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \sin^{-1} x + c$

Ans) B $e^x \sin^{-1} x + c$

46. Question

Mark (✓) against the correct answer in each of the following:

$\int e^x (\tan x + \log \sec x) dx = ?$

A. $e^x \log \sec x + C$

B. $e^x \tan x + C$

C. $e^x (\log \cos x) + C$

D. none of these

Answer

To find: Value of $\int e^x (\tan x + \log(\sec x)) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int e^x (\tan x + \log(\sec x)) dx \dots (i)$

$\Rightarrow I = \int e^x (\tan x - \log(\cos x)) dx$

$$\text{Here } f(x) = -\log(\cos x)$$

$$\Rightarrow f'(x) = \tan x$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = -e^x \log(\cos x) + c$$

$$\Rightarrow I = e^x \log(\sec x) + c$$

$$\text{Ans) A } e^x \log(\sec x) + c$$

47. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x (\tan x + \log \sec x) dx = ?$$

- A. $e^x \log \sec x + C$
- B. $e^x \tan x + C$
- C. $e^x (\log \cos x) + C$
- D. none of these

Answer

To find: Value of $\int e^x (\tan x + \log(\sec x)) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

$$\text{We have, } I = \int e^x (\tan x + \log(\sec x)) dx \dots (i)$$

$$\Rightarrow I = \int e^x (\tan x - \log(\cos x)) dx$$

$$\text{Here } f(x) = -\log(\cos x)$$

$$\Rightarrow f'(x) = \tan x$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = -e^x \log(\cos x) + c$$

$$\Rightarrow I = e^x \log(\sec x) + c$$

$$\text{Ans) A } e^x \log(\sec x) + c$$

48. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x (\cot x + \log \sin x) dx = ?$$

- A. $e^x \log (\sec x + \tan x) + C$

- B. $e^x \sec x + C$
 C. $e^x \log \tan x + C$
 D. none of these

Answer

To find: Value of $\int e^x (\cot x + \log(\sin x)) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x (\cot x + \log(\sin x)) dx \dots (i)$

Here $f(x) = \log(\sin x)$

$$\Rightarrow f'(x) = \cot x$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \log(\sin x) + c$$

Ans) D None of these

49. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \left\{ \tan^{-1} x + \frac{1}{(1+x^2)} \right\} dx = ?$$

A. $e^x \cdot \frac{1}{(1+x^2)} + C$

B. $e^x \tan^{-1} x + C$

C. $-e^x \cot^{-1} x + C$

D. none of these

Answer

To find: Value of $\int e^x \left(\tan^{-1} x + \frac{1}{(1+x)^2} \right) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x \left(\tan^{-1} x + \frac{1}{(1+x)^2} \right) dx \dots (i)$

Here $f(x) = \tan^{-1} x$

$$\Rightarrow f'(x) = \frac{1}{(1+x)^2}$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x (\tan^{-1} x) + c$$

$$\text{Ans) B } e^x (\tan^{-1} x) + c$$

50. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x (\tan x - \log \cos x) dx = ?$$

- A. $e^x \tan x + C$
- B. $e^x \log \cos x + C$
- C. $e^x \log \sec x + C$
- D. none of these

Answer

To find: Value of $\int e^x (\tan x - \log(\cos x)) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int [f'(x) \int g(x)dx] dx$$

We have, $I = \int e^x (\tan x - \log(\cos x)) dx \dots (i)$

Here $f(x) = -\log(\cos x)$

$$\Rightarrow f'(x) = \tan x$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = -e^x \log(\cos x) + c$$

$$\Rightarrow I = e^x \log(\sec x) + c$$

$$\text{Ans) C } e^x \log(\sec x) + c$$

51. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x (\cot x - \operatorname{cosec}^2 x) dx = ?$$

- A. $-e^x \operatorname{cosec}^2 x + C$
- B. $e^x \cot x + C$
- C. $-e^x \cot x + C$
- D. None of these

Answer

To find: Value of $\int e^x(\cot x - \operatorname{cosec}^2 x) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x(\cot x - \operatorname{cosec}^2 x) dx \dots (i)$

Here $f(x) = \cot x$

$$\Rightarrow f'(x) = -\operatorname{cosec}^2 x$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \cot x + c$$

Ans) B $e^x \cot x + c$

52. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x (\sin x + \cos x) dx = ?$$

- A. $e^x \sin x + C$
- B. $e^x \cos x + C$
- C. $e^x \tan x + C$
- D. None of these

Answer

To find: Value of $\int e^x(\sin x + \cos x) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x(\sin x + \cos x) dx \dots (i)$

Here $f(x) = \sin x$

$$\Rightarrow f'(x) = \cos x$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \sin x + c$$

Ans) A $e^x \sin x + c$

53. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \sec x (1 + \tan x) dx = ?$$

- A. $e^x (1 + \tan x) + C$
- B. $e^x \sec x + C$
- C. $e^x \tan x + C$
- D. none of these

Answer

To find: Value of $\int e^x \sec x (1 + \tan x) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x \sec x (1 + \tan x) dx \dots (i)$

$$I = \int e^x (\sec x + \sec x \tan x) dx$$

Here $f(x) = \sec x$

$$\Rightarrow f'(x) = \sec x \tan x$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \sec x + c$$

Ans) B $e^x \sec x + c$

54. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \left(\frac{1+x \log x}{x} \right) dx = ?$$

- A. $e^x \cdot \frac{1}{x} + C$
- B. $e^x \log x + C$
- C. $x e^x \log x + C$
- D. None of these

Answer

To find: Value of $\int e^x \left(\frac{1+x \log x}{x} \right) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x \left(\frac{1+x \log x}{x} \right) dx \dots (i)$

$$I = \int e^x \left(\frac{1}{x} + \log x \right) dx$$

Here $f(x) = \log x$

$$\Rightarrow f'(x) = \frac{1}{x}$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \log x + c$$

Ans) B $e^x \log x + c$

55. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \cdot \frac{x}{(1+x)^2} dx = ?$$

A. $e^x \cdot \frac{1}{(1+x)} + C$

B. $e^x \cdot \frac{1}{x} + C$

C. $e^x \cdot \frac{x}{(1+x)} + C$

D. None of these

Answer

To find: Value of $\int e^x \frac{x}{(1+x)^2} dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x \frac{x}{(1+x)^2} dx \dots (i)$

$$I = \int e^x \left(\frac{x+1-1}{(1+x)^2} \right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{(1+x)} - \frac{1}{(1+x)^2} \right) dx$$

$$\text{Here } f(x) = \frac{1}{(1+x)}$$

$$\Rightarrow f'(x) = -\frac{1}{(1+x)^2}$$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \frac{1}{(1+x)} + c$$

Ans) A $e^x \frac{1}{(1+x)} + c$

56. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx = ?$$

A. $e^x \sin \frac{x}{2} + C$

B. $e^x \cos \frac{x}{2} + C$

C. $e^x \tan \frac{x}{2} + C$

D. None of these

Answer

To find: Value of $\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$

Formula used:

$$(i) \int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$$

We have, $I = \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx \dots (i)$

$$I = \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{1 + \cos x} + \frac{\sin x}{1 + \cos x} \right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{2 \cos^2 \frac{x}{2}} + \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right) dx$$

Here $f(x) = \tan \frac{x}{2}$

$$\Rightarrow f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$$

$$\Rightarrow I = \int e^x \left(f(x) + f'(x) \right) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \tan \frac{x}{2} + c$$

Ans) C $e^x \tan \frac{x}{2} + c$